

Contents

| | |
|--|---------------|
| Preface | (vii) |
| About the Author | (xiii) |
| 1. Origin | 1 |
| 1.1 Historical Development | 2 |
| 1.2 Discovery of Neptune Planet | 3 |
| 1.3 Mixed term theory | 4 |
| 1.4 Small denominator problem | 5 |
| 1.5 Theory of Oscillations | 5 |
| 1.6 Averaging Method | 7 |
| 1.7 Appendix-A | 7 |
| 1.7.1 The approximate nature of Kepler's laws and two body Problem | 7 |
| 1.7.2 Two-body problem | 8 |
| 1.8 Appendix-B | 10 |
| 1.8.1 Three-body problem | 10 |
| 2. Regular Perturbation Theory | 13 |
| 2.1 Taylor expansion and Perturbation Theory | 13 |
| 2.1.1 Taylor's Theorem (Generalized Mean Value Theorem) | 13 |
| 2.1.2 Taylor's Series | 14 |
| 2.1.3 Regular Perturbation Expansion | 14 |
| 2.2 Perturbation Method for Solution of Algebraic Equation | 15 |
| 2.2.1 Illustrative Example 1 | 15 |
| 2.2.2 Illustrative Example 2 | 17 |
| 2.3 Perturbation Theory of Differential Equation | 19 |
| 2.3.1 Illustrative Example | 19 |
| 2.3.2 Projectile equation | 21 |
| 2.3.3 Eigenvalue problem | 24 |
| 2.3.4 Fluid Mechanics Problem | 25 |
| 3. Singular Perturbation Theory | 31 |
| 3.1 Scaling and balancing | 34 |
| 3.2 The outer solution | 40 |
| 3.3 The inner solution | 41 |
| 3.4 Matching the inner and outer solutions | 42 |
| 3.5 Geometric singular perturbation theory and the outer solution | 43 |
| 3.6 A simple physical example | 46 |
| 3.6.1 Time scales | 47 |
| 3.6.2 A secular-type problem | 47 |
| 3.6.3 A layer-type problem | 49 |
| 3.7 General concepts in perturbation problems | 51 |
| 3.7.1 Coordinates and parameters; approximations | 52 |
| 3.7.2 Sequence of approximations | 53 |

| | | |
|--------|--|-----|
| 3.7.3 | Regular and singular perturbations | 54 |
| 3.7.4 | Validity of an approximation | 56 |
| 3.7.5 | Domains of validity | 58 |
| 3.7.6 | Order classes | 58 |
| 3.7.7 | Overlap | 59 |
| 3.7.8 | Limit processes | 60 |
| 3.7.9 | A fundamental lemma | 60 |
| 3.7.10 | Matching | 61 |
| 3.7.11 | The extension theorem | 61 |
| 3.8 | First model equation | 62 |
| 3.8.1 | Formulation of problem | 62 |
| 3.8.2 | Perturbation methods | 63 |
| 3.8.3 | Formal matching | 65 |
| 3.8.4 | Higher order approximations | 65 |
| 3.8.5 | Comparison with exact solution | 67 |
| 3.8.6 | Matching reconsidered | 68 |
| 3.8.7 | Formal limits of equations and formal domains of validity | 69 |
| 3.8.8 | Outer and inner limits and equations | 71 |
| 3.8.9 | Heuristic principle for domains of validity | 72 |
| 3.8.10 | Higher order terms | 74 |
| 3.8.11 | Composite expansion | 77 |
| 3.8.12 | Position of boundary layer | 78 |
| 3.8.13 | Inner and outer expansions and limit-process expansions | 78 |
| 3.8.14 | Note on terminology | 79 |
| 3.8.15 | Techniques of matching | 80 |
| 3.9 | Second model equation | 83 |
| 3.9.1 | The case $n = 3$ | 86 |
| 3.9.2 | Higher order terms | 88 |
| 3.9.3 | Calculation of f_2 | 89 |
| 3.9.4 | Switchbacks, integrated effects | 90 |
| 3.9.5 | The case $n = 2$ | 92 |
| 3.9.6 | Higher order terms | 92 |
| 3.9.7 | Outer and inner expansions versus limit-process expansions | 95 |
| 3.9.8 | Inner limit of the outer expansion | 97 |
| 3.10 | Third model equation | 98 |
| 3.10.1 | Change of coordinates for $n = 2$ | 100 |
| 3.10.2 | Integrated effects | 103 |
| 3.10.3 | Matching reconsidered | 104 |
| 3.11 | Examples from fluid dynamics | 105 |
| 3.11.1 | The Navier-Stokes equations | 105 |
| 3.11.2 | Flow at low Reynolds numbers | 106 |
| 3.11.3 | Flow at high Reynolds numbers | 107 |
| 3.11.4 | Role of sub characteristics | 108 |
| 3.11.5 | Nonuniqueness of the Euler solution | 109 |
| 3.11.6 | Limiting flow inside a finite domain | 110 |

| | |
|--|------------|
| 3.11.7 Optimal coordinates | 114 |
| 3.11.8 Coordinate-type expansions, artificial parameters | 115 |
| 3.12 Appendix | 117 |
| 4. Artificial Parameter Method | 119 |
| 4.1 Artificial Parameter Method | 121 |
| 4.1.1 An illustrative example | 128 |
| 4.2 Solution given by some previous analytic techniques | 129 |
| 4.2.1 Perturbation method | 129 |
| 4.2.2 Lyapunov's artificial small parameter method | 129 |
| 4.2.3 Adomian's decomposition method | 131 |
| 4.2.4 The δ -expansion method | 132 |
| 4.3 Homotopy analysis method | 134 |
| 4.4 Homotopy analysis solution | 136 |
| 4.4.1 Zero-order deformation equation | 136 |
| 4.4.2 High-order deformation equation | 139 |
| 4.4.3 Convergence theorem | 140 |
| 4.4.4 Some fundamental rules | 142 |
| 4.4.5 Solution expressions | 144 |
| 4.4.6 The role of the auxiliary parameter \hbar | 160 |
| 4.4.7 Homotopy-Pade' method | 169 |
| 5. Homotopy Perturbation Method | 175 |
| 5.1 Basic Idea of Homotopy Perturbation Method (HPM) | 175 |
| 5.2 Guidelines for choosing homotopy equation | 177 |
| 5.3 The classic view on HPM | 178 |
| 5.4 Proposed choices for L and v_0 | 182 |
| 5.5 Examples: | 183 |
| 5.5.1 Evolution equations | 183 |
| 5.5.2 Cauchy reaction-diffusion equations | 185 |
| 5.5.3 Emden-Fowler type equations | 187 |
| 5.5.4 Klein-Gordon equations | 189 |
| 5.6 Schrodinger Equation | 193 |
| 5.7 Nonlinear Partial Differential Equations | 196 |
| 5.8 Unsteady non-Newtonian Fluid between Stationary and Oscillating Plates | 196 |
| Bibliography | 201 |