Chapter 1

Per Unit Representation

1.0 Introduction

The per unit value of any quantity is defined as the ratio of its actual value to another arbitrarily selected value of quantity of same dimensions. It is convenient to express impedances, voltages, powers and currents in terms of p.u. in the process of computation in power systems.

$$\text{Per unit value } \Delta = \frac{\text{Actual}}{\text{Base}}$$

i.e ratio of actual value to the base value of the same dimension.

$$\text{For any quantity X, } X_{p.u} = \frac{X}{X_{Base}}$$

Dividing the numerical value by a chosen base value of the same dimensions, any quantity can be converted to a per unit quantity. Percent quantities and per unit quantities are different by a factor of 100. Per unit quantities are dimensionless.

Suppose, a base voltage of 10 KV is considered, voltages of 10, 15, 20 KV can be shown as, 1.00, 1.50, 2.00 per unit respectively. In terms of percentage, 100, 150, 200 percent.

Let us consider,

I_A = Actual current, Amperes

I_B = Base current, Amperes

V_A = Actual voltage, Volts

V_B = Base voltage, Volts

Z_Ω = Actual impedance, Ohms

Z_b = Base impedance, Ohms

S_{va} = Actual Volt ampere

S_b = Base Volt ampere

Per unit current, $I_{pu} = I_A/I_B$

Per unit voltage, $V_{pu} = V_A/V_B$

Per unit impedance, $Z_{pu} = Z_{\Omega}/Z_B$

$Z_{\Omega} = R_{\Omega} + j\,X_{pu}$; $Z_{pu} = Z_{\Omega}/Z_b = R_{\Omega}/Z_B = jX_{\Omega}/Z_B$

or $Z_{pu} = R_{pu} + jX_{pu}$

$R_{pu} = R_{\Omega}/Z_B$ and $X_{pu} = X_{\Omega}/Z_B$

Per unit volt-ampere, $S_{pu}\,\Delta = S_{va}/S_B$

$S = P + jQ = VI\times$; $S_{pu} = S_{VA}/S_B = P/S_B + jQ/S_B$

or $S_{pu} = P_{pu} + jQ_{pu}$

$P_{pu} = P_{watt}/S_B$ and $Q_{pu} = Q_{watt}/S_B$

For the single phase circuit,

$Z_B = V_B/I_B$ and $S_B = V_B I_B$

and,

Base current $I_B = S_B/V_B$

$$= \frac{\text{Base KVA}}{\text{Base KV}}$$

Formulae for one base to another base:

$$Z_{pu.2} = Z_{pu.1} \times (S_{b2}/S_{b1}) \times (V_{b1}/V_{b2})^2$$

Example 1: A single phase Transformer is rated 110/440V, 4 KVA having Leakage reactance at LT side is 0.2 Ω, Determine the Leakage reactance in p.u.

Solution: Given: $V_B = 110V$, $S_B = 4$ KVA, $Z_{Act} = 0.2\Omega$

We know that,

$$Z_B = \frac{(\text{BaseKV})^2}{\text{BaseMVA}}$$

$$Z_B \text{ at LT side} = \frac{\left(110\times10^{-3}\right)^2\times1000}{4} = 3.025\ \Omega$$

$$Z_{p.u.} = \frac{z_{Act}}{z_{Base}} = \frac{0.2}{3.025} = 0.0661 \text{ p.u.}$$

For Secondary side:

$$Z_1' = Z_1\left(\frac{N_2}{N_1}\right)^2$$ where, N_1 & N_2 are the no. of turns on Primary and Secondary side.

$$Z_1' = 0.2\left(\frac{440}{110}\right)^2 = 3.2$$

and $$Z_B = \frac{\left(440\times10^{-3}\right)^2\times1000}{4} = 48.4\Omega$$

$$Z_{p.u} = \frac{3.2}{48.4} = 0.0661 \text{ p.u.}$$

Example 2: The following figure represent a part of electrical network with component. Determine the p.u. reactance of the components:

Synchronous Generator: 25MVA, 11KV, $X'' = 0.15$ p.u.

Transformer T_1: 40MVA, 12.5Δ/132Y KV, X =0.1 p.u

Line: 200 + j500Ω

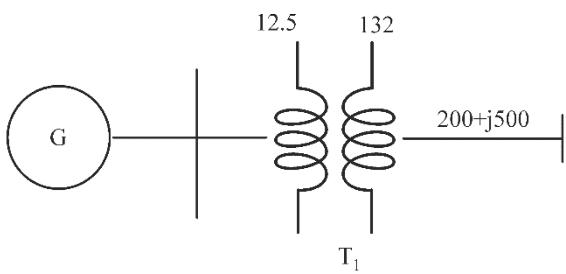

Fig.1.1

Draw the impedance diagram for the system shown in Fig.1.1. For Transmission line choose the base value as 132KV and 25MVA.

Solution: For **Generator:**

The Base uV of transmission line is 132 uV. Hence, base KV at the Generator side 12.5 KV

$$X'' = j0.15\times\frac{25}{25}\times\left(\frac{11}{12.5}\right)^2$$

$$= j0.116 \text{ p.u}$$

For **Transformer 1:**

Base KV at the transformer (T_1) secondary side is 132 uV

$$X'' = j0.1\times\frac{25}{40}\times\left(\frac{132}{132}\right)^2$$

$$= j0.0625\text{p.u}$$

For **Transformer 2:**

$$X'' = j0.1\times\frac{25}{30}\times\left(\frac{132}{132}\right)^2$$

$$= j0.083\text{p.u.}$$

For Motor:

$$X'' = j0.15 \times \frac{25}{20} \times \left(\frac{11}{11}\right)^2$$

$$= j0.187 \text{ p.u.}$$

For Transmission Line:

$$Z_B = \frac{(\text{BaseKV})^2}{\text{BaseMVA}}$$

$$= \frac{(132)^2}{25} = 696.96\Omega$$

$$Zp.u = \frac{Z_{Act}}{Z_{Base}} = \frac{200 + j500}{696.96} = 0.286 + j0.717$$

For a Load of 5MVA and 0.8 p.f lagging,

P = S cosØ = 5 × 0.8 = 4 MW

Q = S sinØ =5 × 0.6 = 3 MVAr

If load is represented as series impedance,

$$R_{p.u.} = V_{p.u}^2 . S_B . \frac{P}{P^2 + Q^2} \text{ and}$$

$$X_{p.u.} = V_{p.u}^2 . S_B . \frac{Q}{P^2 + Q^2}$$

Here, $V_{p.u} = \frac{11}{11} = 1$ p.u

S_B = 25 MVA

So,

$$R_{p.u} = (1)^2 \times 25 \times \frac{4}{4^2 + 3^2} = 4 \text{ p.u}$$

$$X_{p.u} = (1)^2 \times 25 \times \frac{3}{4^2 + 3^2} = 3 \text{ p.u}$$

P.U Impedance Diagram:

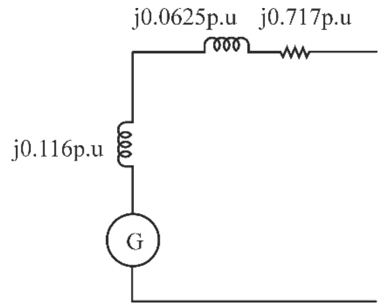

Fig.1.2

Example 3: A 3 bus system is given in Fig.1.3. The ratings of the various components are listed below:

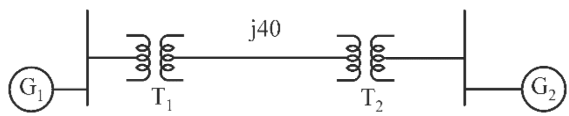

Fig.1.3

Generator 1:40MVA, 12.8KV, $X'' = 0.2$p.u

Generator 2:50MVA, 12.2KV, $X'' = 0.15$p.u

Transformer1:40MVA, 12KV/100KV Y/Y,X = 0.1p.u

Transformer2:20MVA, 13.5KV/125KV Y/Y,X = 0.20p.u

The line impedance are shown in Fig.1.3. Determine the reactance diagram based on 40 MVA and 12.8 KV as base quantities in generator 1.

Solution: Base MVA = 40; Base KV = 12.8

Generator 1:

$$Z_{\text{p.u.new}} = Z_{\text{p.u.given}} \times \left(\frac{\text{Base KV given}}{\text{Base KV new}}\right)^2 \times \left(\frac{\text{Base MVA new}}{\text{Base MVA given}}\right)$$

Per Unit Reactance = $0.2 \times (12.8/12.8)^2 \times (40/40)$

= 0.0.2

Transmission Line 1:

Base voltage along the transmission line whose impedance is j40Ω = 12.8 × 100/12 =106kV

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$= 106^2/40 = 280.9\ \Omega$

Per unit impedance = 40/280.9

= 0.1423p.u

Generator 2:

Base voltage = 106 × 12.5/10.5

=12 KV

$X_{G2} = 0.15 \times (12.2/12)^2 \times 40/50$

= 4.4652p.u

Transformer 1:

40MVA, 12KV, X = 0.1p.u

Base values; 40MVA, 12.8KV

$X_{T1} = 0.1 \times (12/12.8)^2 \times 40/40$

= 0.0878p.u

Transformer 2:

20MVA, 11.5KV, X = 0.20p.u

Base Voltage = 106 × 13.5/125

= 11KV

$X_{T2} = 0.1 \times (11.5/11)^2 \times 40/20$

= 0.2185p.u

Transformer 3:

Base Voltage on the H.T side of transformer 3 is 10 KV

$X_{T3} = 0.2 \times (115/108)^2 \times 40/35$

= 0.25916p.u

Generator 3:

Base voltage on the bus = 106 × 13.5/125

= 11 KV

20 MVA, 12KV, $x'' = 0.20$ p.u

Base values: 40MVA, 11KV

$X_{G3} = 0.20 \times (12/11)^2 \times 40/20$

= 0.476p.u

Transmission line 2:

Base KV = 106

Base MVA = 40

$$\text{Base impedance} = \frac{106^2}{40}$$

$$= 280.9\ \Omega$$

Per unit impedance = j20/280.9

= j0.0711

Reactance Diagram:

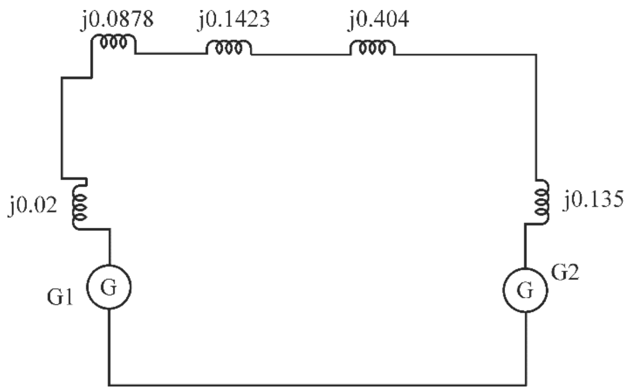

Fig.1.4

Example 4: Draw the impedance diagram for the power system shown in Fig.1.Use a Base of 45KVA,20KVat generator G1. The ratings of the generators and transformers are:

Generator 1: 20MVA, 18KV, X″ =20%,

Generator 2: 20MVA, 20KV, X″ =20%,

Synchronous motor 3, 35 MVA, 20KV, X″ =20%,

Transformer 1: Three phase Y-Y Transformers: 20 MVA, 148 Y/20 Y KV, X = 10%,

Transformer 2: Three phase Y-Δ Transformers: 10MVA,148Y/20ΔKV, X = 10%,

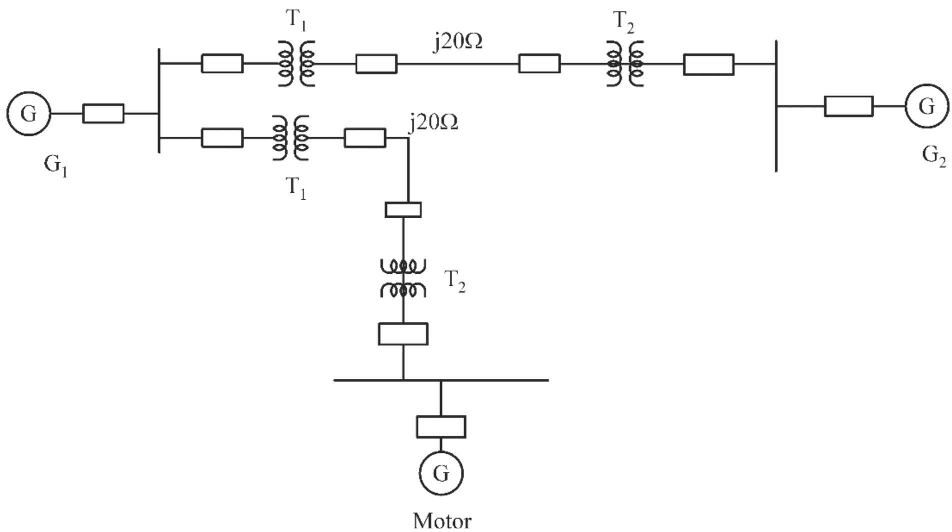

Fig.1.5

Solution: Generator 1:

Given: 25MVA, 18KV, x″ = 20%

Base values: 45KVA, Base KV = 20

$$Z_{p.unew} = Z_{p.ugiven} \times \left(\frac{\text{Base KV given}}{\text{Base KV new}}\right)^2 \times \left(\frac{\text{Base MVA new}}{\text{Base MVA given}}\right)$$

$X_{G1} = 0.2 \times (18/20)^2 \times (45/20 \times 10^3)$

= =0.2916 p.u.

Transformer 1:

Given: 25MVA, 20KV, X = 10%

Base values: 45KVA, 20KV

$X_{T1} = 0.1 \times (45/20 \times 10^3)$

= 0.00025p.u

Transmission Lines:

Base impedance = $(\text{Base KV})^2$/Base MVA

Base impedance = $148^2/45 \times 1000$

= 547600Ω

Per unit impedance = Actual impedance/Base impedance

p.u impedance = 0.0000821p.u

p.u impedance of the other transmission line = 20/547600 = 0.0000807p.u

Generator 2:

$X_{G2} = X_{G1} = 0.0003645$p.u

Y-Δ transformer

Given: 10MVA, 128Y/12.8kΔKV, X = 10%,

Base values; 45KVA, 128KV

$X = (0.1 \times 45/10 \times 10^3)$

$= 0.0005$p.u

Motor:

Given: 35MVA, 12.8KV, $x'' = 20\%$

Base values: 45KVA, 128KV

$X_m = 0.2 \times 45/35 \times 10^3$

$= 0.0000257$p.u

Impedance diagram:

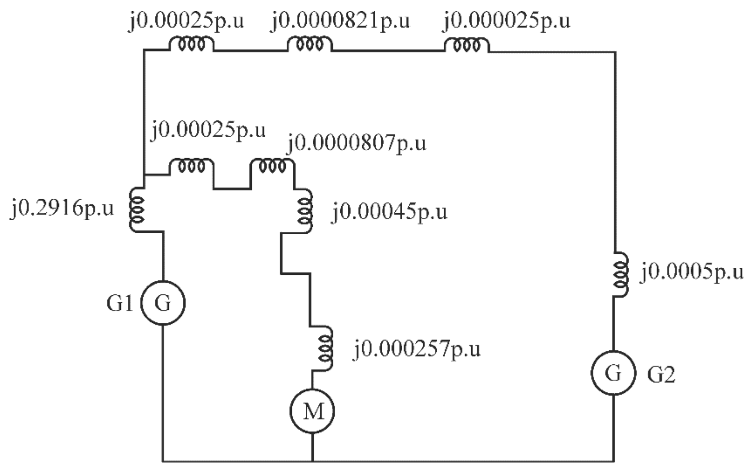

Fig.1.6

Example 5: Three generators are rated as follows:

Generator 1 – 120MVA, 40KV, reactance 14%.

Generator 2 – 170 MVA, 35KV, reactance 9%.

Generator 3 – 140MVA, 31KV, reactance 16%.

Determine the reactance of the generator corresponding to base values of 250MVA and 45 KV.

Solution:

Generator 1:

120MVA, 40kV, $x'' = 0.14$

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{\text{Base KV given}}{\text{Base KV new}}\right)^2 \times \left(\frac{\text{Base MVA new}}{\text{Base MVA given}}\right)$$

$X'' = 0.14 \times (40/45)^2 \times 250/120$

$= 0.230$ p.u.

Generator 2:

170MVA, 35KV, $X'' = 0.09$

$X'' = 0.09 \times (35/45)^2 \times 250/170 = 0.080$ p.u.

Generator 3:

140MVA, 31KV, $X'' = 0.16$

$X'' = 0.16 \times (31/45)^2 \times (250/140)$

$= 0.1350$

Example 6: Determine the generator voltage for the given system shown in Fig. 1.7.

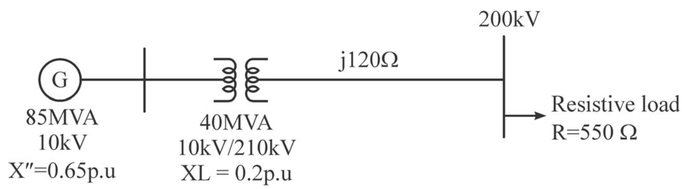

Fig.1.7

Solution: Assume a base of 120MVA and 200KV in the transmission line

Generator:

$$Z_{p.u.new} = Z_{p.u} \times \left(\frac{\text{Base KV given}}{\text{Base KV new}}\right)^2 \times \left(\frac{\text{Base MVA new}}{\text{Base MVA given}}\right)$$

$x''_G = 0.65 \times (10/9.523)^2 \times (120/85)$

$= 1.0118$ p.u.

Transformer:

$X_T = 0.2 \times (210/200)^2 \times (120/40)$

$= 0.6615$

Transmission lines:

Base impedance = $(\text{BaseKV})^2/\text{Base MVA}$

Base impedance = $(200)^2/120$

= 333 Ω

Per unit impedance = Actual impedance/Base impedance

P.U impedance = 120/333 = 0.3603 p.u.

Resistive load of 550Ω

$R_{p.u}$= 550/333 = 1.6516 p.u.

Impedance diagram:

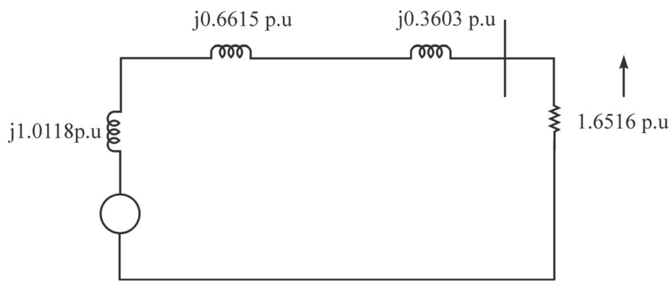

Fig.1.8

At the load bus:

$V_{p.u}$ = 1 p.u.

$R_{p.u}$ = 1.6516 p.u.

$I_{p.u} = V_{p.u.}$

= 1/1.6516 = 0.605 p.u.

Let $I_{p.u.}$ be taken as the reference phasor.

$I_{p.u.}$ = 0.605 + j0 = 0.605<0°

By Kirchhoff's voltage law,

Voltage drop in the network = $I_{p.u} [R_{p.u} + j (X_G + X_T + X_L)]$

= 0.605 [1.6516 + j (1.0118 + 0.6615 + 0.3603)]

= 0.605 [1.6516 + j2.0336]

= 0.605 × 2.6197<50.9159

= 1.589 p.u.

Actual generator terminal voltage:

$V_G = V_{p.u} \times$ Base voltage at the generator terminals

= 1.5849 × 9.523

= 15.0930 KV

Example 7: A 200 MVA, 66 KV Three phase generator has a subtransient reactance of 20%. The generator is connected to 3 motors through a transmission line and two transformers. The motors have rated input of 40 MVA, 30 MVA and 45 MVA at 33 KV with 15% subtransient reactance. The 3phase transformer are rated at 220 MVA, 64/220 KV 'Y' with leakage reactance of 10%. The line has a reactance of 45 Ohms. Selecting the generator rating as the quantities in generator circuit, determine the base quantities in other parts of the system and evaluate the corresponding per unit values.

Solution:

Transmission line diagram for given statement is:

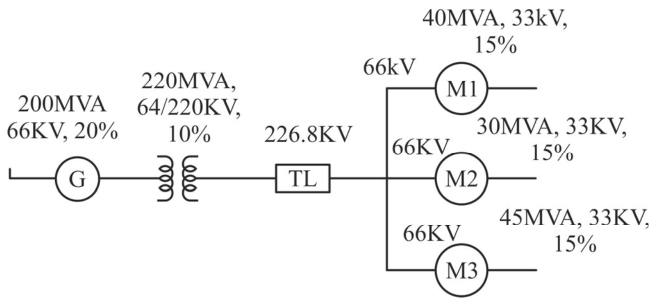

Fig.1.9

The base voltage in transmission line = 66 × (220/64)

= 226.875 KV

In the motor circuit, The base voltage = 226.875 × (64/220)

= 66 KV

Base impedance in the transmission line = (Base KV^2 / Base MVA) Ω

= $((226.875)^2/200)$ Ω

= 257.36 Ω

Per unit impedance = (j45 / j257.686) p.u.

= 0.1746 p.u

Motor 1:

Rating: 40 MVA, 33 KV, $X'' = 0.15$

$$Z_{p.u.new} = Z_{p.u\ given} \times \left(\frac{\text{Base KV given}}{\text{Base KV new}}\right)^2 \times \left(\frac{\text{Base MVA new}}{\text{Base MVA given}}\right)$$

Zp.u new = 0.15 × $[(33/66)^2 \times (200/30)]$ p.u.

= 0.1875 p.u.

Motor 2:

Rating: 30 MVA, 33KV, $X'' = 0.15$.

$Z_{p.u.\ new} = 0.15 \times (33/66)^2 \times (200/45)$p.u

$= 0.25$ p.u

Motor 3:

Rating: 45 MVA, 33kV, $X'' = 0.15$

$Z_{p.u\ (new)} = 0.15 \times (33/66)^2 \times (200/\ 45)$ p.u

$= 0.16$ p.u

Transformer:

Rating: 220 MVA, 64kV, $X'' = 0.1$

$X''(T) = 0.1 \times (64\ /\ 66)^2 \times (200/220) = 0.08548$ p.u

Example 8: Draw the per unit impedance diagram for the power system below. Neglecting the reactance and use a base of 150 MVA, 110 KV in 40 ohms line. The rating of the generator, motor and transformers are:

Generator: 45 MVA, 32 KV, X = 15%

Motor: 42 MVA, 11 KV, X = 25%

Y-Y Transformer: 35 MVA, 22/110 KV, 13%

Y- Transformer: 30 MVA, 20/110 KV, 13%

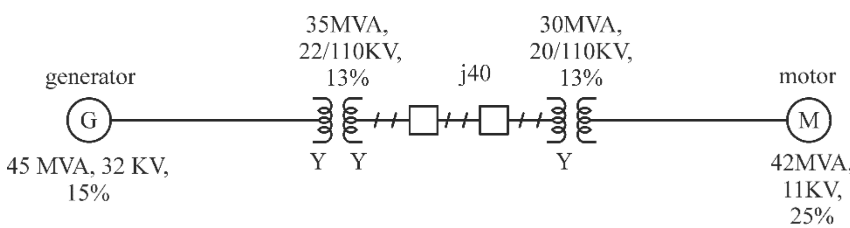

Fig.1.10

Solution:

$$Z_{p.u(new)} = Z_{p.u(given)} \times \left(\frac{\text{Base KV given}}{\text{Base KV new}}\right)^2 \times \left(\frac{\text{Base MVA new}}{\text{Base MVA given}}\right)$$

Generator:

New base voltage in generator = (110 × 22 /110) KV = 22 KV

$X'' = 0.15 \times (32/22)^2 \times (150\ /45)$p.u $= 1.0578$ p.u

Transformer 1:

New voltage in Transformer 1 = (110 × 22/110) KV = 22 KV

$X'' = 0.13 \times (22\ /\ 22)^2 \times (150\ /\ 35)$ p.u. $= 0.5571$ p.u.

Transmission Line:

$$Z_B = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$= (110)^2 / 150\ \Omega$$

$Z_B = 80.66\ \Omega$

$Z_{p.u.}$ = (Actual impedance / Base impedance) p.u.

= 40 /80.66 p.u.

= 0.49 p.u.

Transformer 2:

New base voltage in Transformer 2 = (110 × 20 / 110) KV = 20 KV

$X'' = 0.13 \times (20 / 20)^2 \times (150 / 30)$ p.u.

= 0.65 p.u.

Motor:

$X'' = 0.25 \times (11 / 11)^2 \times (150 / 42)$ p.u.

= 0.8928 p.u.

Impedance diagram:

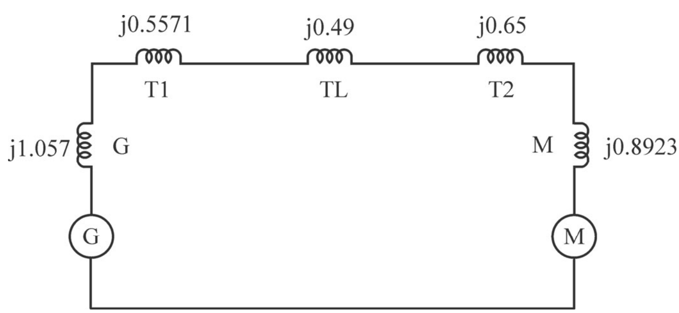

Fig.1.11

Example 9: 3 bus system is given in fig.1.12. The ratings of the various components are listed below:

Generator 1: 45 MVA, 12.5 KV, $X'' = 0.12$ p.u.

Generator 2: 30 MVA, 12.3 KV, $X'' = 0.3$ p.u.

Generator 3:20 MVA, 10 KV, $X'' = 0.2$ p.u.

Transformer 1: 40 MVA, 11/110 KV Y/Y, X = 0.15 p.u.

Transformer 2: 20 MVA, 12.5/115 KV Y/Y, X = 0.2 p.u.

Transformer 3: 30 MVA, 12.5/115 KV Y/Y, X = 0.1 p.u.

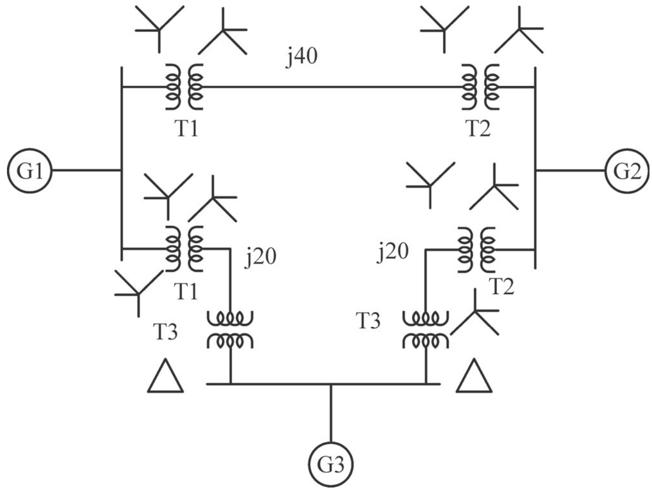

Fig.1.12

The line impedances are shown in Fig.1.12. Determine the reactance based on 45 MVA and 12.5 kV as base quantities in generator 1.

Solution:

Base MVA = 45 MVA; Base KV = 12.5 KV;

For Generator 1:

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{\text{Base KVgiven}}{\text{Base KVnew}}\right)^2 \times \left(\frac{\text{BaseMVAnew}}{\text{BaseMVAgiven}}\right)$$

$$Z_{p.u\ new} = 0.12 \times \left(\frac{12.5}{12.5}\right)^2 \times \left(\frac{45}{45}\right) = 0.12 \text{ p.u.}$$

Transformer 1:

40 MVA, 11 KV, X = 0.15p.u.

Base value 45 MVA, 12.5 KV

$$X_{T1} = Z_{p.u\ new} = 0.15 \times \left(\frac{11}{12.5}\right)^2 \times \left(\frac{45}{40}\right)$$

$$= 0.13068 \text{ p.u.}$$

Transformer 2:

20 MVA, 12.5 KV, X = 0.2 p.u.

Base voltage on the LV side of Transformer 2 = $12.5 \times \left(\frac{110}{11}\right)$

$= 125$ KV

$= 125 \times \left(\frac{12.5}{115}\right)$

$= 13.5$ KV

$$X_{T2} = 0.2 \times \left(\frac{12.5}{13.5}\right)^2 \times \left(\frac{45}{20}\right)$$

$= 0.3858$ p.u.

Transformer 3:

Base voltage on the HV side of transformer 3 $= 12.5 \times \left(\frac{115}{12.5}\right)$

$= 115$KV

$= 15 \times \left(\frac{115}{12.5}\right)$

$= 138$ KV

X = 0.1 p.u, 115 KV, 30 MVA

$$X_{T3} = 0.1 \times \left(\frac{115}{138}\right)^2 \times \left(\frac{45}{30}\right) = 0.10416\text{p.u}$$

Generator 2:

Base voltage $= 125 \times \left(\frac{12.5}{115}\right)$

$= 13.5$ KV

x'' = 0.3 p.u., 12.3 KV, 30 MVA

$$X_{G2} = 0.3 \times \left(\frac{12.3}{13.5}\right)^2 \times \left(\frac{45}{30}\right)$$

$= 0.3735$p.u

Generator 3:

Base voltage on the bus $= 138 \times \left(\frac{12.5}{115}\right) = 15$ KV

20 MVA, 10 kV, $X'' = 0.2$ p.u.

$$X_{g3} = 0.2 \times \left(\frac{10}{15}\right)^2 \times \left(\frac{45}{20}\right)$$

$$= 0.2\text{p.u.}$$

Transmission Line 1:

Base voltage along the transmission line whose impedance is j40Ω $= 12.5 \times \left(\frac{110}{11}\right)$

$$= 125 \text{ KV}$$

$$\text{Per unit impedance} = \left(\frac{\text{Actual impedance}}{\text{Base impedance}}\right)$$

$$\text{Base impedance} = \frac{(\text{BaseKV})^2}{\text{BaseMVA}}$$

$$= \frac{(125)^2}{45}$$

$$= 347.22\ \Omega$$

$$\text{Per unit impedance} = \left(\frac{40}{347.22}\right) = 0.1152 \text{ p.u.}$$

Transmission Line 2:

Base KV = 125, base MVA = 45

$$\text{Base impedance} = \frac{(125)^2}{45} = 347.22\ \Omega$$

$$\text{Per unit impedance} = \left(\frac{20}{347.22}\right) = 0.0576 \text{ p.u.}$$

Transmission Line 3:

Base KV = 125, Base MVA = 45

$$\text{Base impedance} = \frac{(125)^2}{45} = 347.22\ \Omega$$

$$\text{Per unit impedance} = \left(\frac{20}{347.22}\right) = 0.0576 \text{ p.u.}$$

Reactance Diagram:

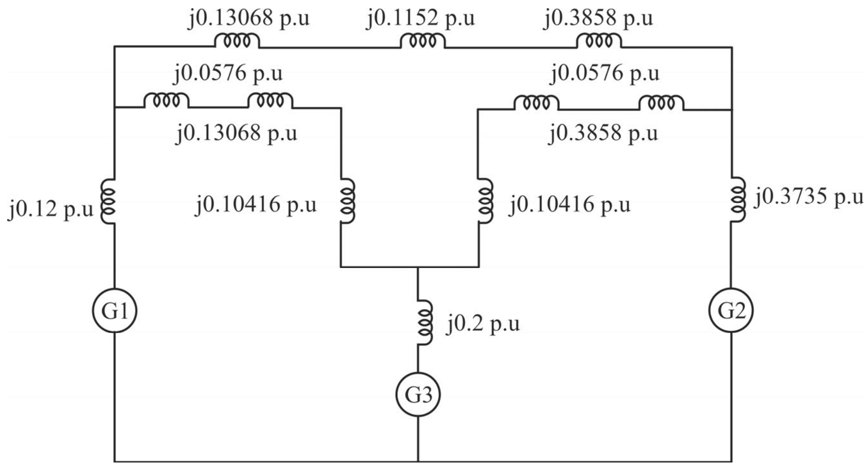

Fig.1.13

Example 10: 13MVA, 8 KV, 3-φ Generator has a subtransient reactance of 40%. It is connected through a Δ-Y transformer to a high voltage transmission line having a total series reactance of 60Ω.At the load end of the line is a Y-Y step down transformer .Both transformer bank are composed of 1- φ transformer connected for 3- φ operation. Each of the two transformer comprising each bank is rated 5050 KVA,11/110 KV with a reactance 10%. The load represented as a motor which is drawing 12MVA at 10.2KV with 60% reactance. Draw the impedance diagram showing all impedance in per unit. Choose a base of 12MVA, 12.5KV at the motor.

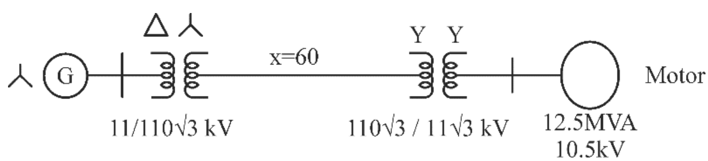

Fig.1.14

Solution:

Base MVA= 12; Base KV = 12.5

Load reactance = 0.6pu.

Total MVA rating of the transformer 1 and 2 separately = 3 x5050/1000

= 15.15MVA

Base KV on the transmission line side $=10.2 \times \left(\frac{110\sqrt{3}}{11\sqrt{3}}\right)$ =102 kV

$$\text{Base impedance} = \frac{(\text{BaseKV})^2}{\text{BaseMVA}}$$

Base impedance in the transmission line $=102^2/12 = 867\Omega$

$$\text{Per unit impedance} = \left(\frac{\text{Actual impedance}}{\text{Base impedance}}\right)$$

Per unit impedance=60/867=0.0653 p.u

Transformer 1:

$$\text{Base KV on the generator side is given by} = \left(\frac{11}{110\sqrt{3}}\right) \times 102 = 5.9 \text{ KV}$$

$$\text{Per unit of transformer } X_{t_1} = 0.1 \times \left(\frac{11}{5.9}\right)^2 \times \left(\frac{12}{15.15}\right) = 0.2753 \text{ p.u}$$

Generator 1:

Given: 13 MVA, 8 KV, $X'' = 0.4$ p.u.

Base value = 12 MVA, 5.9 KV

$$X_{g_1} = 0.4 \times \left(\frac{8}{5.9}\right)^2 \times \left(\frac{12}{13}\right) = j0.6788 \text{p.u}$$

Transformer 2:

Given:15.15 MVA, $110\sqrt{3}$ KV, $X'' = 0.1$

Base value =10.5 MVA,102 KV,

$$X_{t_2} = 0.1 \times \left(\frac{110\sqrt{3}}{102}\right)^2 \times \left(\frac{10.5}{15.15}\right)$$

$$= 0.235 \text{ p.u.}$$

Impedance diagram:

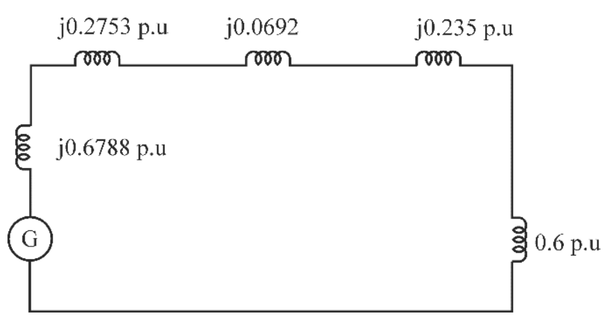

Fig.1.15

Example 11: Figure 1.16 shows a single line diagram of an unloaded three generator power system with interconnection between the generators by means of three transformers and a transmission line with two sections, with their impedances marked on the diagram. The ratings of the generators and transformers are given below:

Generator	MVA	KV	Reactance p.u
1	50	13.8	0.15
2	40	13.2	0.2
3	30	11	0.25

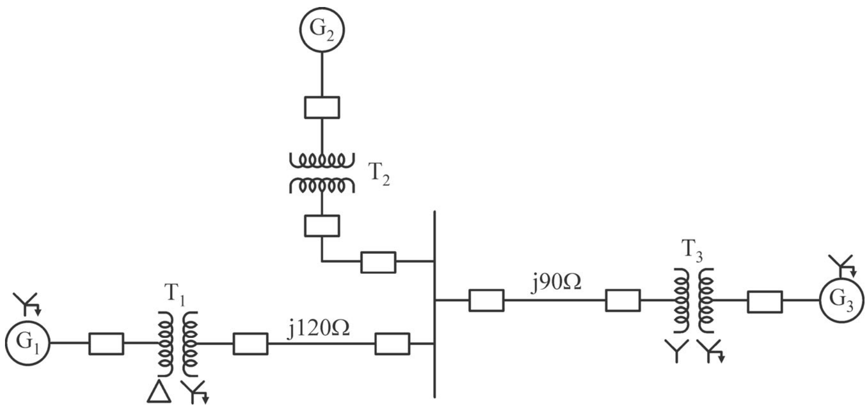

Fig.1.16

Transformer 1: 45MVA, 11Δ–110Y KV, X = 10%

Transformer 2: 25MVA, 12.5(del) – 115Y KV, X = 15%

Transformer 3: Single phase units, each rated 10 MVA, 6.9/69 KV, X = 10%

Draw the impedance diagram and mark all values in p.u. choosing a base of 30MVA, 13.8KV in the generator 1 circuit.

Solution:

Base MVA = 30; Base KV = 13.8KV

Generator 1:

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{(\text{Base KV}_{given})}{\text{Base KV}_{new}}\right)^2 \times \left(\frac{(\text{Base MVA}_{new})}{\text{Base MVA}_{given}}\right)$$

$$X_{g1} = 0.15 \times (13.8/13.8)^2 \times 30/50$$

$X_{g1} = 0.09$ p.u.

Transformer 1:

Given : 45MVA, 11Δ /110Y KV, X = 10%

$X_{T1} = 0.1 \times (11/13.8)^2 \times (30/45)$

$X_{T1} = 0.042$ p.u.

Transmission Line 1:

Base voltage on the transmission line with impedance

j120Ω = 13.8 × (110/11) = 138 KV

$$\text{Base Impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$\text{Base impedance} = \frac{(138)^2}{30} = 6.348\Omega$$

$$\text{Per unit impedance} = \frac{\text{Actual Impedance}}{\text{Base Impedance}}$$

Per unit impedance = j120/634.8 = j0.18903 p.u.

Transmission Line 2:

Per unit impediance = j90/634.8 = 0.14177 p.u.

Transformer 2:

Given: 25 MVA, 12.5 Δ /115Y KV, X = 0.15 p.u.

Base value: 30MVA, 13.8KV

$X_{T2} = 0.15 \times (115/13.8)^2 \times 30/25$

$X_{T2} = 12.5$ p.u.

Generator 2:

Base KV = 13.8 × 12.5/115 = 1.5 p.u.

Given : 25 MVA, 1.5 KV, X = 0.15 p.u.

$X_{G2} = 0.15 \times (1.5/1.5)^2 \times 30 / 25$

= 0.18. p.u.

Transformer 3:

Line to line voltage when three single phase transformers are used as a three phase transformer is

$\sqrt{3} / \sqrt{3} \times 6.9/6.9$ = 11.95KV/1119.51KV

Given: 30MVA, 11.95/119.51 KV, X = 0.1p.u

Base values: 30MVA, 13.8 KV

$X_{T3} = 0.1 \times (119.51/13.8)^2 \times 30/30$

$X_{T3} = 0.499$ p.u.

Generator 3:

Base KV = 13.8 × 11.95 / 119.51 = 13.79 KV

Given: 30MVA, 11KV, X = 0.25

Base value: 30, 1.379

$X = 0.25 \times (11 /1.379)^2 \times 30/30$

X = 1012.1 p.u.

Impedance Diagram:

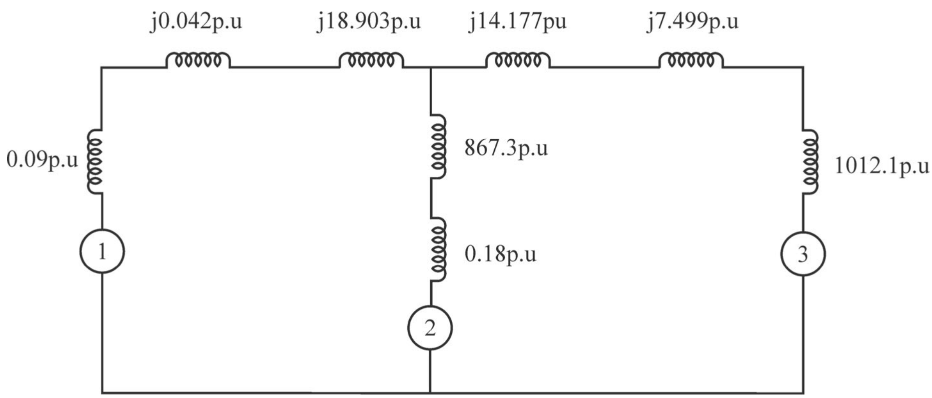

Fig.1.17

Example 12: The one line diagram of an unloaded power system is shown in Fig. 1.18. Reactances of the sections of transmission line are shown in the diagram. The generators and transformers are rated as follow:

Generator 1: 20 MVA, 13.8 KV, $X'' = 0.2$p.u

Generator 2: 30 MVA, 18 KV, $X'' = 0.2$p.u.

Generator 3:30 MVA, 20 KV, $X'' = 0.2$p.u.

Transformer T1:20 MVA, 20 KV, $X'' = 0.2$p.u.

Transformer T2: Single phase units each rated 30MVA, 127/18KV, X = 10%

Transformer T3:35MVA, 220Y/22Y KV, X = 10%

Draw the impedance diagram with all reactance's marked in p.u. and with letters to indicate points corresponding to the one–line diagram. Choose a base of 50MVA, 13.8 KV in the circuit of generator 1.

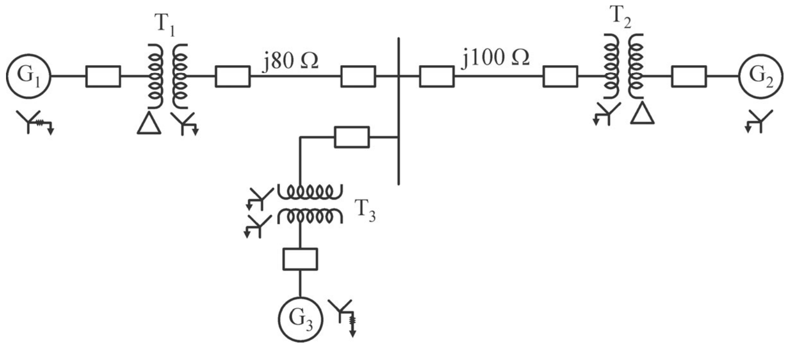

Fig.1.18

Solution:

50MVA, 13.8KV – Base values

Generator 1:

20MVA, 13.8KV, X″ = 0.2.

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{(\text{Base KV}_{given})}{\text{Base KV}_{new}}\right)^2 \times \frac{\text{Base MVA}_{new}}{\text{Base MVA}_{given}}$$

$X_{G1} = 0.2 \times (13.8/13.8)^2 \times 50/20$

$= 0.5$ p.u.

Transmission line 1:

Base voltage on the transmission line using the transformation ratio = 220kV

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

Base impedance = (220 × 220)/50 = 968Ω

$$\text{Per unit impedance} = \frac{\text{Actual Impedance}}{\text{Base Impedance}}$$

Per unit impedance = 80/968 = 0.0826 p.u.

Transmission line 2:

Per unit impedance = 100 / 968 = 0.1033 p.u.

Transformer 1:

Base MVA = 50, KV = 13.8

Given MVA = 25.13 MVA, 13.8KV, X = 10p.u.

$X_{t1} = 0.1 \times (13.8 / 13.8)^2 \times 50 /25$

$= 0.2$ p.u.

Transformer 2:

Voltage rating of the transformers when they are put to form a three phase transformer

$= \sqrt{3} \times 127/18$ KV = 220/18 KV

$X_{t2} = 0.1 \times (18/18)^2 \times 50/30$

$= 0.166$ p.u.

Generator 2:

Given: 30MVA, 18KV, $X'' = 0.2$

Base values: 50 MVA, 18KV

$X_{g2} = 0.2 \times (18/18)^2 \times 50/30$

$= 0.3333$ p.u.

Generator 3:

Given: 30MVA, 20KV, $X'' = 0.2$ p.u.

Base Values: 50MVA, 22KV

$X_{g3} = 0.2 \times ((20/22)^2 \times 50/30$

$= 0.275$ p.u.

Transformer 3:

$X_{t3} = 0.1 \times 50/35 = 0.1429$ p.u.

Impedance diagram:

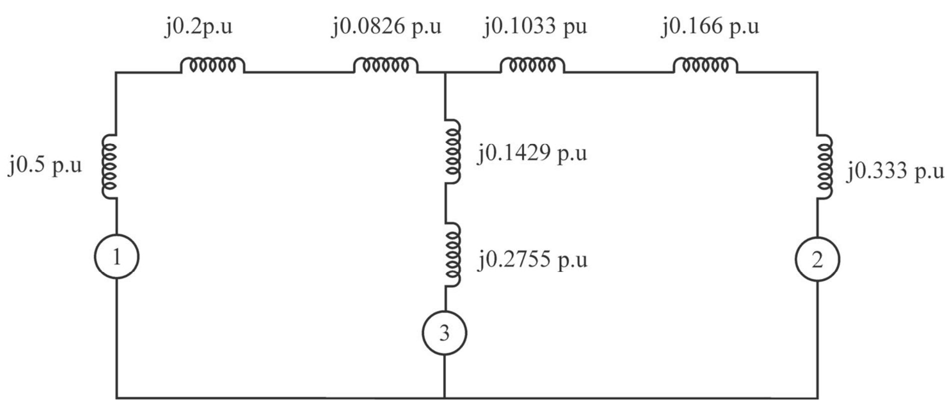

Fig.1.19

Example 13: Figure 1.20 shows a single line diagram of an unloaded three generator power system with interconnection between the generators by means of three transformers and a transmission line with two sections, with their impedances marked on the diagram. The rating of the generator and transformer are given below:

Generator	MVA	KV	Reactance p.u.
1	30	7.7	0.3
2	20	7.7	0.20
3	35	15.4	0.20

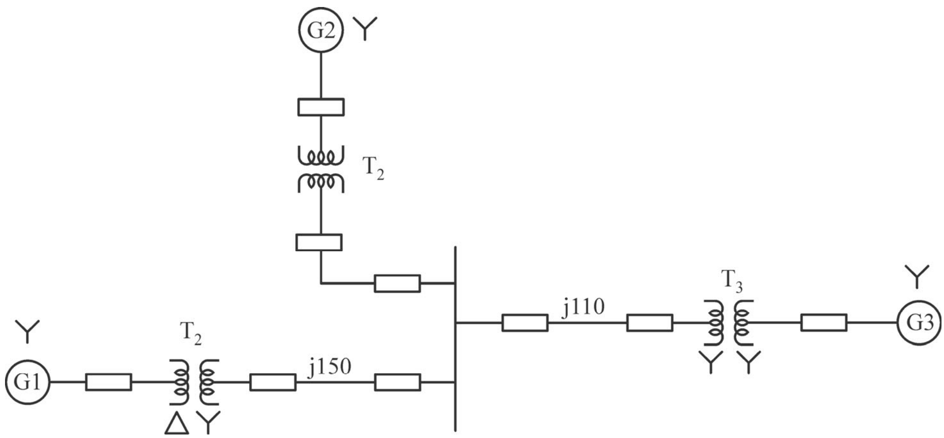

Fig.1.20

Transformer 1: 35 MVA, 8.0Δ – 120Y, X = 10%

Transformer 2: 20 MVA, 8.0Δ – 120Y, X = 10%

Transformer 3: Single phase units, each rated 15 MVA, 8.0/80KV, X = 10%

Draw an impedance diagram and mark all values in p.u. choosing a base of 35MVA, 7.7KV in generator 1 circuit.

Solution:

Base MVA=35; Base KV=7.7KV

Generator 1:

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{\text{Base KV}_{given}}{\text{Base KV}_{new}}\right)^2 \times \left(\frac{\text{BaseMVA}_{new}}{\text{BaseMVA}_{given}}\right)$$

$$X_{G1} = 0.3 \times \left(\frac{7.7}{7.7}\right)^2 \times \frac{35}{30} = 0.35\text{p.u.}$$

Transformer 1:

Given: 35MVA, 8Δ – 120Y KV, X = 10%

$$X_{T1} = 0.1 \times \left(\frac{8.0}{7.7}\right)^2 \times \frac{35}{35} = 0.1 \times 1.0794 = 0.10794\text{p.u.}$$

Transmission line 1:

Base voltage on the transmission line with impedance j150 Ω = $7.7\times \frac{12}{8} = 115.5$

$$\text{Base Impedance} = \left(\frac{\text{BaseKV}}{\text{BaseMVA}}\right)^2$$

$$\text{Base Impedance} = \frac{(115.5)^2}{35} = 381.15\ \Omega$$

$$\text{Per unit impedance} = \left(\frac{\text{Actual impedance}}{\text{Base impedance}}\right)$$

$$\text{Per unit impedance} = \frac{j150}{381.15} = j0.39354\text{p.u.}$$

Transmission line 2:

$$\text{Per unit impedance} = \frac{j110}{381.15} = j0.2886\text{p.u.}$$

Transformer 2:

Given: 20MVA, 8.0Δ – 120Y KV, X = 0.1p.u.

Base values: 35 MVA, 115.5 KV

$$X_{T2} = 0.1 \times \left(\frac{120}{115.5}\right)^2 \times \frac{35}{20}$$

$$= 0.1889\text{p.u.}$$

Generator 2:

$$\text{Base KV} = 115.5 \times \frac{8}{120} = 7.7\text{ KV}$$

Given: 20 MVA, 7.7 KV, X = 0.20p.u.

$$X_{G2} = 0.2\times \left(\frac{7.7}{7.7}\right)^2 \times \frac{35}{20} = 0.35\text{p.u.}$$

Transformer 3:

line to line voltage when three 1-∅ transformer are used as a 3-∅ transformer is

$$\frac{\sqrt{3}}{\sqrt{3}} \times \frac{8}{80} = \frac{13.856\text{kV}}{138.564\text{kV}}$$

Given: 45MVA, 13.865/138.564 KV, X = 0.1p.u.

Base values: 35MVA, 115.5KV

$$X_{T3} = 0.1 \times \left(\frac{(138.56)}{115.5}\right)^2 \times \frac{35}{45} = 0.1119 \text{ p.u.}$$

Generator 3:

$$\text{Base KV} = 115.5 \times \frac{13.856}{138.564} = 11.549\text{KV}$$

Given: 35MVA, 15.4KV, X = 0.2p.u.

Base values: 35MVA, 11.549KV

$$X_{G3} = 0.2 \times \left(\frac{15.4}{11.549}\right)^2 \times \frac{35}{35} = 0.3556\text{p.u.}$$

Impedance diagram:

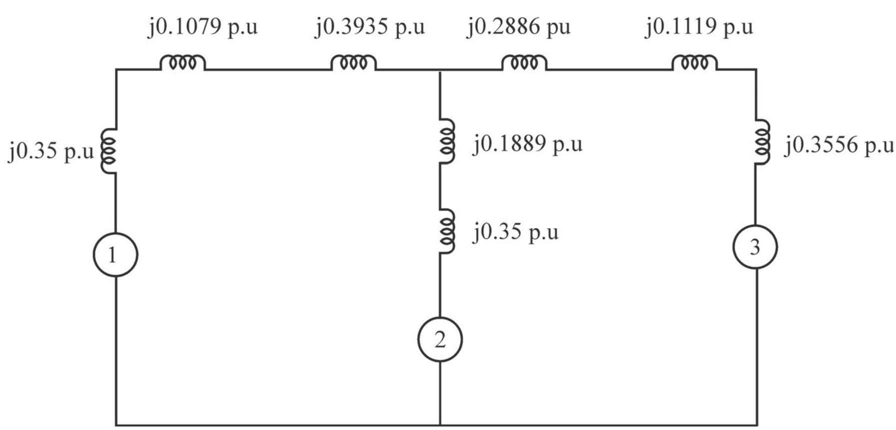

Fig.1.21

Example 14: The one line diagram of an unloaded power system is shown in Fig. 1.22. Reactance of two sections of transmission line are shown in the diagram. The generators and transformers are rated as follows:

Generator 1: 30MVA, 15.8KV, X″ = 0.2p.u.

Generator 2: 40MVA, 20KV, X″ = 0.2p.u.

Generator 3: 40MVA, 22KV, $X'' = 0.2$p.u.

Transformer 1: 30MVA, 220KV, $X'' = 10\%$

Transformer 2: Single phase unit each rates 10MVA, 130/20 KV, X = 10%

Transformer 3: 40MVA, 220Y/25Y KV, X = 10%

Draw the impedance diagram with all reactance marked in p.u. and with letters to indicate points corresponding to the one-line diagram. Choose a base of 60MVA, 15.8KV in the circuit of generator 1.

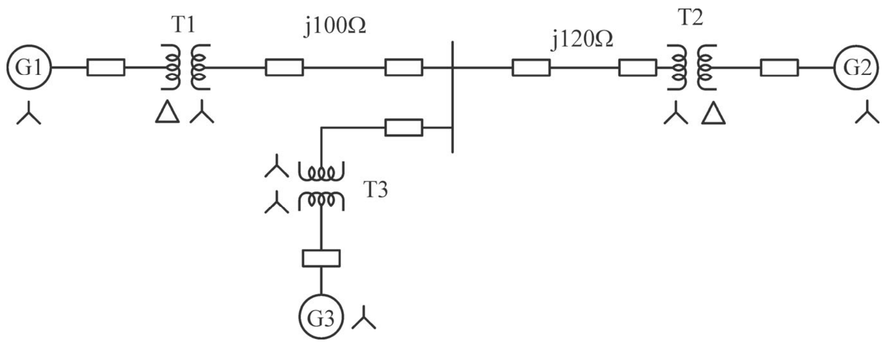

Fig.1.22

Solution:

Base values: 60MVA, 15.8KV

Generator 1:

30MVA, 15.8KV, $x'' = 0.2$

$$Z_{p.u\,new} = Z_{p.u\,given} \times \left(\frac{\text{Base KV}_{given}}{\text{Base KV}_{new}}\right) \times \left(\frac{\text{BaseMVA}_{new}}{\text{BaseMVA}_{given}}\right)$$

$$X_{G1} = 0.2 \times \left(\frac{15.8}{15.8}\right)^2 \times \left(\frac{60}{30}\right)$$

$$= 0.4\text{p.u.}$$

Transmission Line 1:

Base voltage on the transmission line using the transformation ratio = 220 KV

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$\text{Base impedance} = \frac{(220)^2}{60}$$

$$= 806.67\Omega$$

$$\text{Per unit impedance} = \left(\frac{\text{Actual impedance}}{\text{Base impedance}}\right)$$

$$= \frac{100}{806.67} = 0.1239\text{p.u.}$$

Transmission Line 2:

$$\text{Per unit impedance} = \frac{120}{806.67} = 0.1487\text{p.u.}$$

Transformer 1:

Base values: 60MVA, 15.8KV

Given: 30MVA, 15.8KV, X = 10%

$$Z_{\text{p.u new}} = Z_{\text{p.u given}} \times \left(\frac{(\text{BasekVgiven})}{\text{BasekVnew}}\right)^2 \times \left(\frac{\text{BaseMVAnew}}{\text{BaseMVAgiven}}\right)$$

$$X_{T1} = 0.1 \times \left(\frac{15.8}{15.8}\right)^2 \times \left(\frac{60}{30}\right)$$

$$X_{T1} = 0.2\text{p.u}$$

Transformer 3:

$$X_{T3} = 0.1 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{60}{40}\right)$$

$$= 0.15\text{p.u}$$

Transformer 2:

Voltage of the transformer when they are put to form a 3-Ø transformer

$$= \frac{\sqrt{3}}{220} \times 130\text{ KV} = \frac{225}{220}\text{KV}$$

$$X_{T2} = 0.1 \times \left(\frac{225}{220}\right)^2 \times \left(\frac{60}{30}\right) = 0.209\text{p.u.}$$

Generator 2:

Given: 40MVA, 20KV, $X'' = 0.2$

Base values: 60MVA, 19.5KV

$$X_{G2} = 0.2 \times \left(\frac{20}{19.5}\right)^2 \times \left(\frac{60}{40}\right) = 0.315\text{p.u.}$$

Generator 3:

Given: 40MVA, 22KV, $X'' = 0.2$

Base values: 60MVA, 25KV

$$X_{g3} = 0.2 \times \left(\frac{22}{25}\right)^2 \times \left(\frac{60}{40}\right) = 0.232\text{p.u.}$$

Impedance diagram:

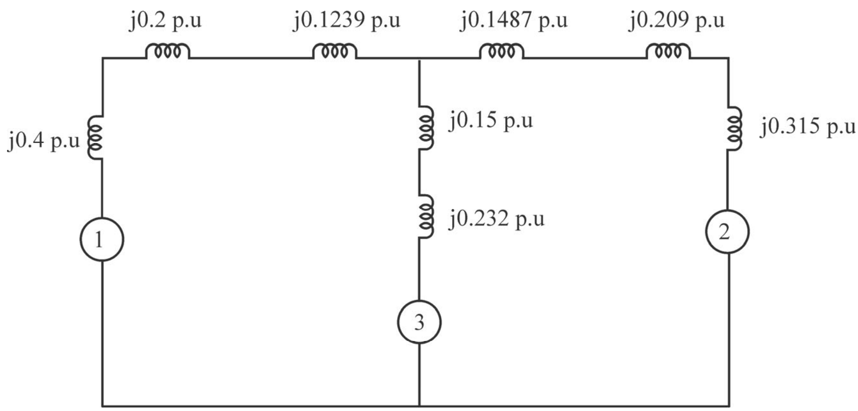

Fig.1.23

Example 15: A 3 bus system is given in Fig.1.24.The ratings of the various components are listed below:

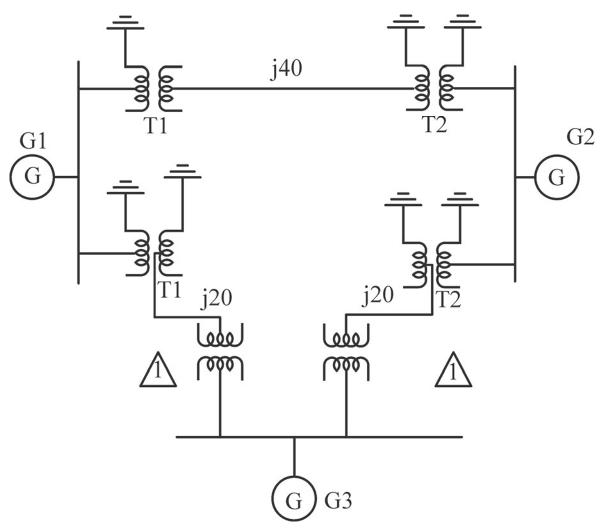

Fig.1.24

Generator 1: 40 MVA, 13.8 KV, $X'' = 0.2$p.u.

Generator 2: 50 MVA, 14.2 KV, $X'' = 0.15$p.u.

Generator 3: 30 MVA, 12 KV, $X'' = 0.25$p.u.

Transformer 1: 50 MVA, 10 KV/100KV Y/Y, X = 0.12p.u.

Transformer 2:30 MVA, 11.5 KV/110KV Y/Y, X = 0.14p.u.

Transformer 3:35 MVA, 11.5 KV/110KV Y/Y, X = 0.2p.u.

The line impedances are shown in the figure. Draw the reactance diagram based on 40 MVA and 13.8 kV as base quantities in Generator 1.

Solution:

Base MVA = 40; Base KV = 13.8

Generator 1:

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{\text{Base KV}_{given}}{\text{Base KV}_{new}}\right)^2 \times \frac{\text{Base MVA}_{new}}{\text{Base MVA}_{given}}$$

Per unit reactance = $0.2 \times (13.8/13.8)^2 \times 40/40 = 0.2$p.u.

Transmission line 1:

Base voltage along the transmission line whose impedance is j40Ω=13.8 × 100/10 = 138KV

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$\text{Base impedance} = \frac{138^2}{40} = 476.1\Omega$$

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

Per unit impedance = 40/476.1= 0.0840p.u.

Generator 2:

Base Voltage = 138 × 11.5/110 =14KV

$X_{G2} = 0.15 \times (14.2/14)^2 \times 40/50$

$= 0.15 \times (14.2/14)^2 \times 0.8 = 0.1234$p.u.

Transformer 1:

50 MVA, 10 KV X = 0.12p.u.

Base values: 40 MVA, 13.8 KV

$X_{T1} = 0.12 \times (10/13.80)^2 \times 40/50 = 0.05040$p.u.

Transformer 2:

30 MVA, 11.5 KV X = 0.14p.u.

Base voltage on the L.T. side of the transformer 2 = 138 × 11.5/110 = 14KV

$X_{T2} = 0.14 \times (11.5/14)^2 \times 40/30 = 0.14 \times 0.674744 \times 1.3333 = 0.1259$p.u.

Transformer 3:

Base voltage on H.T. side of the transformer 3 is 138 KV.

$X_{T3} = 0.2 \times (110/138)^2 \times 40/35$

$= 0.2 \times 1.428571429 \times 12100/19044$

$= 0.14522$p.u.

Generator 3:

Base voltage on the bus= 138 × (11.5/110) =14KV

30 MVA, 12KV, $X'' = 0.25$p.u.

Base values: 40 MVA, 14KV

$= 0.25 \times (12/14)^2 \times 40/30$

$= 0.25 \times 144/196 \times 1.3333$

$= 0.244$p.u.

Transmission Line 2:

Base KV = 138

Base MVA = 40

Base impedance = $(138)^2/40 = 476.1\Omega$

Per unit impedance = j20/476.1 = j0.04200

Reactance Diagram

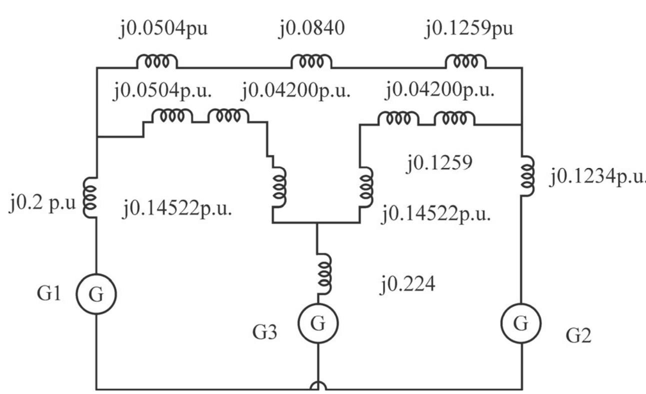

Fig.1.25

Example 16: A 20 MVA, 9.5KV 3-ϕ Generator has a sub-transient reactance of 20%. It is connected through a Δ-Y transformer to a high voltage transmission line having a total series reactance of 60Ω . At the load end of the line is a Y-Y step down transformer .Both transformers banks are composed of 1-ϕ transformers connected for 3-ϕ operation. Each of the two transformers comprising each bank is rated 5667 KVA, 11/110 KV with a reactance of 10 %. The load represented as impedance is drawing 11 MVA, 13.5 KV at 70 % power factor lagging. Draw the impedance diagram showing all impedances in per unit. Choose a base of 11 MVA, 13.5KV in the load circuit.

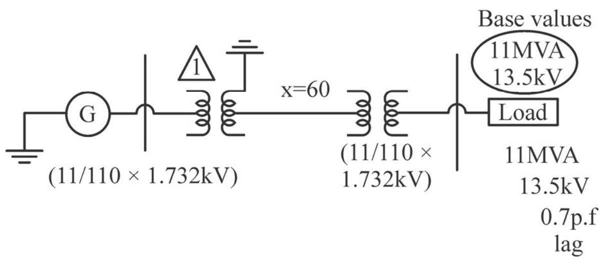

Fig.1.26

Solution:

MVA = 11; KV = 13.5

Load impedance = 0.7 + j0.6p.u. (Using the formula for p.u. impedance)

Total MVA rating of the transformers 1 and 2 separately = 3 × 5667/1100 = 15.4545 MVA

Base KV on the transmission line side = $13.5 \times 110\sqrt{3}/11\sqrt{3}$ = 135 KV

Base impedance = $(\text{Base KV})^2/\text{Base MVA}$

Base impedance in the transmission line side = $(115)^2/11$ = 1656.8181

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

Per unit impedance = 60/1656.8181 = 0.036213

Transformer 1:

Base KV on the generator side is given by

$= 115 \times 11/110\sqrt{3}$ = 6.6 KV

Therefore, per unit impedance of transformer 1 with reference to low voltage winding is given by

$X_{T1} = 0.1 \times (11/6.6)^2 \times 11/15 = 0.2037$p.u.

Generator 1:

Given, 20 MVA, 9.5 KV, $X'' = 0.2$

Base values: 11 MVA, 6.6 KV

$X'' = 0.2 \times (110\sqrt{3}/115)^2 \times 11/15 = 0.4025$p.u.

Transformer 2:

Given 15 MVA, $110\sqrt{3}$ KV, $X'' = 0.3$p.u.

Base values: 11 MVA, 115 KV

$X_{T2} = 0.3 \times (110\sqrt{3}/115)^2 \times 11/15 = 0.3 \times 36300/13225 \times 0.73333 = 0.06038$p.u.

Impedance Diagram:

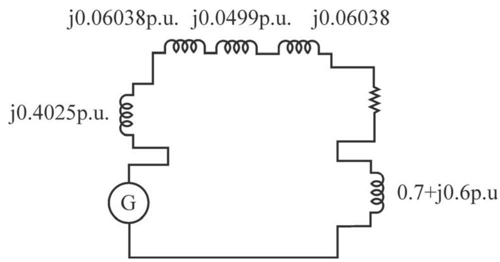

Fig.1.27

Example 17: Fig.1.28 shows a single line diagram of an unloaded three generator power system with interconnection between the generators by means of three transformers and a transmission line with two sections, with their impedances marked on the diagram. The rating of the generators and transformers are given below:

Generator 1: 30MVA, 6.9KV, $X'' = 20$p%.

Generator 2: 20MVA, 6.9KV, $X'' = 10\%$.

Generator 3: 35MVA, 13.4KV, $X'' = 10\%$.

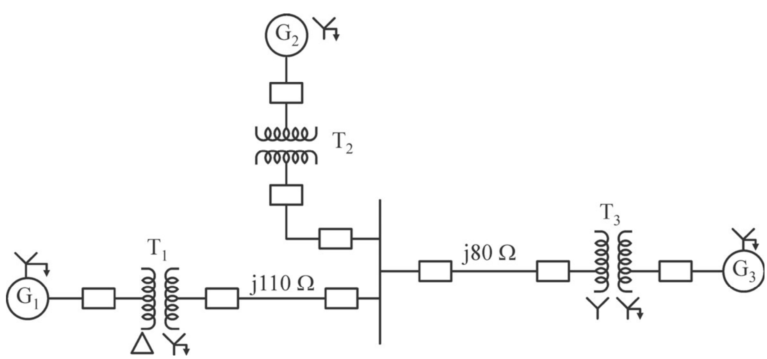

Fig.1.28

Transformer 1: 35MVA, 7.2 Δ-120YKV, X = 10%

Transformer 2: 20MVA, 7.2 Δ -120 YKV, X = 10%

Transformer 3: Single phase units, each rated 10MVA, 7.9/79KV, X = 10%

Draw an impedance diagram and mark all values in p.u. choosing a base of 35MVA, 6.9KV in the generator 1 circuit.

Solution:

Take Base MVA = 35 and Base KV = 6.9

Generator 1:

$X_{G1} = 0.2 \times (35/30) \times (6.9 \times 6.9)^2 = 0.2 \times (35/30) = 0.233$p.u.

For Transformer 1:

$X_{T1} = 0.1 \times (35/35) \times (7.2/6.9)^2 = 0.1088$ p.u.

Transmission Line 1:

Base Voltage with impedance j110 **Ω** = 6.9 × (120/7.2) = 115KV

Now, $Z_{p.u.} = Z_{actual} \times \left\{\dfrac{\text{Base MVA}}{(\text{Base KV})^2}\right\}$

$= 110 \times \{35/(115)^2\} = j0.2911$ p.u.

Transmission Line 2:

Base Voltage with impedance j80 **Ω** = 115KV

Now, Per unit impedance $= Z_{actual} \times \left\{\dfrac{\text{Base MVA}}{(\text{Base KV})^2}\right\}$

$= 80 \times \{35/(115)^2\} = j0.2117$ p.u.

Transformer 2:

$X_{T2} = 0.1 \times (35/20) \times (120/115)^2 = 0.1905$ p.u.

Generator 2:

Base KV = (115 × 7.2)/120 = 6.9KV

$X_{G2} = 0.1 \times (35/20) \times (6.9/6.9)^2 = 0.175$ p.u.

Transformer 3:

Line to Line Voltage when three 1 –∅ Transformers are used as a 3 –∅ transformer is

$(79 \times \sqrt{3})/(7.9 \times \sqrt{3}) = 136.83/13.68$KV

Base MVA in 3 – ∅ = 10 × 3 = 30MVA

$X_{T3} = 0.1 \times (35/10) \times (136.83/115)^2 = 0.4954$ p.u.

Generator 3:

Base KV = (115 × 13.68)/136.83 = 11.49KV

$X_{G3} = 0.1 \times (35/35) \times (13.4/11.49)^2 = 0.1360$ p.u.

Impedance Diagram:

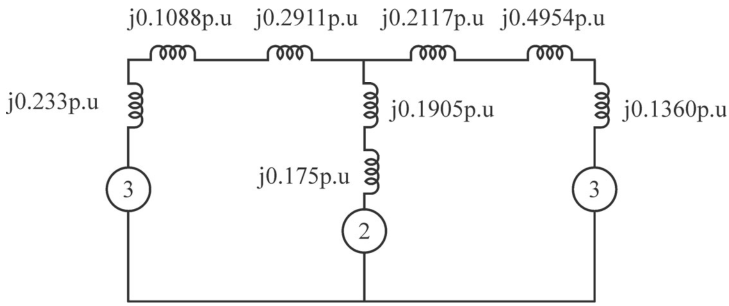

Fig.1.29

Example 18: The one line diagram of an unloaded power systems is shown in Fig.1.30. Reactance's of the two sections of transmission line are shown in the diagram. The generators and transformers are rated as follows:

G_1: 25MVA, 14.8KV, $X'' = 0.2$ p.u.

G_2: 35MVA, 19KV, $X'' = 0.2$ p.u.

G_3: 35MVA, 21KV, $X'' = 0.2$ p.u.

T_1 : 30MVA, 220KV, X = 10%

T_2 : Single Phase units each rated 10MVA, 127/18KV, X = 10%

T_3 : 40MVA, 220Y/21Y KV, X = 10%

Draw the impedance diagram with all reactance's marked in p.u. and with letters to indicate points corresponding to the one-line diagram. Choose a base of 60MVA, 14.8KV in the circuit of generator 1.

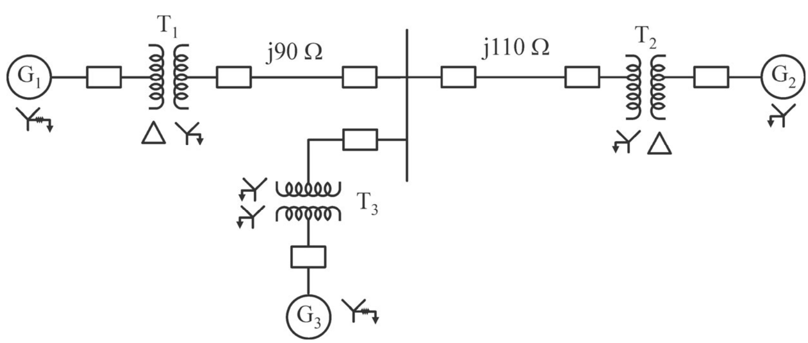

Fig.1.30

Solution:

50MVA, 14.8KV – Base Values

$$G_1:\ X_{G1} = 0.2 \times \left(\frac{50}{25}\right) \times \left(\frac{14.8}{14.8}\right)^2 = 0.4 \text{ p.u.}$$

$$T1: X_{T2} = 0.1 \times \left(\frac{50}{30}\right) \times \left(\frac{14.8}{14.8}\right)^2 = 0.16 \text{ p.u.}$$

TL_1:

$$\text{Base voltage on the transmission line} = 14.8 \times \left(\frac{220}{14.8}\right) = 220\text{KV}$$

$$\text{Per unit impedance} = 90 \times \left(\frac{50}{220 \times 220}\right) = 0.0929 \text{ p.u.}$$

TL_2:

$$\text{Per unit analysis} = 110 \times \left(\frac{50}{220 \times 220}\right) = 0.1136 \text{ p.u.}$$

T_2: Voltage rating of the transformers when they are put to form a 3–∅ transformer

$$= \frac{\sqrt{3} \times 127}{18}\text{kV} = \frac{207.84}{18}\text{kV}$$

MVA given for 3–∅ will be 10 × 3 = 30 MVA

$$X_{T2} = 0.1 \times \left(\frac{50}{30}\right) \times \left(\frac{127 \times \sqrt{3}}{220}\right)^2 = 0.16661 \text{ p.u.}$$

G_2: Base voltage = (220 × 18)/(127 × $\sqrt{3}$) = 19.05 KV

$$X_{G2} = 0.2 \times \left(\frac{50}{35}\right) \times \left(\frac{19}{19.05}\right)^2 = 0.2842 \text{ p.u.}$$

For T_3:

Base Voltage = 220KV

$$X_{T3} = 0.1 \times \left(\frac{50}{40}\right) \times \left(\frac{220}{220}\right)^2 = 0.125 \text{ p.u.}$$

G3: Base Voltage = (220 × 21)/220 = 21KV

$$X_{G3} = 0.2 \times \left(\frac{50}{35}\right) \times \left(\frac{21}{21}\right)^2 = 0.2857 \text{ p.u}$$

Reactance Diagram:

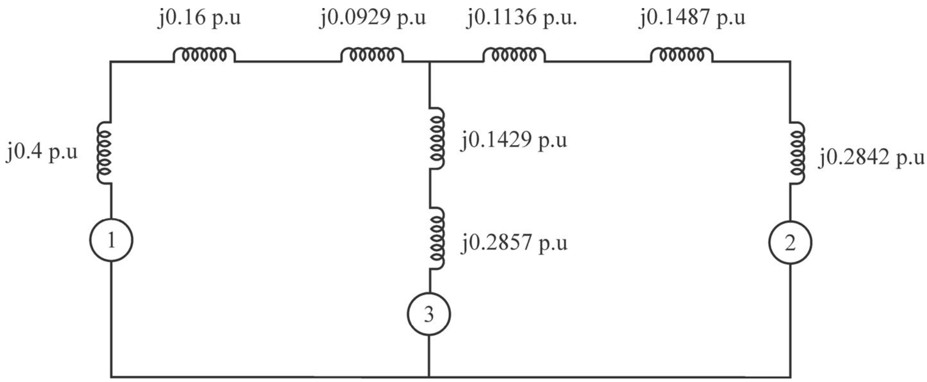

Fig.1.31

Example 19: Two Generators rated at 11.2 MVA, 13 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to two motors of input 10 MVA, and 12 MVA respectively. The Operating Voltage of motors is 11.2KV. Assuming base quantities as 40 MVA and 14.8 KV, Draw the reactance diagram. The percentage Reactance for Generators is 13% and that for motors is 18%.

Solution:

Generator 1:

$Z_{p.u.given} = 0.13$, Base KV_{given} =13KV, Base KV Actual = 14.8 KV,

Base MVA_{Actual} = 40MVA, Base MVA $_{Given}$ = 11.2MVA

$Zp.u._{new} = 0.13 \times (13/14.8)^2 \times 40/11.2 = 0.358218$ p.u.

Generator 2:

$Zp.u._{given} = 0.13$, Base KV $_{given}$ = 13.2 KV, Base KV $_{Actual}$ = 14.8 KV,

Base MVA_{Actual} = 40MVA, Base MVA_{given} = 15MVA

$Zp.u._{new} = 0.13 \times (13.2/14.8)^2 \times 40/15 = 0.275763$ pu

Motor 1:

$Zp.u._{given} = 0.18$, Base KV $_{given}$ = 11.2KV, Base KV_{Actual} = 14.8KV,

Base MVA_{Actual} = 40MVA, Base MVA_{given} = 10MVA

$Zp.u._{new} = 0.18 \times (11.2/14.8)^2 \times 40/10 = 0.41233$ p.u.

Motor 2:

$Zp.u._{given} = 0.18$, Base KV_{given} = 11.2KV, Base KV Actual = 14.8KV, Base MVA $_{Actual}$ = 40 MVA, Base MVA$_{given}$ = 13.5 MVA

Zp.u. new = $0.18 \times (11.2/14.8)^2 \times 40/13.5 = 0.305429$ p.u.

Reactance Diagram:

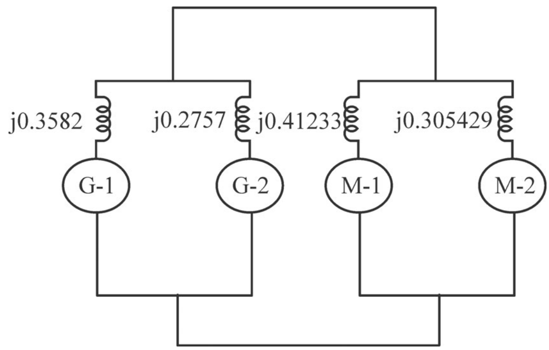

Fig.1.32

Example 20: Draw the Impedance Diagram for the power system shown in fig.1.33 impedance's are in pu. Neglect resistance and use a base of 40 KVA, 120 KV, 40 Ω line. The Ratings of the generators, motors and transformers are

Generator 1: 25 MVA, 15 KV, Reactance % = 20%

Generator 2: 28 MVA, 16 KV, Reactance % = 18%

Synchronous motor 3: 33 MVA, 14.8 KV, Reactance% = 18%

3 phase Y-Y Transformers: 20 MVA, 120Y/15Y KV, Reactance% = 12%

3 phase Y-Δ Transformers: 18 MVA, 120Y/15 ΔKV Reactance% = 12%

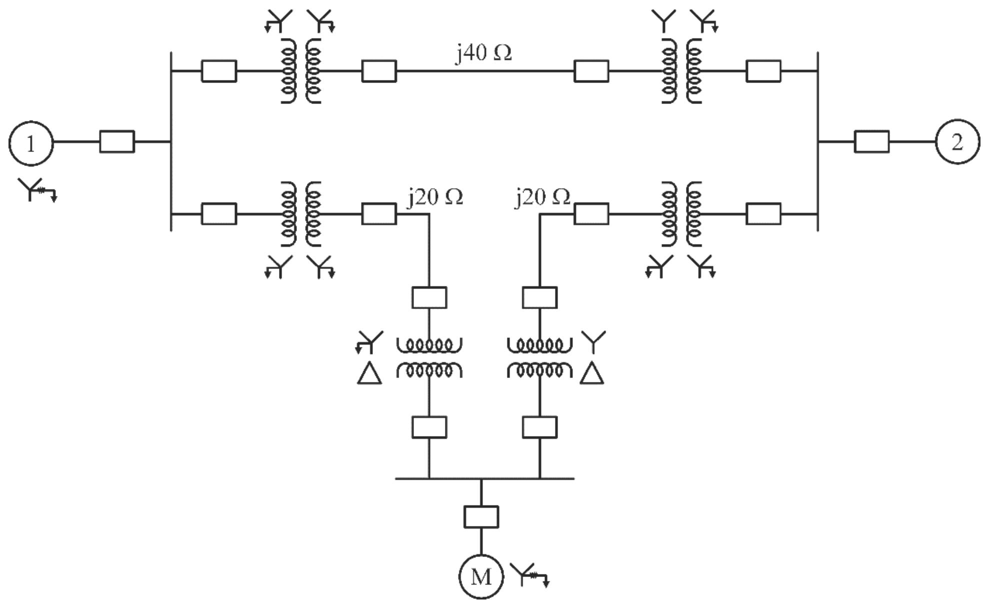

Fig.1.33

Solution:

Generator 1:

$Zp.u._{given} = 0.2$, Base $KV_{given} = 15KV$, Base $KV_{Actual} = 15KV$

Base $MVA_{Actual} = 40$ KVA, Base $MVA_{given} = 25MVA$

$Zp.u._{new} = 0.2 \times (15/15)^2 \times (40/25000) = 0.00032pu$

Transformer 1(Y-Y):

$Zp.u._{given} = 0.12$, Base $KV_{given} = 15$ KV, Base KV $_{Actual} = 15KV$,

Base MVA $_{Actual} = 40$ KVA, Base $MVA_{given} = 20MVA$

$Zp.u._{new} = 0.12 \times (15/15)^2 \times (40/20000) = 0.00024p.u.$

Transmission Lines:

$$\text{Per unit Impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$\text{Base Impedance} = (\text{Base KV})^2/\text{Base MVA}$$

$$= \frac{(120)^2 \times 1000}{40}$$

$$= 360000$$

Per unit Impedance = 40/360000 = 0.000111 p.u.

Per unit Impedance on Other Transmission Line = 20/360000 = 0.000055556 p.u.

Generator 2:

$Zp.u._{given}$ = 0.18, Base KV_{given} = 16 KV, Base KV_{Actual} = 15KV,

Base MVA_{Actual} = 40KVA, Base MVA_{given} = 28MVA

$Zp.u._{new} = 0.18 \times (16/15)^2 \times (40/28000)$

$= 0.00029257$ p.u.

Transformer 2: (Y – Δ)

$Zp.u._{given}$ = 0.12, Base KV_{given} = 15KV, Base KV_{Actual} = 15KV,

Base MVA $_{Actual}$ = 40 KVA, Base MVA_{given} = 18MVA

$Zp.u._{new} = 0.12 \times (15/15)^2 \times (40/18000) = 0.000266$ p.u.

Synchronous Motor 3:

$Zp.u._{given}$ = 0.18, Base KV_{given} = 14.8 KV, Base KV_{Actual} = 15KV,

Base MVA $_{Actual}$ = 40KVA, Base MVA_{given} = 33MVA

$Zp.u._{new} = 0.18 \times (14.8/15)^2 \times (40/33000) = 0.00021204$pu

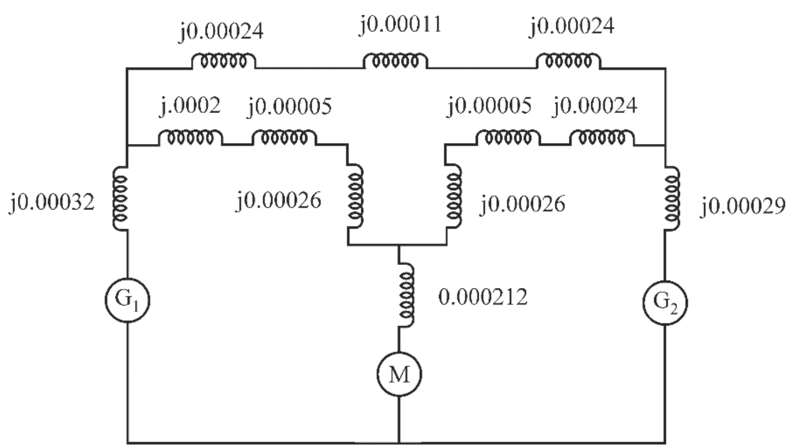

Fig.1.34

Example 21: A transformer having single phase is rated as 110/440V, 3KVA. L.T. side of the transformer has a leakage reactance of 0.05Ω. Calculate reactance in p.u.

Solution:

Actual leakage reactance = 0.05Ω

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$\text{Base impedance in L.T. side} = \frac{\left(110\times10^{-3}\right)^2\times1000}{3}$$

$$= 4.033\ \Omega$$

$$\text{Per unit impedance of the circuit element} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$\text{Per unit impedance} = \frac{0.05}{4.033} = 0.01239\text{p.u.}$$

Example 22: Below diagram shows a two machine system. The ratings are as follows:

Synchronous generator: 25MVA, 11KV, $X'' = 0.17$p.u.

Synchronous motors: 20MVA, 11KV, $X'' = 0.17$p.u.

Transformer T1: 30MVA, 12.5Δ/132YKV, X = 0.12p.u.

Transformer T2: 25MVA, 132Y/11ΔKV, X = 0.12p.u.

Line: 300 + j400

Static load: 4MVA, 0.7 Power factor lagging.

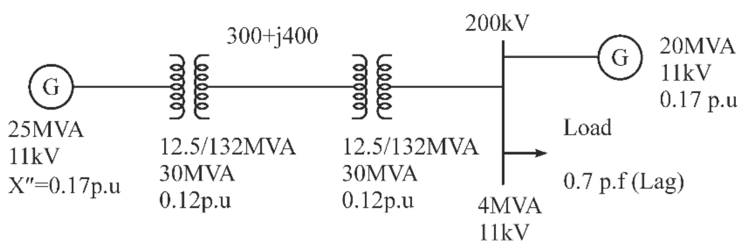

Fig.1.35

Draw the impedance diagram for the system. Choose base voltage of 132KV for the transmission line and a base volt ampere of 30MVA.

Solution:

Generator:

$$\text{Base voltage at Generator side} = \frac{125}{132}\times132 = 12.5\text{KV}$$

$$X'' = j0.17\times\frac{30}{25}\times\left(\frac{11}{12.5}\right)^2 = j0.157\text{p.u.}$$

Transformer 1:

Here we are taking base quantities at H.T. side of transformer.

$$X = j0.12 \times \frac{30}{30} \times \left(\frac{132}{132}\right)^2$$

$$= j0.12 p.u.$$

Transformer 2:

$$X'' = j0.12 \times \frac{30}{25} \times \left(\frac{132}{132}\right)^2$$

$$= j0.144 p.u.$$

Motor:

Base voltage at motor side = $\frac{11}{132} \times 132 = 11$ KV

$$X'' = j0.17 \times \frac{30}{20} \times \left(\frac{11}{11}\right)^2$$

$$= j0.255 p.u.$$

Transmission line:

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}} = \frac{(132)^2}{30} = 580.8\Omega$$

$$Z_{p.u.} = \frac{\text{Actual impedance}}{\text{Base impedance}} = \frac{300 + j40}{580.8} = 0.5165 + j0.688$$

For a load of 5 MVA, 0.7 power factor lagging.

$$\cos\Phi = 0.7$$

$$\Phi = \cos^{-1}(0.7) = 45.57^\circ$$

$$P = S\cos\Phi = 4 \times 0.7 = 2.8MW$$

$$Q = \sin\Phi = 4 \times \sin(45.57^\circ) = 2.86MVAr$$

If load is represented as a series impedance,

$$R_{p.u.} = V_{p.u.}^2 \times S_B \times \frac{P}{P^2 + Q^2}$$

$$X_{p.u.} = V_{p.u.}^2 \times S_B \times \frac{Q}{P^2 + Q^2}$$

Here $V_{p.u.} = \frac{11}{11} = $ p.u.

S_b = 30 MVA

$$R_{p.u.} = 1^2 \times 30 \times \frac{2.8}{2.8^2 + 2.86^2} = 5.24 \text{p.u.}$$

$$X_{p.u.} = 1^2 \times 30 \times \frac{2.8}{2.8 + 2.86^2} = j5.35 \text{p.u.}$$

Per unit impedance Diagram:

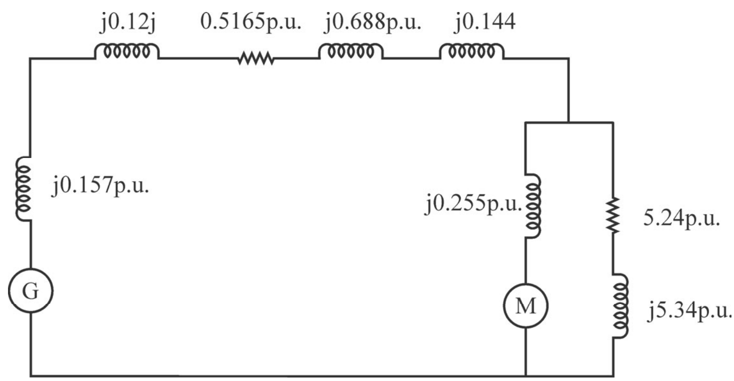

Fig.1.36

Example 23: Ratings of 3 generators are given below

G_1: 75MVA, 30KV, 9% Reactance

G_2: 125MVA, 31KV, 10% Reactance

G_3: 100MVA, 32KV, 7% Reactance

Find generator's new reactance with base values 150MVA and 40KV.

Solution:

$$Z_{\text{p.u. new}} = Z_{\text{p.u. old}} \times \left(\frac{\text{Base KV}_{\text{given}}}{\text{Base KV}_{\text{Actual}}}\right)^2 \times \left(\frac{\text{Base MVA}_{\text{Actual}}}{\text{Base MVA}_{\text{given}}}\right) \times \left(\frac{\text{Base MVA}_{\text{Actual}}}{\text{Base MVA}_{\text{given}}}\right)$$

G_1: $X_{G1} = 0.09 \times (30/40)^2 \times (150/75) = 0.1012$ p.u.

G_2: $X_{G2} = 0.1 \times (31/40)^2 \times (150/125) = 0\ .0721$ p.u.

G_3: $X_{G3} = 0.07 \times (32/40)^2 \times (150/100) = 0.0672$ p.u.

Example 24: Determine the generator voltage for given figure:

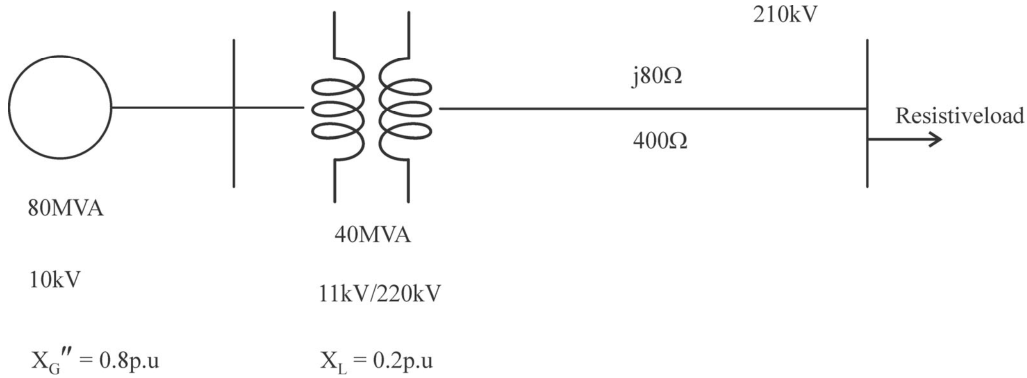

Fig.1.37

Solution: Assume 80MVA as Base power and 210KV as Base voltage in transmission line.

Generator:

Base KV_{new} towards generator side = Transformer ratio× Base KV towards transmission line

Base KV_{new} = (11/220) × 210 =10.5

$$X_G'' = 0.8 \times (10/10.5)^2 \times (80/80)$$

$$X_G'' = 0.7256 \text{ p.u.}$$

Transformer:

$$X_T = 0.2 \times (220/210)^2 \times (80/40) = 0.4390 \text{ p.u.}$$

Transmission line: Base impedance = $\dfrac{(\text{Base KV})^2}{\text{Base MVA}} = \dfrac{210^2}{80}$

$$Z_{Base} = 551.25\ \Omega$$

Per unit impedance = Actual impedance/Base impedance

= 80/551.25 = 0 .1451 p.u.

Resistive load 400Ω:

$$R_{p.u.} = 400/551.25 = 0.7256 \text{ p.u.}$$

Impedance diagram:

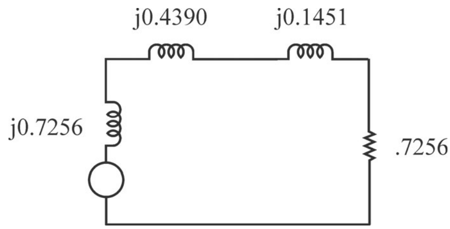

Fig.1.38

$V_{p.u.}$ = 210/210 = 1 p.u.

$R_{p.u.}$ = 0.7256 p.u.

Now, $I_{p.u.} = V_{p.u}/R_{p.u.}$ = 1/0.7256 = 1.3781 p.u.

Let, $I_{p.u.}$ is reference phasor

$I_{p.u.}$ = 1.3781 + j0 = 1.3781 with 0^0 angle

According to KVL, the V_{drop} in network = $I_{p.u.}\{R_{p.u.} + j\,(X_G + X_T + X_{TL})\}$

= 1.3781{0.7256 + j (0.7256 + 0.4390 + 0.1451)}

= 0.9999 + j1.8049

= 2.0634 p.u.

Actual generator voltage:

$V_G = V_{p.u.} \times$ Base Voltage

V_G = 2.0634 × 10.5 = 21.6657 KV (line to line)

Example 25: The one line diagram of an unloaded power system is shown in Fig.1.39. Reactance of the two section of transmission line are shown in the diagram. The generators and transformers ratings are as follows:

Generator 1: 30MVA, 13.8KV, $X'' = 0.1$pu

Generator 2: 20MVA, 18KV, $X'' = 0.1$pu

Generator 3: 30MVA, 20KV, $X'' = 0.1$p.u.

Transformer 1: 30MVA, 13.8/200 KV, $X'' = 0.2$p.u.

Transformer 2: Single phase units each rated 50MVA, 115.4/18KV, $X'' = 0.2$p.u.

Transformer 3: 40MVA, 200Y/22YKV, $X'' = 0.2$p.u.

Draw the impedance diagram with all reactance marked in p.u. and with letters to indicate points corresponding to one-line diagram. Choose a base of 60MVA, 13.8KV in the circuit of generator 1.

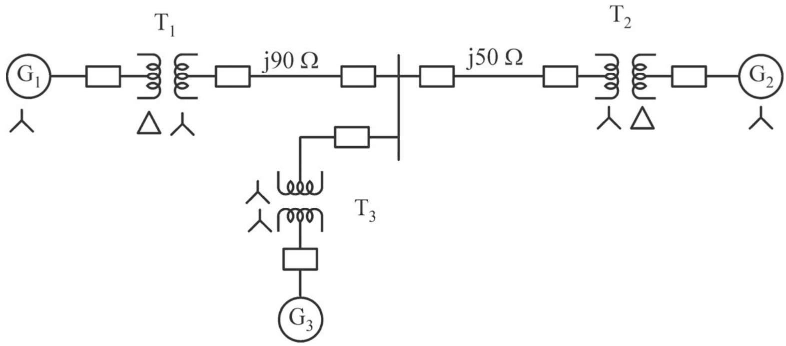

Fig.1.39

Solution:

Generator 1: 30MVA, 13.8KV, $X'' = 0.1$pu

Base MVA = 60MVA, Base KV = 13.8KV

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{\text{Base KV}_{given}}{\text{Base KV}_{new}}\right)^2 \times \left(\frac{\text{Base KMVA}_{new}}{\text{BaseMVA}_{given}}\right)$$

$$X_{G1} = 0.1 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{60}{30}\right)$$

$$= 0.2 \text{ p.u.}$$

Transmission Line 1:

Base voltage on the transmission line using the transformation ratio = 200 KV.

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$\text{Base impedance} = \frac{(200)^2}{60} = 666.67\Omega$$

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$= \frac{90}{666.67} = 0.134\text{p.u}$$

Transmission Line 2:

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$= \frac{50}{666.67} = 0.0749 \text{ p.u}$$

Transformer 1: Base MVA = 60 MVA, Base KV = 13.8 KV

Base MVA given = 30, Base KV_{given} 13.8KV, $X'' = 0.2$ p.u

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{(\text{Base KV}_{given})}{\text{Base KV}_{new}}\right)^2 \times \left(\frac{\text{Base MVA}_{new}}{\text{Base MVA}_{given}}\right)$$

$$X_{T1} = 0.2 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{60}{30}\right) = 0.4 \text{ p.u.}$$

Transformer 3:

Base MVA = 60 MVA, Base KV = $200 \times \left(\frac{22}{200}\right) = 22$ KV

Given: 40 MVA, 220Y/22Y KV, $X'' = 0.2$p.u

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{(\text{BaseKVgiven})}{\text{BaseKVnew}}\right)^2 \times \left(\frac{\text{BaseMVAnew}}{\text{BaseMVAgiven}}\right)$$

$$X_{T3} = 0.2 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{60}{40}\right)$$

$$= 0.3 \text{ p.u.}$$

Transformer 2:

Voltage rating of transformer when they are put to form 3-Φ transformer,

$$= \frac{\sqrt{3}}{18} \times 115.4 \text{ KV} = \frac{200}{18} \text{KV}$$

$$\text{Base KV} = \frac{18}{200} \times 200 = 18\text{KV}$$

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{(\text{BaseKVgiven})}{\text{BaseKVnew}}\right)^2 \times \left(\frac{\text{BaseMVAnew}}{\text{BaseMVAgiven}}\right)$$

$$X_{T2} = 0.2 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{60}{50}\right) = 0.24 \text{ p.u.}$$

Generator 2:

Given: 20MVA, 18KV, $X'' = 0.1$p.u

Base MVA = 60 MVA, Base KV = 18 KV

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{(\text{BaseKVgiven})}{\text{BaseKVnew}}\right)^2 \times \left(\frac{\text{BaseMVAnew}}{\text{BaseMVAgiven}}\right)$$

$$X_{G2} = 0.1 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{60}{20}\right) = 0.3 \text{ p.u}$$

Generator 3:

Given: 20 MVA, 20KV, $X'' = 0.1$p.u.

Base MVA = 60 MVA, Base KV = 22 KV

$$Z_{p.u\ new} = Z_{p.u\ given} \times \left(\frac{(\text{BaseKVgiven})}{\text{BaseKVnew}}\right)^2 \times \left(\frac{\text{BaseMVAnew}}{\text{BaseMVAgiven}}\right)$$

$$X_{G3} = 0.1 \times \left(\frac{20}{22}\right)^2 \times \left(\frac{60}{20}\right) = 0.247 \text{p.u}$$

Impedance Diagram:

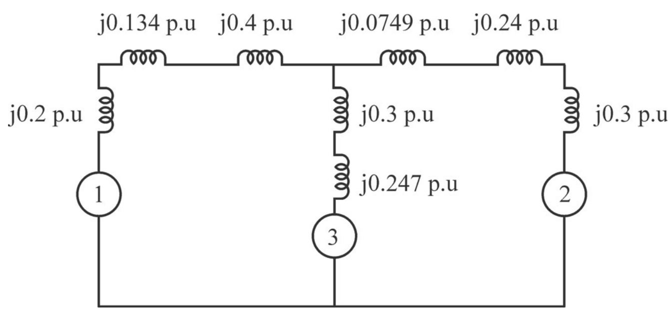

Fig.1.40

Impedance Diagram:

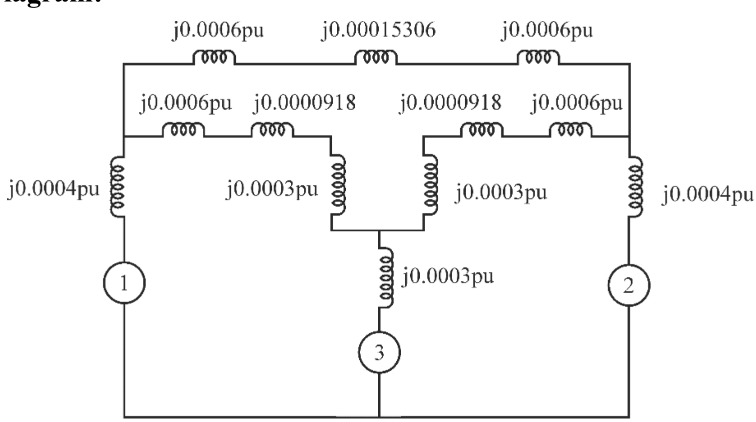

Fig. 1.41

Example 26: A 3 bus system is given in fig.1.41. The ratings of various components are listed below.

Generator 1: 70 MVA, 15.8 KV, X″ = 017 p.u.

Generator 2: 60 MVA, 15.2 KV, X″ = 0.4 p.u.

Generator 3: 50 MVA, 13 KV, X″ = 0.35 p.u.

Transformer 1: 55MVA, 13KV/120KV Y/Y, X = 0.2 p.u.

Transformer 2: 35 MVA, 14.5 KV/125 KV Y/Y, X = 0.16 p.u.

Transformer 3: 50 MVA, 14.5 KV/125 KV Y/Y, X = 0.2 p.u.

The line impedance are shown in figure. Determine the reactance base on 70 MVA and 15.8 KV as base quantities in generator 1.

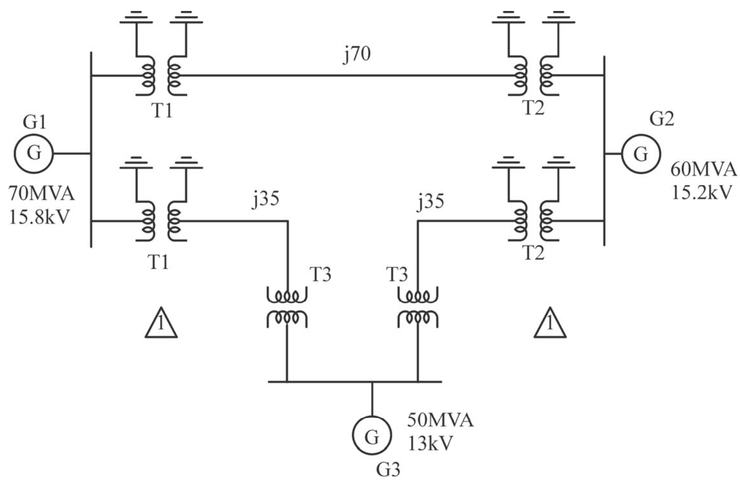

Fig.1.42

Solution: Base MVA = 70, Base KV = 15.8

Generator 1:
Per unit reactance = $0.17 \times (15.8/15.8)^2 \times 70/70$

$= 0.17$ p.u.

Generator 2:

Base voltage = $145.8 \times 14.5/125 = 16.9$ KV
60 MVA, 15.2 KV, X″ = 0.4 p.u.

Base value 70 MVA, 16.9 KV

$X_{G2} = 0.4 \times (15.2/16.9)^2 \times 70/60$

$= 0.3774$ p.u.

Generator 3: Base voltage on the bus = 145.8 × 14.5/125 = 16.9 kV

50MVA, 13KV, $X'' = 0.35$ p.u.

Base value 70MVA, 16.9 KV

$X_{G3} = 0.35 \times (13/16.9)^2 \times 70/50 = 0.2899$ p.u.

Transformer 1: 55 MVA, 13 KV, X = 0.2 p.u

Base value 70 MVA and 15.8 KV

$X_{T1} = 0.1 \times (13/15.8)^2 \times 70/55$

$= 0.0861$ p.u

Transformer 2: 35 MVA, 14.5 KV, X = 0.16 p.u.

Base value = 145.8 x 14.5/125 = 16.9 KV

$X_{T2} = 0.16 \times (14.5/16.9)^2 \times 70/35$

$= 0.2355$ p.u

Transformer 3: 50MVA, 12.5 KV, X = 0.2 p.u.

Base value = 16.9 × 125/14.5

= 145 KV

$X_{T3} = 0.2 \times (125/145)^2 \times 70/50$

$= 0.2080$ p.u.

Transmission line 1: base voltage along the transmission line whose impedance is j70Ω = 15.8 × 120/13

= 145.8 KV

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$= 145^2/70 \quad = 303.68\ \Omega$$

$$\text{Per unit impedance} = \frac{\text{Actua lim pedance}}{\text{Base impedance}}$$

$$= 70/303.68 = 0.2305 \text{ p.u}$$

Transmission line 2: Base KV = 145.8, Base MVA = 70

Base impedance = j 37.5/303.68

= j 0.1152 p.u

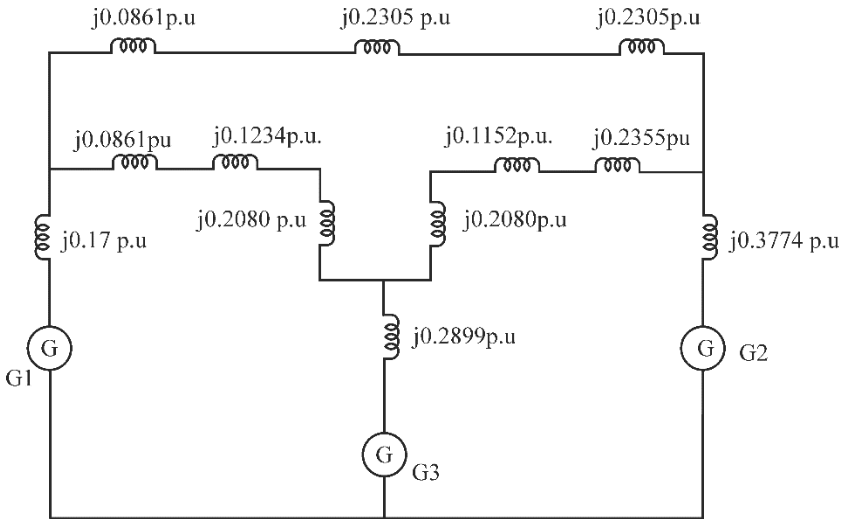

Fig.1.43

Example 27: A 25 MVA, 9.5 KV, 3-Φ generator has subtransient reactance of 20%. It is connected through a Δ-Y transformer to a high voltage transmission line having a total series reactance of 70 Ω. At the load end of the line is a Y-Y step down transformer. Both transformer banks are composed of 1-Φ transformers connected for 3–Φ operation. Each of the two transformers comprising each bank is rated 7682 KVA, 10/100 KV with reactance of 10%. The load represented as impedance is drawing 12 MVA at 14.5 KV at 80% power factor lagging. Draw the impedance diagram showing all impedance in per unit. Choose a base of 12 MVA, 14.5 KV in the load circuit.

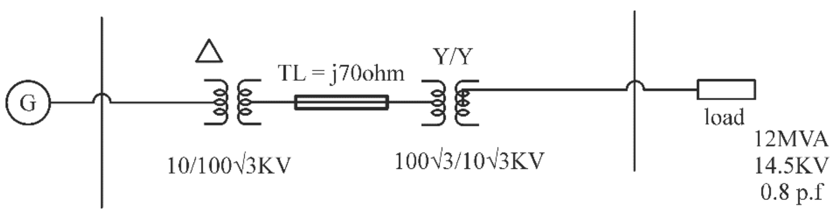

Fig.1.44

Solution:

Load:

MVA = 12, KV = 14.5

Load impedance = 0.8 + j o.6 p.u.

Total MVA rating of transformer 1 and 2 separately = 3 × 7628/1000

= 23 MVA

Base KV on the transmission line side = $14.5 \times \dfrac{100\sqrt{3}}{10\sqrt{3}}$

= 145 KV

Base impedance = $(\text{Base KV})^2$/Base MVA

Base impedance in the transmission line = $145^2/12$

= 1752.08 Ω

Per unit impedance = Actual impedance/Base impedance

= 70/1752.08 = 0.0399 p.u.

Transformer 1: Base KV on the generator side is given by

= $145 \times 10/100\sqrt{3}$ = 8.3 KV

$X_{T1} = 0.1 \times (10/8.3)^2 \times (12/23)$

= 0.075 p.u

Generator 1: given 25 MVA, 9.5KV, $X'' = 0.2$ p.u.

Base value = 12MVA, 8.3 KV

$X'' = 0.2 \times (9.5/8.3)^2 \times (12/25)$

= j 0.1257 p.u

Transformer 2: given 23MVA, $100\sqrt{3}$ KV, $X'' = 0.1$ p.u.

Base value = 12 MVA, 145KV

$X_{T2} = 0.1 \times (100\sqrt{3}/145)^2 \times 12/23$

= 0.0743 p.u

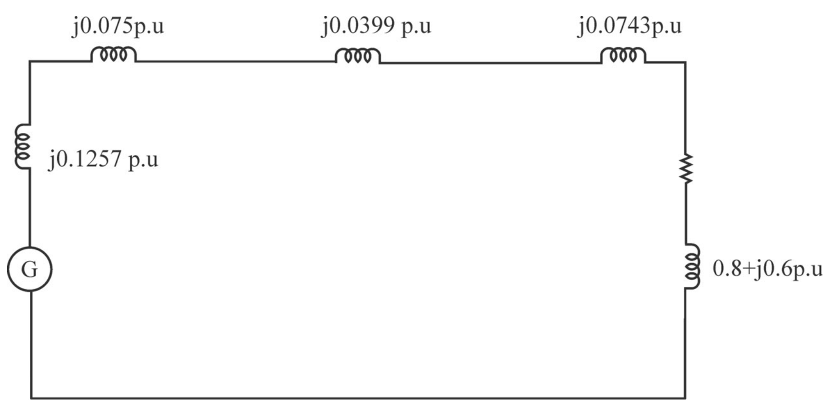

Fig.1.45

Example 28: A 200 MVA,44KV, 3-ϕ generator has a subtransient reactance of 25%. The generator is connected to three motors through a transmission line and two Transformers. Each motor have rated 40MVA with 30% subtransient reactance. The 3-ϕ transformers are rated at 110MVA,32KV/110KV with leakage reactance 9%. The line has a reactance of 50 ohms. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in other parts of the system and evaluate the corresponding p.u. values.

Solution:

The base voltage in the transmission line = 44 × 110/32 = 151.25KV

In the motor circuit, the base voltage = 151.25 × 32/110 = 44KV

Base impedance in the transmission line = $(\text{Base KV})^2/\text{base MVA}$

$$= (151.25)^2/200 = 114.382\ \Omega$$

p.u. impedance = j50/j114.382 = 0.4371 p.u.

Motor 1:

40 MVA, 44KV, $X'' = 0.3$

$$X_1'' = 0.3 \times (40/44)^2 \times 200/40 = 0.4371 \text{ p.u.}$$

Motor 2:

30MVA, 40KV, $X'' = 0.3$

$$X_2'' = 0.3 \times (40/44)^2 \times 200/30 = 1.6528 \text{ p.u.}$$

Motor 3:

60MVA, 40KV, $X'' = 0.3$

$$X_3'' = 0.3 \times (40/44)^2 \times 200/60 = 0.8264 \text{ p.u.}$$

Transformer:

110MVA, 32KV, $X'' = 0.09$

$$X_T'' = 0.09 \times (32/44)^2 \times 200/110$$

$$= 0.0865 \text{ p.u.}$$

Example 29: Fig.1.45 shows a two machine system. The ratings are as follows:

Synchronous generator: 30MVA, 11KV, $X'' = 0.25$ p.u.

Synchronous motor: 25MVA, 11KV, $X'' = 0.25$ p.u.

Transformer T_1 = 35MVA, 13.5Δ/132Y KV, X = 0.2 p.u

Transformer T_2 = 30MVA, 132Y/11ΔKV, X = 0.2 p.u

Line: 100 + j400Ω

Static load: 10MVA, 0.75 power factor lagging.

Draw the impedance diagram for the system. Choose a base voltage of 132KV for the transmission line and a base volt ampere of 30MVA.

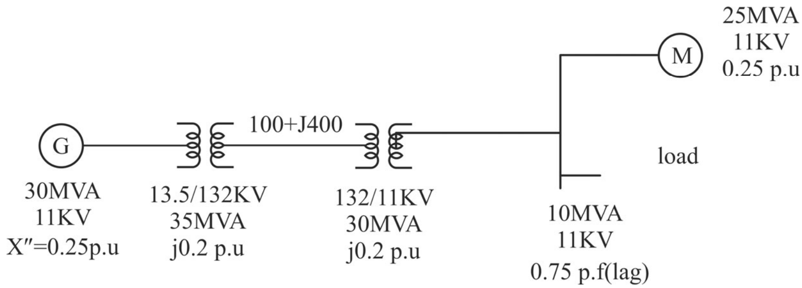

Fig.1.46

Solution:

Generator:

$$X'' = j0.25 \times 30/30 \times (11/13.5)^2 = j0.1659 \text{ p.u.}$$

Transformer 1:

$$X = j0.2 \times 30/35 \times 132/132 = j0.1714 \text{ p.u.}$$

Transformer 2:

$$X'' = j0.2 \times 30/30 \times (132/132)^2 = j0.2 \text{ p.u.}$$

Motor:

$$X'' = 0.25 \times 30/25 \times (11/11)^2 = 0.3 \text{ p.u.}$$

Transmission line:

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$$= 132^2/30 = 580.8\Omega$$

$$Z_{p.u.} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$= 100 + j400/580.8$$

$$= 0.172 + j0.688$$

For a load of 10MVA, 0.75 power factor lagging,

$$P = S\cos\phi = 10 \times 0.75 = 7.5\text{MW}.$$

$$Q = S\sin\phi = 10 \times 0.66 = 6.6\text{MVAr}$$

If load is represented as a series impedance,

$$R_{p.u} = \frac{V_{p.u}^{\ 2}.S_b.P}{P^2 + Q^2} \text{ and } X_{p.u} = \frac{V_{p.u}^{\ 2}.S_b.Q}{P^2 + Q^2}$$

Here, $V_{p.u.} = 11/11 = 1$p.u

$S_b = 30$MVA

Therefore, $R_{p.u} = \frac{(1.0)^2 \times 30 \times 7.5}{(7.5)^2 + (6.6)^2} = 2.25$ p.u.

$$X_{p.u} = \frac{(1.0)^2 \times 30 \times 6.6}{(7.5)^2 + (6.6)^2} = 1.983 \text{ p.u.}$$

P.U. impedance diagram:

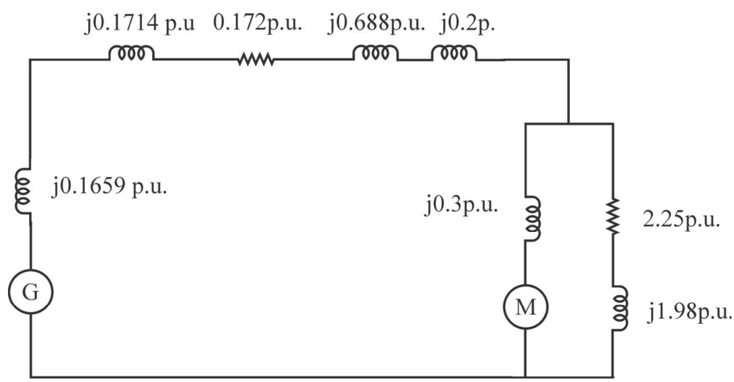

Fig.1.47

Example 30: Three generators are rated as follows

Generator 1: 120MVA, 32KV, reactance 9%.

Generator 2: 140MVA, 30KV, reactance 7%.

Generator 3: 160MVA, 29KV, reactance 10%.

Determine the reactance of the generators corresponding to base values of 250MVA and 40KV.

Solution:

Generator 1:

120MVA, 32KV, $X'' = 0.09$

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{\text{Base KVgiven}}{\text{Base KVnew}}\right)^2 \times \left(\frac{\text{Base KMVAnew}}{\text{Base MVAgiven}}\right)$$

$$X'' = 0.09 \times (32/40)^2 \times 250/120$$

$= 0.09 \times 0.64 \times 2.083$

$= 0.1199$ **p.u.**

Generator 2:

140MVA, 30kV, $X'' = 0.07$

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{\text{Base KVgiven}}{\text{Base KVnew}}\right)^2 \times \left(\frac{\text{Base KMVAnew}}{\text{Base MVAgiven}}\right)$$

$X'' = 0.07 \times (30/40)^2 \times 250/140$

$= 0.07 \times 0.5625 \times 1.785$

$= 0.0702$ p.u.

Generator 3:

160MVA, 29KV, $X'' = 0.1$

$$Z_{p.u.new} = Z_{p.u.given} \times \left(\frac{\text{Base KVgiven}}{\text{Base KVnew}}\right)^2 \times \left(\frac{\text{Base KMVAnew}}{\text{Base MVAgiven}}\right)$$

$X'' = 0.1 \times (29/40)^2 \times 250/160$

$= 0.1 \times 0.5256 \times 1.5625$

$= 0.0821_{p.u.}$

Example 31: Determine the generator voltage for the network as shown in the figure below.

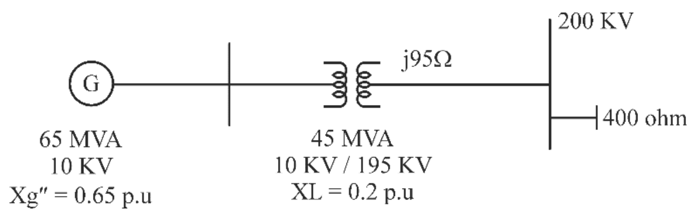

Fig.1.48

Solution:

Let us assume a Base value of the transmission line is 95MVA & 200KV.

Generator: Base voltage of the Generator: (10/195) × 200 kV = 10.2564kV

$$Z_{p.u.new}=Z_{p.u.given} \times \left(\frac{\text{Base KVgiven}}{\text{Base KVnew}}\right)^2 \times \left(\frac{\text{Base MVAnew}}{\text{Base MVAgiven}}\right)$$

$X''_G = 0.65 \times 95/65 \times (10/10.2564)^2$

$= 0.65 \times 1.4615 \times 0.9506$

$= 0.90304$p.u.

Transformer:

$X_T = 0.2 \times 95/45 \times (195/200)^2$

$= 0.2 \times 2.111 \times 0.9506$

$= 0.4013$ p.u.

Transmission Line:

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

$= (200)^2/95$

$= 40000/95$

$= 421.05\ \Omega$

$$\text{Per unit impedance} = \frac{\text{Actual limpedance}}{\text{Base impedance}}$$

$= 95/421.05$

$= 0.225$p.u.

Resistive load of 400 Ω:

$R_{P.U} = 400/421.05$

$= 0.95$p.u.

Impedance diagram:

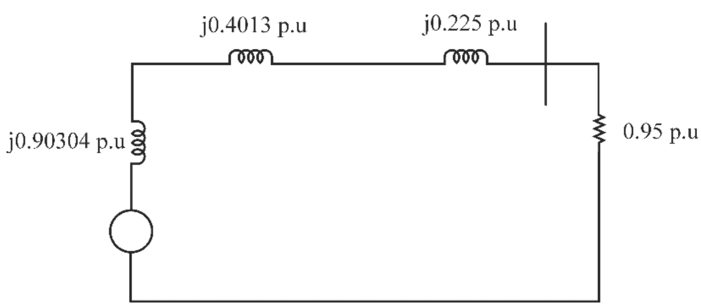

Fig.1.49

At the load bus,

$V_{p.u} = 1$p.u.

$R_{p.u.} = 0.95$p.u.

Therefore,

$I_{p.u.} = V_{p.u.}/R_{p.u.}$

$=1/0.95$

$=1.052$p.u.

Let us take $I_{p.u.}$ as the reference phasor.

$I_{p.u}=1.052p.u+j0$

$=1.052$ with angle 0^0

Applying KVL, the voltage drop in the network

$= I_{p.u}[R_{p.u} + j(X_G + X_t + X_L)]$

$= 1.052[0.95 + j(0.90304 + 0.4013 + 0.225)]$

$=1.052[0.95 + j1.52934]$

$= 1.052 \times 1.8003$

$= 1.8939$p.u.

Actual generator terminal voltage,

$V_G = V_{p.u} \times$ Base voltage at the generator terminals

$= 1.8939 \times 10.2564$

$= 19.425$ kV (Line to Line)

Example 32: A 3 bus system is given in figure. The ratings of the various components are listed below:

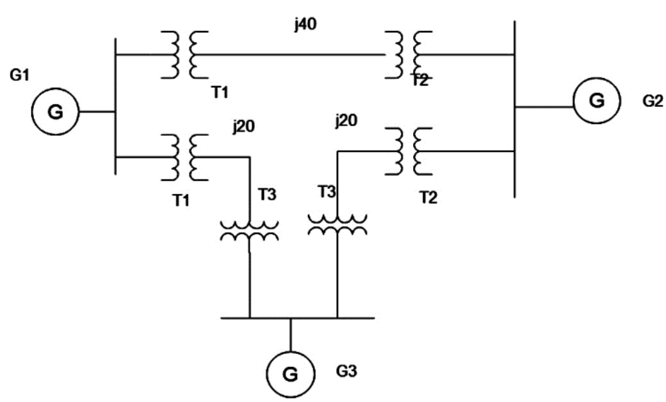

Fig.1.50

Generator 1: 40MVA, 12.8kV, X" = $0.2_{p.u}$

Generator 2: 50MVA, 12.2Kv, X" = $0.15_{p.u}$

Generator 3: 20MVA, 12Kv, X" = $0.20_{p.u}$

Transformer 1: 40MVA, 12kV/100kV Y/Y,X = $0.1_{p.u}$

Transformer 2: 20MVA, 13.5kV/125kV Y/Y,X = $0.20_{p.u}$

Transformer 3: 35NVA, 10.5Kv/115kV Y/Y,X = $0.2_{p.u}$

The line impedances are shown in figure 50. Determine the reactance diagram based on 40MVA and 12.8kV as base quantities in generator 1.

Solution:

Base MVA=40; BasekV =12.8

Generator 1:

$Z_{p.u.new} = Z_{p.u.given}(\text{Base kV}_{given}/\text{Base kV}_{new})^2 \text{x Base MVA}_{new}/\text{Base MVA}_{given}$

Per Unit Reactance = $0.2 \times (12.8/12.8)^2 \times (40/40)$

$= 0.0156_{p.u}$

Transmission Line 1:

Base voltage along the transmission line whose impedance is j40Ω = 12.8 × 100/12 =106kV

Base impedance = $(\text{Base kV})^2/\text{Base MVA}$

Base impedance = $106^2/40 = 280.9\Omega$

Per unit impedance = $40/280.9 = 0\ .\ 1423_{p.u}$

Generator 2:

Base voltage =106 × 12.5/10.5 = 12 kV

$X_{G2} = 0.15 \times (12.2/12)^2 \times 40/50$

$= 4.4652$ p.u

Transformer 1:

40MVA, 12Kv, $X = 0.1_{p.u}$

Base values; 40MVA, 12.8Kv

$X_{T2} = 0.1 \times (12/12.8)^2 \times 40/40$

$= 0.0878_{p.u}$

Transformer 2:

20MVA, 11.5Kv, $X = 0.20_{p.u}$

Base Voltage = 106 × 13.5/125 = 11kv

$X_{T2} = 0.1 \text{ X } (11.5/11)^2 \times 40/20$

$= 0.2185_{p.u}$

Transformer 3:

Base Voltage on the H.T side of transformer 3 is 106kv

$X_{T3} = 0.2 \times (115/108)^2 \times 40/35$

$= 0.25916$ P.U

Generator 3:

Base voltage on the bus = 106 × 13.5/125 = 11kv

20 MVA, 12 kV, X" = 0.20p.u

Base value: 40MVA, 11kv

X_{G3} = 0,20 × $(12/11)^2$ × 40/20

= 0.476 P.U

Transmission line 2:

Base kV =106

Base MVA = 40

Base impedance = $106^2/40$ = 280.9

Per unit impedance = j20/280.9 = j0.0711

Reactance Diagram:

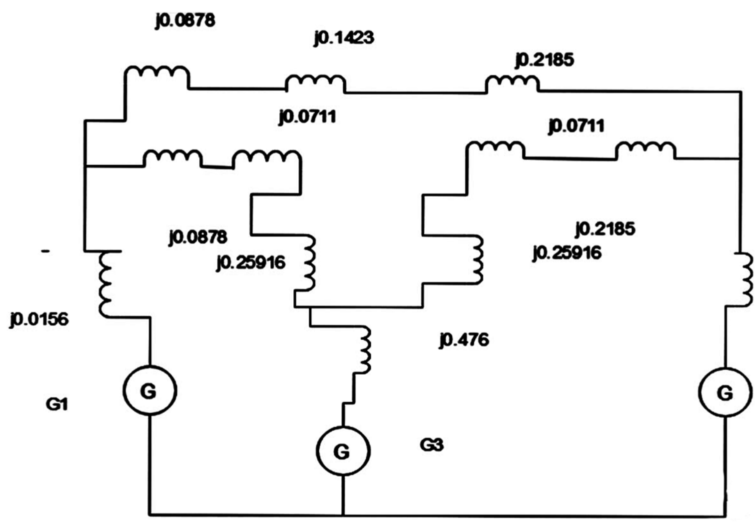

Fig.1.51

Example 33: Draw the impedance for the power system shown in fig. 1.49. Impedances are in p.u. Neglect resistances. Use a base of 45kVA, 148 KV, 20Ω line. The ratings of the generators, motors and transformers are:

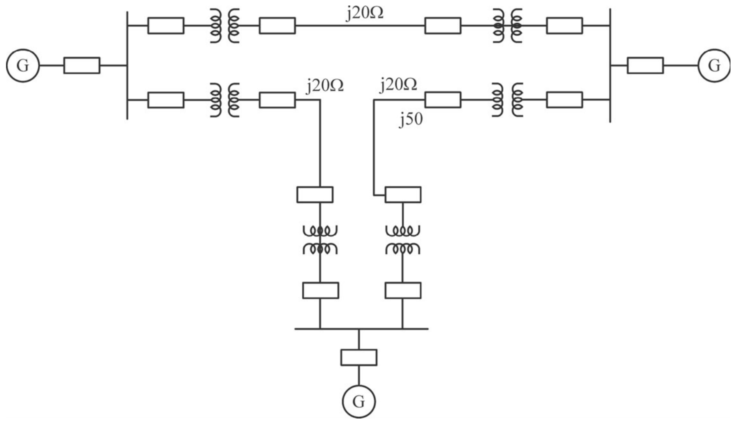

Fig. 1.52

Generator 1: 25MVA, 18KV, $X'' = 20\%$

Generator 2: 25MVA, 18KV, $X'' = 20\%$

Synchronous motor 3; 35 MVA, 12.8KV, $X'' = 20\%$

Three phase Y-Y Transformers: 10MVA, 128Y/20YKV, X = 10%

Three phase Y-Δ Transformers: 10MVA, 128Y/12.8ΔKV, X = 10%

Solution:

Generator 1:

Given: 25MVA, 18 KV, $X'' = 20\%$

Base values: 45 KVA

$$Z_{p.unew} = Z_{p.ugiven} \times \left(\frac{\text{Base KVgiven}}{\text{Base KVnew}}\right)^2 \times \left(\frac{\text{Base KMVAnew}}{\text{Base MVAgiven}}\right)$$

$X_{G1} = 0.2 \times (18/20)^2 \times (45/20 \times 10^3)$

$= 0.000364$ P.U

Transformer 1:

Given: 25MVA, 20KV, X = 10%

Base values; 45KVA, 20KV

$X_{T1} = 0.1 \times (45/20 \times 10^3) = 0.00025$ P.U

Transmission Lines:

$$\text{Base impedance} = \frac{(\text{Base KV})^2}{\text{Base MVA}}$$

Base impedance = $148^2/40 \times 1000$

= 547600Ω

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

p.u impedance = 0.0000821 P.U

p.u impedance of the other transmission line = 20/547600

= 0.0000807 P.U

Generator 2:

$X_{G2} = X_{G1} = 0.0003645$ P.U

Y-Δ transformer

Given: 10MVA, 128Y/12.8 ΔKV, X = 10%

Base values; 45kVA, 128KV

$X = (0.1 \times 45/10 \times 10^3)$

= 0.000045 P.U

Motor:

Given: 35MVA, 12.8 KV, $X'' = 20\%$

Base values: 45 KVA, 128KV

$X_m = 0.2 \times 45/35 \times 10^3$

= 0.0000257 P.U

Impedance diagram

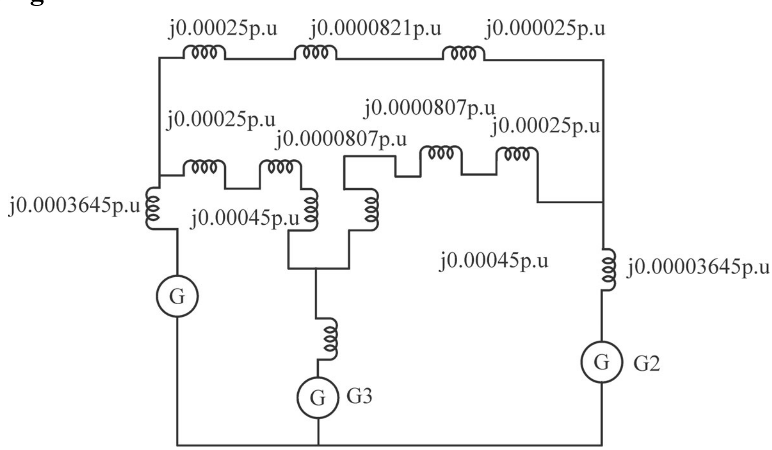

Fig.1.53

Exercise

Multiple Choice Questions

1. What is the value of base impedance for a 20kV, 100MVA, 3-ϕ System?

   a) 5 Ω b) 7 Ω c) 4 Ω d) 1 kΩ

2. In a system, base quantities are 40 kV and 200 MVA. What the value of p.u. admittance of j0.7S?

   a) j4S b) j5.6S c) j5.9S d) j6.4S

3. What is the per unit synchronous reactance on the base value of 300 MVA and 30kV if a 200 MVA with 40 kV synchronous generator has 2p.u synchronous reactance.

   a) 5.33pu b) 5.56pu c) 5.79pu d) 5.2pu

4. Which one is correct: Per unit impedance of a transformer is ______

   a) higher if calculated from primary side than from secondary side.

   b) both are same whether calculate from primary side or secondary side.

   c) always zero.

   d) always unity

5. A synchronous machine's p.u. value is 0.484. What will the p.u. value be if the base voltage is increased by 1.5 times?

   a) 0.266 b) 0.242 c) 0.220 d) 0.215

6. A synchronous generator is capable of operating at 21 kV, 50 MVA and 50Hz. The generator's base impedance is going to be

   a) 8.82 Ω b) 8.9 Ω c) 8.1Ω d) 8.25Ω

7. The synchronous reactance of an 22kV, 300MVA synchronous generator is 0.5 p.u. According to the basis values of 200MVA and 11kV, the p.u. synchronous reactance is

   a) 1.64 b) 1.33 c) 1.43 d) 1.54

8. For a 275MVA machine operating on its own base, the p.u. parameters are inertia M = 13 pu and reactance X = 9 pu. On a 70 MVA common base, the pu values for inertia and reactance will be

   a) 51.07, 2.29 b) 40, 1.9 c) 60, 3.4 d) 49,10

9. A circuit element has a per-unit impedance of 0.25. The new value of the per-unit impedance of the circuit element will be if the base kV is reduced by one third and the base MVA is reduced by half

   a) 1.161 b) 1.125 c) 1.191 d) 1.233

10. The per unit impedance value of an element is represented by X for a given base voltage and base volt ampere. What will this element's per-unit impedance value be if the voltage and volt-amp bases are both doubled?

a) 2X b) 4X c) 0.5X d) X

Answers

1. (c), 2. (b), 3. (a), 4. (b), 5. (d), 6. (a), 7. (b), 8. (a), 9. (b), 10. (c)

Subjective Questions

1. What are the advantages of Per Unit system?

2. Three generators are rated as follows:

Generator 1 – 120 MVA, 40KV, reactance 14%.

Generator 2 - 170 MVA, 35KV, reactance 9%.

Generator 3 – 140 MVA, 31KV, reactance 16%.

Determine the reactance of the generator corresponding to base values of 250 MVA and 45 KV.

3. A 3 bus system is given in fig.1.51. The ratings of the various components are listed below:

Generator 1: 50 MVA, 13.8 KV, X″ = 0.4 P.U
Generator 2: 60 MVA, 12 KV, X″ = 0.25 P.U
Generator 3: 40 MVA, 11 KV, X″ = 0.30 P.U
Transformer 1: 50 MVA, 10 KV/100KV Y/Y, X = 0.24 P.U
Transformer 2: 30 MVA, 11.5 KV/110KV Y/Y, X = 0.30 P.U
Transformer 3: 35 MVA, 11.5 KV/110KV Y/Y, X = 0.5 P.U

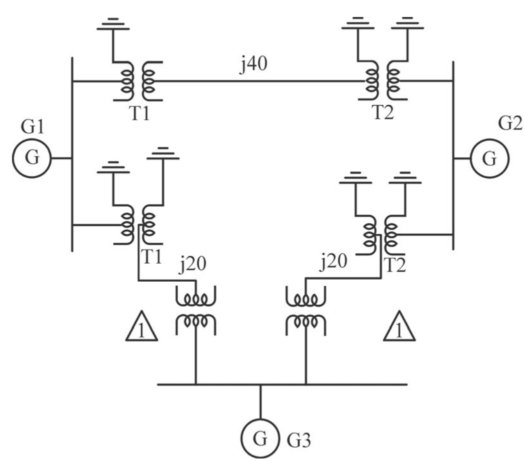

Fig.1.54

The line impedances are shown in the figure. Determine the reactance diagram based on 50 MVA and 13.8 KV as base quantities in Generator 1.

4. Two Generators rated at 15 MVA, 13 KV and 20 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to two motors of input 10 MVA, and 12 MVA respectively. The Operating Voltage of motors is 11.2KV. Assuming base quantities as 25 MVA and 15KV, Draw the reactance diagram. The percentage Reactance for Generators is 13% and that for motors is 20%.

5. Ratings of 3 generators are given below:

G_1: 125 MVA, 20KV, 9% Reactance

G_2: 100 MVA, 35KV, 10% Reactance

G_3: 150 MVA, 30KV, 7% Reactance

Find generator's new reactance with base values 150MVA and 40KV.

6. A 300MVA, 44KV, 15%, 3-ϕ generator has a subtransient reactance of 20%.The generator is connected to three motors through a transmission line and two Transformers. The motors have rated inputs of 50MVA, 20MVA and 70MVA with 30% subtransient reactance. The 3-ϕ transformers are rated at 100 MVA, 32KV/110KV with leakage reactance 7%. The line has a reactance of 40 ohms. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in other parts of the system and evaluate the corresponding p.u. values.

7. Three generators are rated as follows

Generator 1 - 100MVA, 30KV, reactance 9%.

Generator 2 - 150MVA, 35KV, reactance 11%.

Generator 3 - 200MVA, 40KV, reactance 12%.

Determine the reactance of the generators corresponding to Base values of 200MVA and 40KV.