CHAPTER 1
ELECTROMECHANICAL ENERGY CONVERSION

1.1 Electromechanical Energy Conversion Devices

An electromechanical energy conversion device is the one which converts mechanical energy into electrical energy or vice-versa.

Generator - Converts mechanical energy into electrical energy

Motor - Converts electrical energy into mechanical energy

The block diagram in Fig.1.1 represents an electromechanical energy conversion system. The electrical and the mechanical systems are interconnected through a magnetic field coupling. Such a system can perform bi-lateral conversion i.e., conversion of mechanical energy into electrical energy or vice versa.

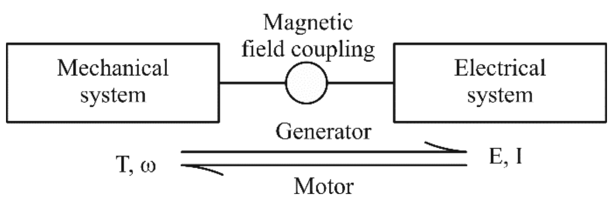


Figure 1.1 Energy Conversion

1.1.1 Generator Action

An electromechanical energy conversion device converting mechanical energy into electrical energy is shown in Fig.1.2.

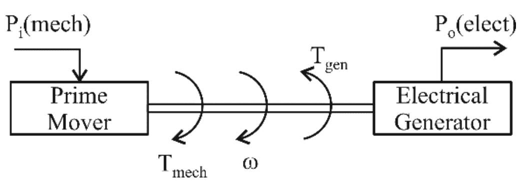


Figure 1.2 Generator Action

The mechanical energy input $P_{i(mech)}$ is supplied to a prime mover. The prime mover is mechanically coupled with the electrical generator to transmit mechanical energy to the generator. A mechanical torque T_{mech} is developed by the prime mover which rotates the generator at a speed ω rad/sec. Electrical energy output $P_{o(elect)}$ is developed by the generator. The generator torque T_{gen} balances the mechanical torque T_{mech}. The energy balance equation is given by

$P_{i(mech)}$	=	$P_{o(elect)}$	+	P_{field}	+	P_{heat}	
(Input)		(output)		(Increase in field energy)		(Cu, iron, mechanical losses converted into heat)	…..(1.1)

1.1.2 Motor Action

In this case, electrical power drives a motor and the motor drives a mechanical load as shown in Fig.1.3.

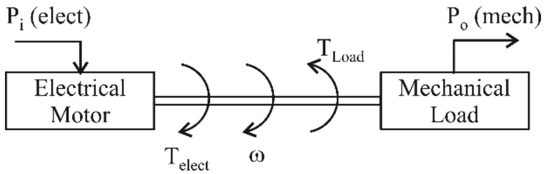


Figure 1.3 Motor Action

The energy balance equation is given by

$P_{i(elect)}$	=	$P_{o(mech)}$	+	P_{field}	+	P_{heat}	
(Input)		(output)		(Increase in field energy)		(Cu, iron, mechanical losses converted into heat)	…..(1.2)

1.2 Principle of Electromechanical Energy Conversion

It is clear from above discussion that a bilateral flow of energy is possible between an Electrical system and a Mechanical system through a common medium called Magnetic field coupling.

In all rotating machines there is an important role of electromagnetic energy conversion. A rate of change of flux is essentially needed to produce dynamically induced emf. In general,

$$\text{Induced emf} = Blv$$

where 'B' is flux density in wb/m^2, 'l' is the length of conductor in metre placed in magnetic field and 'v' is the velocity of the movement of that conductor in m/sec.

Lorentz Law: If a particle, having charge of 'q' coulombs is moved in a pure electric field E volt/m, then the force experienced is given by

$$F = qE \text{ (in pure electric field)} \quad .....(1.3)$$

Now, if the same charged particle is moved in a pure magnetic field having flux density of 'B' wb/m^2, with a velocity of 'v' m/sec, then the force experienced is given by

$$F = gv \times B \text{ (in pure magnetic field)} \quad .....(1.4)$$

The Lorentz law states that the force experienced by a particle charge in the presence of an electric and a magnetic field together is given by

$$F = q\,(E + v \times B) \qquad \text{.....(1.5)}$$

If the number of charged particles moving is large, then the force is given by

$$F_V = \rho\,(E + v \times B)$$

where F_V is force density (force/velocity is N/m^3) and 'ρ' is charge density (coloumb/m^3)

Also, $$J = \rho v \qquad \text{.....(1.6)}$$

where J is the current density (Amp/m^2)

Thus, in a pure magnetic field, the force is given by

$$F_V = J \times B \qquad \text{.....(1.7)}$$

1.3 Electromechanical Force

An electromechanical force is commonly seen to appear in following cases:

1. In alignment of magnetic fields:
   (a) When a magnetic material is placed under the influence of a magnetic field Fig.1.4 (a) or
   (b) When a weak magnet is placed under the influence of a stronger magnetic field Fig.1.4(b)

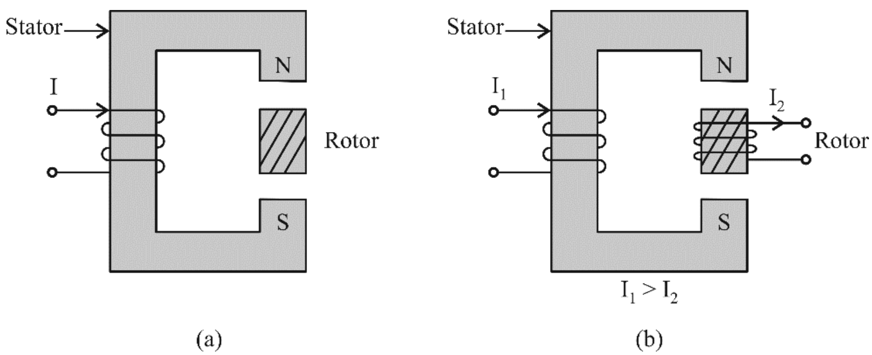


Figure 1.4 Rotor Alignment

2. When a current carrying conductor is placed in a magnetic field (motor action) or when a conductor is moved in a magnetic field (generator action). The above actions are explained through Flemings Left Hand Rule and Fleming's Right Hand Rule respectively.

1.4 Energy Flow in Electromechanical Systems

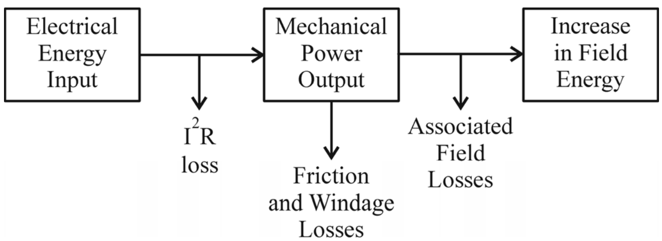
The flow of energy can be understood by following:

$$\begin{bmatrix}\text{Electrical}\\ \text{Energy Input}\end{bmatrix} \equiv \begin{bmatrix}\text{Mechanical}\\ \text{Energy}\\ \text{Output}\end{bmatrix} + \begin{bmatrix}\text{Increase}\\ \text{in Field}\\ \text{Energy}\end{bmatrix} + \begin{bmatrix}\text{Energy lost}\\ \text{in the form of}\\ \text{heat } (I^2R)\end{bmatrix} \quad \text{.....(1)}$$

The above relationship can be rearranged as

$$\begin{bmatrix}\text{Electrical}\\ \text{Energy}\\ \text{Input}\\ \text{minus } (I^2R)\\ \text{Losses}\end{bmatrix} \equiv \begin{bmatrix}\text{Mechanical}\\ \text{Energy}\\ \text{Output}\\ \text{Plus}\\ \text{Friction and}\\ \text{Windage losses}\end{bmatrix} + \begin{bmatrix}\text{Increase}\\ \text{in}\\ \text{Field}\\ \text{Energy}\end{bmatrix} \quad \text{.....(2)}$$

The above can also be represented by following diagram.



.....(3)

Based on the flow diagram (3), the differential energy balance equation can be written as follows.

$$dW_e = dW_m + dW_f \quad \text{.....(1.8)}$$

Here dW_e is the differential electrical energy supplied in time 'dt'

dW_m is the differential mechanical energy output in time 'dt'

dW_f is the differential field energy output in time 'dt'

Under steady state condition, the energy balance equation becomes

$$W_e = W_m + W_f \quad \text{.....(1.9)}$$

1.5 Energy Balance in Electromechanical Systems

This topic would be studied under following subcategories.

Singly Excited System (Single Source of Electrical Power)

A - Without a rotor

B - With a rotor

B.1 - static energization

B.2 - Dynamic energization

B.2.1 - Slow rotor movement

B.2.2 - Fast (instantaneous rotor movement)

B.2.3 - Transient rotor movement

Doubly Excited System (Two Sources of Electrical Power)

1.5.1 Singly Excited System (without a Rotor)

The electrical energy supplied by a source gets either dissipated, converted into other form, or stored in magnetic or in static (like in battery, capacitor etc.) forms.

The current flowing in circuit shown in Fig.1.5 results in dissipation of a part of the energy in the resistance of coil, while the rest of the energy gets stored in magnetic field.

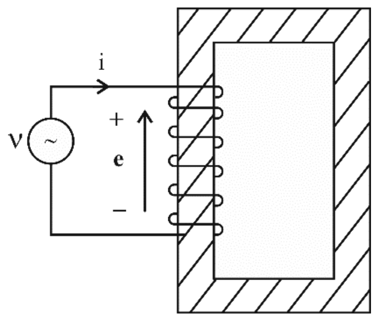


Figure 1.5 Singly Excited System without Rotor

The KVL gives $v = e + iR$ …..(1.10)

where R is the coil resistance

The power equation can be given as

$$vi = ei + Ri^2 \quad \text{…..(1.11)}$$

The energy for duration 'dt' is given as

$$vidt = eidt + Ri^2\,dt \quad \text{…..(1.12)}$$

Here, vidt is the input energy, eidt is the energy stored in magnetic field, and Ri^2 dt is the energy lost in the form of heat.

Thus, the energy stored $dw_f = eidt$

But, $e = \dfrac{d\psi}{dt}$, where $\psi = N\phi$ …..(1.13)

$$\therefore \quad dw_f = \frac{d\psi}{dt} \,.\, i \,.\, dt = i\ d\psi \quad \text{…..(1.14)}$$

Or,

$$\text{energy stored} = \int dw_f = \int id\psi = \int Nid\phi \quad \text{…..(1.15)}$$

At the same time, since

$$E = L\frac{di}{dt} \quad \text{…..(1.16)}$$

Eq. (1.13) can be written as

$$dW_f = L\frac{di}{dt}.idt = Lidi$$

or, energy stored = $\int dW_f = \int Lidi$ .....(1.17)

1.5.1.1 Concept of Energy and Coenergy

Eq. (1.15) can be graphically represented as follows.

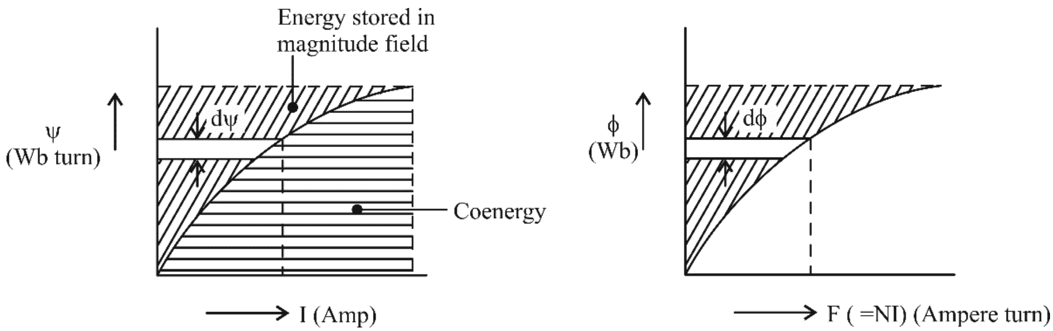


Figure 1.6 Energy - Coenergy

The variations of ' ψ ' and 'ϕ' are plotted against 'I' and 'F' respectively. The area above the curve represents the amount of energy stored in magnetic field, while that under the curve is called the 'Coenergy".

Energy and Co-energy for Linear B-H curve

For Linear systems $LI = N\phi$

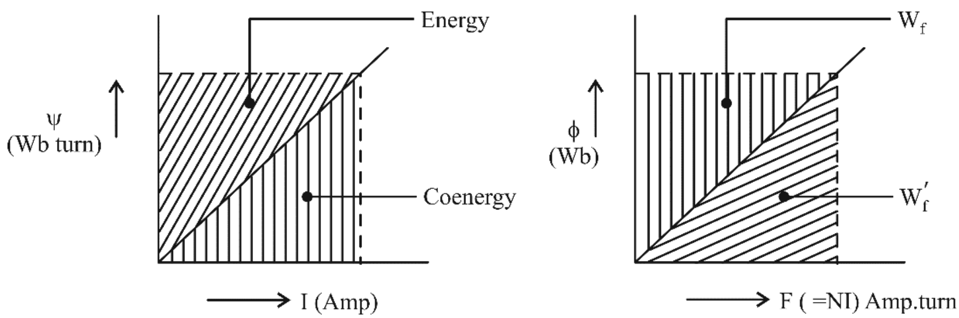


Figure 1.7 Energy – Coenergy (Linear)

Let the Energy be represented by W_f and the Coenergy by W_f'.

1.5.2 Singly Excited System (with Rotor)

The ferromagnetic rotor is placed at position A (Fig. 1.8). The magnetic material is energized by allowing a current to flow through the winding as a result of it, the rotor moves by angle θ_m and acquired position B. The torque acting on the Motor to make this movement is called saliency (Reluctance) torque. It is the torque exerted on rotor to bring the rotor to a position where the reluctance of magnetic path is minimum.

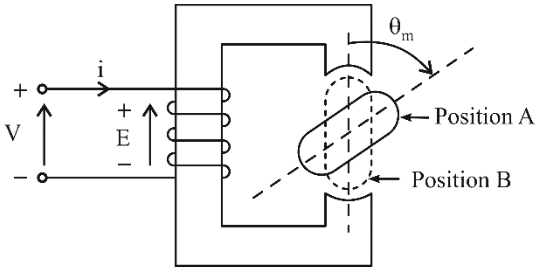


Figure 1.8 Singly excited system with rotor

The assumptions made in the analysis to follow are as given below.

1. ψ - i relationship is linear

   That is $\psi = L(\theta)\, i$
2. No iron loss takes place
3. Leakage flux is zero
4. Electrostatic field is not present
5. The core material of stator and rotor has $\mu_r = \infty$. That is, the reluctance offered by magnetic path is zero. In other words, the net reluctance in the magnetic path is caused by the air gap only.
6. The flux crossing the air gap is uniform, undistorted and radial with no fringing effect.
7. The air gap is very thin so that constant flux density is maintained in the air gap.

If there is no reluctance in the magnetic core, then the total available mmf will be utilized equally in creating air gap flux in the two parts of airgap.

The voltage equation is given by

$$V = Ri + {}^{d\psi}\!/_{dt} \qquad \text{.....(1.18)}$$

The energy is represented by

$$W_e = \underbrace{\int_0^t vi\,dt}_{\substack{\text{Energy}\\ \text{Input}}} = \underbrace{\int_0^t i^2 R\,dt}_{\substack{\text{Energy}\\ \text{lost}(=W_l)}} + \underbrace{\int_0^t \frac{id\psi}{dt}\,dt}_{\substack{\text{Useful}\\ \text{Energy}(=W_u)}} \qquad \text{.....(1.19)}$$

$W_e = W_l + W_u$

$W_u = W_f + W_m$

where W_f = energy stored in magnetic field

W_m = energy converted to mechanical energy output

1.6 Static Energization

Let us analyze the situation where the rotor occupies two static positions, position A and position B, as shown in Fig.1.8. The $\psi - I$ characteristics for these two positions are drawn in Fig.1.9.

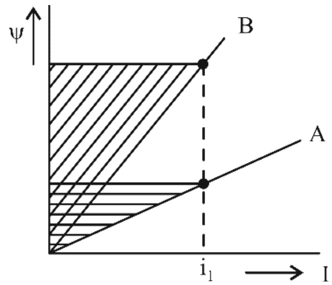


Figure 1.9 Energy Stored for Fig.1.8

The energy stored in magnetic field when the rotor occupies positions A and B are shown by the shaded regions in Fig.1.9.

As the rotor is stationary, $W_m = 0$; No mechanical energy exchange is involved. Therefore

$$\int_0^{\psi} i d\psi = W_f \text{ ; energy stored}$$

or

$$W_f = \int_0^{\psi} i d\psi = \int_0^{\psi} \frac{\psi}{L} d\psi \text{ ; as } \psi = N\phi = Li$$

$$= \frac{\psi^2}{2L} = \frac{L^2 i^2}{2L} = \frac{1}{2} Li^2$$

Thus, the energy stored is given by $\frac{1}{2}Li^2$. Further the slope of ψ - I characteristic is given by

$$\frac{\psi}{i} = \frac{N\phi}{i} = L \qquad \text{.....(1.20)}$$

Figure 1.9 shows that $L_A < L_B$.

It means that the inductance and hence the energy stored, is maximum when the rotor is aligned with the static magnetic field (Position B). The value of inductance and energy stored keep reducing as the rotor occupies positions away (e.g. Position A) from the axis of static magnetic field, for the same amount of current in the magnetic coil.

1.7 Dynamic Energization

A situation is considered where the rotor moves from position A to position B in Fig.1.10.

This movement can take place either slowly or instantaneously.

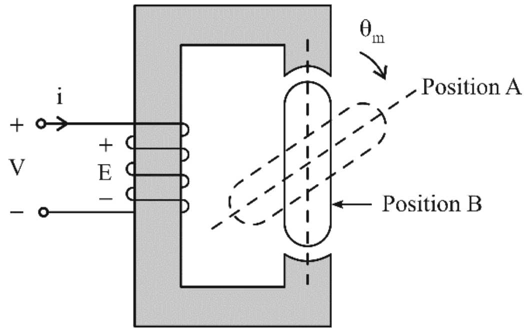


Figure 1.10 Dynamic Energization

1.7.1 Slow Movement of Rotor

The rotor slowly moves from position A to B. This movement is so slow that the rate of change of flux experienced by it is small. Therefore, no emf gets induced. Hence there is no change in current. The change in energy magnetic field (ΔW_f) is the difference between the energy stored at rotor position A (W_{fA}) and that at rotor position B (W_{fB}). As shown in Fig.1.11.

$$\Delta W_f = W_{fB} - W_{fA}$$

$$= \Delta Oef - \Delta Odc$$

$$= \frac{1}{2}[oe \times ef) - (od \times ef)]$$

$$= \frac{1}{2} ef\,(oe - od) = \frac{1}{2}\ ef \times de \qquad \text{.....(1.21)}$$

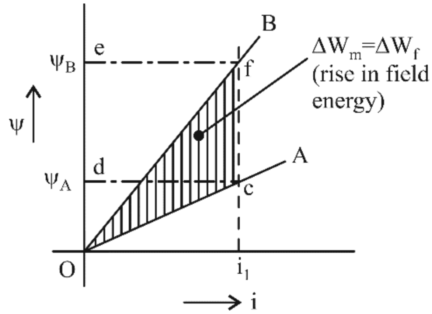


Figure 1.11 Energy Stored – Slow motion

$$\Delta W_e = \int_{\psi_A}^{\psi_B} i_1 d\psi = i_1(\psi_B - \psi_A) = ef \times de \qquad \text{....(1.22)}$$

This change in electrical change input (ΔW_e) leads to rise in magnetic field energy (ΔW_f) and in mechanical energy (ΔW_m) to shift the rotor position.

Thus, the mechanical energy ΔW_m spent in slowly moving the rotor from position A to B results in rise of ΔW_f.

Expression for Electromechanical Torque T_m

$$T_m = \lim_{\theta_m \to 0} \left\{ \frac{\Delta W_f}{\Delta \theta_m} \right\} \Bigg|_{\text{constnat } i} \qquad \text{.....(1.23)}$$

$$= \frac{\partial W_f}{\partial \theta_m} \Bigg|_{\text{constant } i}$$

$$= \frac{\partial}{\partial \theta_m} (W_f) \Bigg|_{\text{constant } i}$$

We know that $W_f = \frac{1}{2} L i^2$

Also, the system is assumed to be linear,

Hence $L \neq f(i)$

$$\therefore \qquad T_m = \frac{i^2}{2} \frac{dL}{d\theta_m} \qquad \text{.....(1.24)}$$

1.7.2 Fast Movement of Rotor

The rotor instantaneously moves from position A to position B as shown in Fig.1.12. The flux linkage however cannot change instantaneously and remains the same as ψ_1 during rotor movement. In Fig.1.12 it is depicted that the initial operating point is at c. With the sudden movement of rotor, it gets shifted to c' at the same flux linkage ψ_1. After the rotor reaches at position B, the operating point shifts from c' to c. Current i_2 is drawn to develop ψ_1 flux linkage and the operating point settles down at c for rotor position B. The expression of electrical energy W_e is given by.

$$W_e = \int_{\psi_1}^{\psi_2} i \, d\psi$$

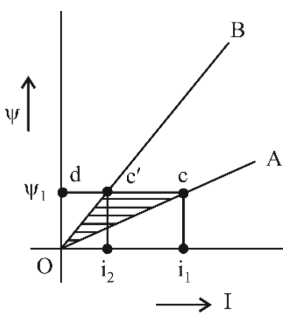


Figure 1.12 Energy Stored – Fast movement

Since due to fast rotor movement, the flux linkage does not change, therefore $\psi_c = \psi_{c'} = \psi_1$ is constant.

So, $W_m = -W_f$ (decrease in stored magnetic field energy)

It shows that no electric energy is taken during fast movement of the rotor. The stored magnetic energy only gets consumed in such a rotor movement. It is shown by occ' in Fig.1.12.

Expression for torque T_m is given by

$$T_m = \lim_{\theta_m \to 0} \left\{ \frac{-\Delta W_f}{\Delta \theta_m} \right\} \Bigg|_{\text{constant } \psi} \quad \text{.....(1.25)}$$

$$= \frac{-\partial W_f}{\partial \theta_m} \Bigg|_{\text{constant } \psi} = \frac{-\partial}{\partial \theta_m} \left(\frac{\psi^2}{2L} \right) \Bigg|_{\text{constant } \psi}$$

$$\therefore \quad T_m = \frac{-\psi^2}{2} \frac{1}{L^2} \frac{\partial L}{\partial \theta_m}$$

$$= \frac{i^2}{2} \frac{\partial L}{\partial \theta_m}$$

For linear system

$$T_m = \frac{i^2}{2} \frac{dL}{d\theta_m} \quad \text{.....(1.26)}$$

1.7.3 Transient Rotor Movement

Between the two extremes, actually when the rotor moves from A to B, the speed is neither very slow nor very fast. The mechanical energy used by the rotor is given by area oce (shaded). The current gets reduced from i_2 to i_1 as the rotor gets aligned to magnetic field and the flux linkage rises from ψ_1 to ψ_2. In Fig.1.13 the graphical relationship gives the following change in electrical energy

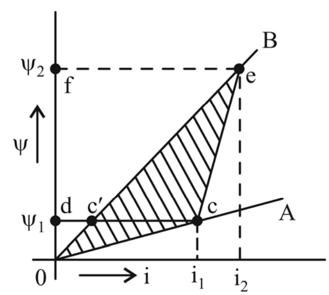


Fig.1.13 Transient Rotor Movement

ΔW_e = fecd (input energy)

change in field energy

ΔW_f = Oef – Ocd (stored energy).

Hence change in mechanical energy = $\Delta W_e - \Delta W_f$ (output)

or $\Delta W_m = \Delta W_e - \Delta W_f$

= fecd – Oef + Ocd = Ocef – Oef

= Oce (shaded portion)

1.8 Doubly Excited System

In doubly excited system both the stator and the rotor are excited as shown in Fig.1.14. The rotor and stator have saliency. The assumptions made in carrying out this analysis are same as given earlier.

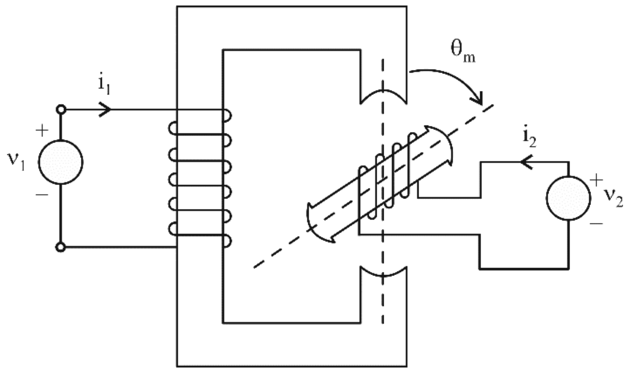


Figure 1.14 Doubly Excited System

Equation set (1) – Flux linkages

$$\psi_1 = L_1\, i_1 + Mi_2$$

$$\psi_2 = L_2\, i_2 + Mi_1$$

Equation set (2) voltages

$$v_1 = R_1 i_1 + \frac{d\psi_1}{dt}$$

$$v_2 = R_2 i_2 + \frac{d\psi_2}{dt}$$

Equation set (3) – Combining equation sets (1) and (2)

$$v_1 = R_1 i_1 + L_1\frac{di_1}{dt} + i_1\frac{dL_1}{dt} + i_2\frac{dM}{dt} + M\frac{di_2}{dt}$$

$$v_2 = R_2 i_2 + L_2\frac{di_2}{dt} + i_2\frac{dL_2}{dt} + i_1\frac{dM}{dt} + M\frac{di_1}{dt}$$

Equation set (4) – Power equations (Multiplying (3) with i)

$$v_1 i_1 + R_1 i_1^2 + L_1 i_1\frac{di_1}{dt} + i_1^2\frac{dL_1}{dt} + i_1 i_2\frac{dM}{dt} + Mi_1\frac{di_2}{dt}$$

$$v_2 i_2 + R_2 i_2^2 + L_2 i_2\frac{di_2}{dt} + i_2^2\frac{dL_2}{dt} + i_1 i_2\frac{dM}{dt} + Mi_2\frac{di_1}{dt}$$

Equation set (5) – Energy equations (integrating and adding equation sets in (4))

$$\int_0^t (v_1 i_1 + v_2 i_2) dt = \int_0^t \underbrace{\left(R_1 i_1^2 + R_2 i_2^2\right)}_{[A]} dt$$

$$+\underbrace{\int_0^t \left(L_1 i_1 \frac{di_1}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_1 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}\right)}_{[B]} dt$$

$$+\underbrace{\int_0^t \left(i_1^2 \frac{dL_1}{dt} + i_2^2 \frac{dL_2}{dt} + 2 i_1 i_2 \frac{dM}{dt}\right)}_{[C]} dt$$

The above equation has three components and they express the energies. [A] gives energy lost as copper loss. [B] is stored energy in magnetic field and [C] is energy converted into mechanical form.

[A] corresponds to energy lost in copper windings on stator and rotor. This is given by

$$\int_0^t \left(R_1 i_1^2 + R_2 i_2^2\right) dt$$

[B] corresponds to energy stored in magnetic field W_f and is given by

$$W_f = \int_0^t \left(L_1 i_1 \frac{di_1}{dt} + L_2 i_2 \frac{di_2}{dt} + M i_1 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}\right) dt$$

[C] corresponds to mechanical energy developed

The term $\left(i_1^2 \frac{dL_1}{dt} + i_2^2 \frac{dL_2}{dt} + 2 i_1 i_2 \frac{dM}{dt}\right)$ is zero because the system considered is linear

Part [B] of the expression is used to obtain expression for torque.

The expression for Electromagnetic torque (T_e)

$$T_e = \frac{dW_f}{dt}$$

$$= \frac{d}{dt}\left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + 2 M i_1 i_2\right]$$

$$i_1 = \left\{\begin{array}{l} \frac{1}{2} L_1 \frac{di_1^2}{dt} + \frac{1}{2} L_2 \frac{di_2^2}{dt} + 2 M i_1 \frac{di_2}{dt} + 2 M i_2 \frac{di_1}{dt} \\ +2 i_1 i_2 \frac{dM}{dt} + \frac{1}{2} i_1^2 \frac{dL_1}{dt} + \frac{1}{2} i_2^2 \frac{dL_2}{dt} \end{array}\right\}$$

$$di_2^2 = \begin{Bmatrix} L_1 i_1 \frac{di_1}{dt} + L_2 i_2 \frac{di_2^1}{dt} + 2Mi_1 \frac{di_2}{dt} + 2Mi_2 \frac{di_1}{dt} \\ +2i_1 i_2 \frac{dM}{dt} + \frac{1}{2} i_1^2 \frac{dL_1}{dt} + \frac{1}{2} i_2^2 \frac{dL_2}{dt} \end{Bmatrix}$$

The expression for electromechanical torque (T_m)

$$T_m = \frac{dW_f}{d\theta_m}$$

$$= \left[\frac{1}{2} i_1^2 \frac{dL_1}{d\theta_m} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta_m} + \left(2 i_1 i_2 \frac{dM}{d\theta_m} \right) \right]$$

In above equation the term $\left(\frac{1}{2} i_1^2 \frac{dL_1}{d\theta_m} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta_m} \right)$ represents the reluctance (or saliency) torque. This term becomes zero in machines having uniform air gap.

The second term $\left(2 i_1 i_2 \frac{dM}{d\theta_m} \right)$ represents the co-alignment torque.

$T_m = \frac{1}{2} i^2 \frac{dL}{d\theta_m}$; if either i_1 or i_2 becomes zero. T_m thus becomes same as that of a singly excited system.

1.9 Physical Concept of Force and Torque Production

1.9.1 DC Machines

The electromagnetic field poles are present on the stator, while the armature winding is placed on the rotor. Fig.1.15 shows the Generator and the Motor action of the machine.

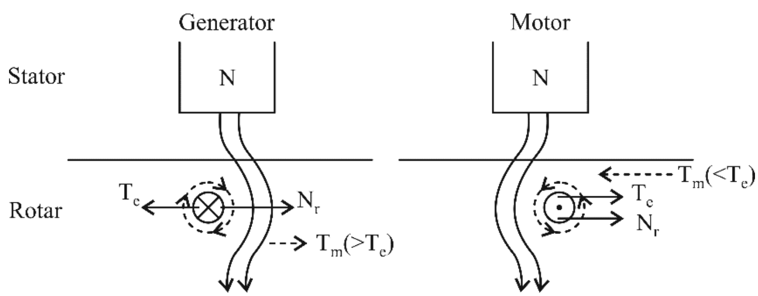


Figure 1.15 DC Machines

In case of Generator the rotor is rotated at N_r rpm. Flemings Right Hand (FRH) rule determines the current direction in armature conductor. The main field flux gets deviated due to magnetic field surrounding the armature conductor.

The magnetic lines of forces are elastic in nature. It can be assumed that they try to straighten themselves, thus developing an electromagnetic torque T_e to work opposite to the direction of rotation. In generators, the mechanical torque T_m applied externally on the rotor. T_m is greater than the electromagnetic torque T_e developed.

T_m and T_e work opposite to each other. Hence in case of generators, the rotation N_r, resulted due to T_m is in opposite direction to T_e.

In case of Motor, the armature conductor carries current in the direction shown. Applying Fleming's Left Hand (FLH) rule, the direction of rotation obtained is in the same direction as that of T_e. As the load on the motor is mechanical, the load torque T_m will work opposite to T_e. However, $T_e > T_m$ makes the motor run at N_r in the same direction as that of T_e.

1.9.2 Synchronous Machines

The arrangement of the field and the armature windings in synchronous machines are opposite to that of DC machines, as shown in Fig.1.16.

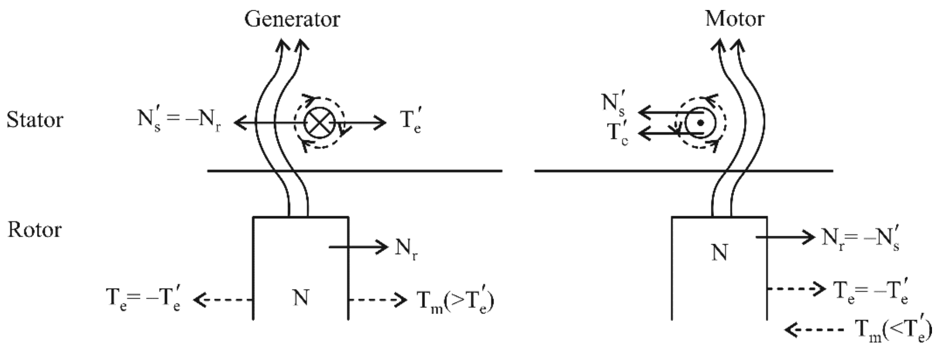


Figure 1.16 Synchronous Machines

In case of Generator, the rotor is rotated at N_r. In other words, the armature conductor moves at a fictitious speed of $N'_S\left(=-N_r\right)$. The FRH rule determines the direction of current in armature conductor. The surrounding magnetic field of which deviates the main field flux as shown. These lines of forces develop a torque T'_e on armature conductors. In other words, an equal and opposite torque T_e acts the rotor. The rotor rotates at N_r because the mechanical input torque T_m is greater than T_e.

Similarly, in case of motor, the armature current carries a current as shown. The FLH rule gives the direction of movement of armature conductor as N'_S (fictitious). The magnetic field surrounding the armature conductor, deviates the main field flux such that a fictitious electromagnetic torque gets developed as T'_e. As the armature conductor cannot move, the rotor moves at N_r $\left(=-N'_S\right)$ due to the torque $T_e(=-T'_e)$ acting indirectly on it. The mechanical torque T_m opposes the electromechanical torque T_e in both the cases.

1.9.3 Induction Machines

The stator and rotor carry three phase ac distributed field and armature windings respectively. Fictitious poles are created in both the stator and rotor. The poles created due to stator winding rotate at N_s rpm (synchronous speed), while the rotor poles rotate at N_r rpm.

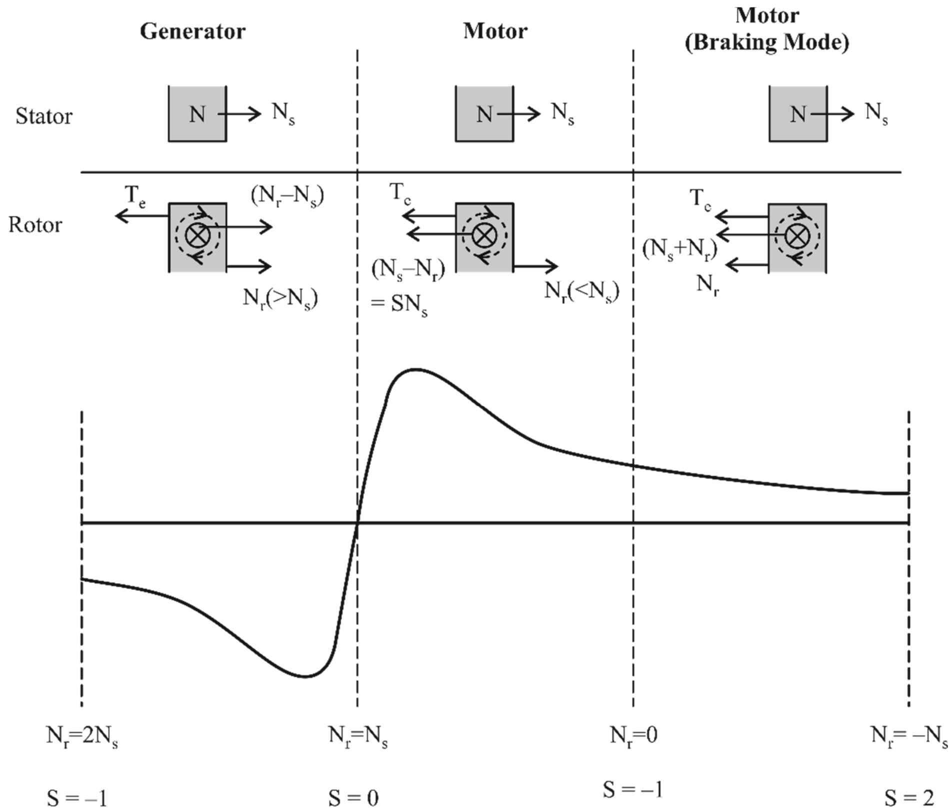


Figure 1.17 Induction Machine

Fig.1.17 explains the operation of induction machine in all its three modes; i.e., Generating, Motoring and Braking modes. The corresponding slip-torque characteristic is also drawn for the three modes of operation of induction machine.

1.9.4 Summary

The above discussed can be summarized as follows.

1. The direction of current in the armature conductor decides the direction of its surrounding magnetic field. This causes deviation in the main field flux. The main field flux is stronger, and due to the basic elastic nature of flux, it tries to straighten itself. Moreover, the weaker field, surrounding the armature conductor, tries to align itself with the main field flux. In this process, the armature conductor experiences force resulting in development of T_e.

2. In case of Generators, the electromechanical (input) torque (T_m) is greater than the electromagnetic (load) torque (T_e). As the rotor rotates due to the T_m (input), the direction of rotation of rotor is the same as that of T_m. Hence T_e and N_r act in opposite directions.
3. In case of Motors, the electromagnetic (input) torque (T_e) is greater than the Electromechanical (load) torque (T_m). The rotor rotates due to T_e, hence T_e and N_r act in the same direction.

REVIEW QUESTIONS

1. What is meant by Electromechanical Energy Conversion?
2. Describe the concept of Energy and Co-energy.
3. Derive the expression for torque developed in doubly excited system. From this expression derive the expression for torque in singly excited system.
4. With the help of diagrams, explain the effect of saturation on stored magnetic energy.
5. How does an Induction machine change its behavior when the rotor speed is varied from negative synchronous speed to positive synchronous speed?

PROBLEMS

1. The following are the inductances in a doubly excited system.

   $L_1 = 10 + 5 \cos 2\theta$ henry

   $L_2 = 9 + 4 \cos 2\theta$ henry

   $M = 15 \cos \theta$ henry

   The currents in the two coils are

   $I_1 = 0.5$ Amp

   $I_2 = 0.7$ Amp

Calculate

(i) Torque as a function of θ and its value for θ = –45°

(ii) Expression for energy stored as a function of θ.

Sol: (i) The expression of torque is as given below

$$T = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}$$

$$= \frac{1}{2}(0.5)^2 \frac{d}{d\theta}(10 + 5\cos 2\theta) + \frac{1}{2}(0.7)^2 \frac{d}{d\theta}(9 + 4\cos 2\theta) + (0.5)(0.7)\frac{d}{d\theta}(15\cos\theta)$$

$$T = \frac{1}{2}(0.5)^2 (-10\sin 2\theta) + \frac{1}{2}(0.7)^2 .(-8\sin 2\theta) + (0.5)(0.7)(-15\sin\theta)$$

$= -3.21 \sin(90) - 5.25 \sin(45)$

or $T = -6.91$ nm

(ii) Energy stored in field as a function of θ

$$Wf_\theta = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$= \frac{1}{2}(10 + 5\cos 2\theta)(0.5)^2 + \frac{1}{2}(9 + 4\cos 2\theta)(0.7)^2$$

$$+ (15\cos\theta)(0.5)(0.7)$$

$$= 3.455 + 1.605\cos 2\theta + 5.25\cos\theta$$