CHAPTER

1 Introduction

Often, the behavior of continuous physical systems can be better understood by constructing a mathematical model. The mathematical model is constructed by selecting various parameters that govern the behavior of the structural system. The mathematical model of a physical system is best understood in terms of the governing ordinary differential equations (ODE's) or partial differential equations (PDE's). These governing equations of the mathematical model need to be solved in order to predict the behavior of the physical system. The physical domain of this analytical solution to the mathematical model has an infinite number of points. The analytical solutions of governing ordinary differential equations (ODEs as well as PDEs) are given by a mathematical expression that yields the values of the desired unknown quantities at infinite number of locations in the body. However, these analytical solutions are not usually obtainable because of the complicated geometries, loadings, and material properties.

Often, it is very difficult to obtain a closed form solution to the governing equations. It is generally not possible to obtain analytical mathematical solutions for problems involving complicated geometries, loadings, and material properties. Hence the use of numerical methods, such as finite element method, needs to be relied upon in order to obtain acceptable solutions. In such cases, the use of finite element method is mandated and is invaluable. While the analytical approach requires the solution of differential equations, the finite element formulation of the problem results in a system of simultaneous algebraic equations for solution. These equations are solved to obtain approximate values of the unknowns at discrete number of points in the continuum.

The name *finite element* was coined by Clough(1960). Finite Element Method is a numerical technique used to solve a system of a system of governing equations over the domain of a continuous physical system. Areas of work where finite element method can be applied include structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential.

Finite element method consists of discretizing the physical domain into simple geometric shapes called finite elements. The process of dividing a continuous physical body into an equivalent system of finite elements is called discretization. These elements are connected to one another at a finite number of points named as nodes. Thus, discretization involves the process of splitting up a continuous domain into an assemblage of finite number of finite elements. The process of dividing a body into an equivalent system of smaller bodies viz. finite elements is called discretization. These finite elements are interconnected at points common to two or more elements by nodes/ boundary lines / surfaces. While analytical solution is sought for the entire physical body as a continuum, in finite element method equations are formulated for each finite element separately and then are combined to obtain solution for the entire body. Thus, the finite element solution for a structure refers to the displacements at each node and stresses within each of the finite elements that make up the entire structure. In non-structural problems, the nodal unknowns may, for instance, be temperatures or fluid pressures due to thermal or fluid fluxes.

1.1 HISTORY OF FINITE ELEMENT METHOD

The basic ideas of the finite element method owe their origin to the seminal contributions made by Turner, Clough, Martin, and Topp (1956) and Argyris and Kelsey (1960). Topp's paper brings out the initial application of simple finite elements for pin-jointed bar and triangular plate with in-plane loads. It represents one of the key contributions in the development of finite element method.

The advent of the digital computer made finite element method practically viable as it provided a rapid means of performing many calculations involved in its implementation. With the development of high-speed digital computers, the application of finite element method also evolved at a rapid and impressive pace.

The pioneering contribution made by Przemieniecki (1970) presents finite element method for the solution of stress analysis problems. A broader interpretation of finite element method its applicability to any general field problem was provided by Zienkiewicz and Cheung (1967). This eventually led to the realization that finite element equations can also be derived using least squares approach or a weighted residual method such as Galerkin method. This created a widespread interest among applied mathematicians in applying the finite element method for the solution of linear and nonlinear differential equations. Over a period of time, there have been many research publications related to finite element method.

In the year 1941, Hrennikoff presented a solution of elasticity problem using one-dimensional elements. A similar problem was handled by Mc Henry (1943) in the year 1943. In the same year, Courant introduced shape functions over triangular subregions to model the whole region. Levy developed the flexibility and stiffness methods for structure problems in the years 1947 and 1953 respectively. Argyris and Kelsey (1960) developed matrix structural analysis methods using energy principles in the year 1954. In 1956, Turner, Clough, Martin and Topp derived stiffness matrices for truss, beam and 2D plane stress elements and direct stiffness method. It was in the year 1960 that Clough coined the name "***Finite Element***". In the same year, Turner et al. took up work on large deflection and thermal analysis. Melosh developed stiffness matrix for plate bending element in the year 1961. Also, Martin developed the tetrahedral stiffness matrix for 3D problems in 1961. During the next year 1962, Gallagher worked on problems involving material nonlinearity. Grafton and Strome (1963) developed element stiffness matrix for a curved-shell bending element in 1963. In the same year, Melosh applied variational formulation to solve non-structural problems. Three dimensional elements of axisymmetric solids were developed by Clough et. al in the year 1965. Zienkiewicz published his first book on finite element method in 1967. He worked on viscoelasticity problems in the succeeding year. In 1969, Szabo and Lee adapted weighted residual methods to structural analysis. A book on nonlinear continua was published by Oden in 1972. Nonlinear dynamic behaviour of large-displacement problems was investigated by Belytschko in 1976. Since then, several new developments have taken place in the field of finite element analysis. These include:

(a) development of several new elements
(b) convergence studies
(c) developments of supercomputers
(d) availability of powerful microcomputers
(e) development of user-friendly general-purpose finite element software packages.

Non-deterministic Finite element models represent the latest developments in Finite element methods. The non-deterministic approach to finite element analysis considers uncertainty in the structural parameters that define the structural system. The structural behaviour of such an uncertain system is sought in terms of non-deterministic structural response. The following are various developments in the area:

(a) Stochastic finite element methods model uncertainty of structural parameters as stochastic variables. The uncertainty is defined using probability distribution functions.

(b) Interval finite element methods model uncertainty of structural parameters by closed intervals. They combine the concepts of interval algebra and classical finite element analysis. The structural response is sought in terms of interval variables.

(c) Fuzzy randomness is incorporated into structural behaviour to construct a fuzzy random model of finite element analysis. Fuzzy randomness is also modelled using probability boxes (p-boxes).

The development of high storage, and faster computers have removed all limitations imposed on earlier implementation of finite element models. The coming years will be sure to witness much rapid progress in the area of finite element method.