

Contents

Preface	(v)
Foreword	(vii)
A note to the Students	(ix)
Acknowledgements	(xi)
List of Symbols	(xiii)

CHAPTER 1

Introduction

1.1 History of Finite Element Method	2
--------------------------------------	---

CHAPTER 2

Elastic Continuum

2.1 Equilibrium conditions for Three-dimensional Continuum	5
2.2 Equilibrium conditions for Two-dimensional Continuum	7
2.3 Strain-displacement relationship	8
2.4 Linear Constitutive Relations	10
2.5 Two-dimensional stress distribution -Plane Stress condition	12
2.6 Two-dimensional stress distribution Plane Strain condition	14

CHAPTER 3

Variational Methods

3.1 Potential Energy Functional	17
3.2 Calculus of Variations	18
3.2.1 Essential and Natural Boundary Conditions	19
3.3 Rayleigh-Ritz Method	20
3.4 Galerkin's Method	20

CHAPTER 4**Discretization of Domain**

4.1	Types of elements	24
4.1.1	One Dimensional Elements	25
4.1.2	Two-Dimensional Elements	25
4.1.3	Axisymmetric Elements	26
4.1.4	Three-Dimensional Elements	26
4.1.5	Higher Order Elements	27
4.2	Interpolation Functions	27
4.2.1	Polynomial Interpolation Functions	28
4.2.2	Criteria for Selection of Interpolation Functions	28
4.2.3	Pascal's Triangle	30
4.3	Natural Coordinates	30
4.3.1	Natural Coordinates for a One-Dimensional Element	31
4.3.2	Area Coordinates for a Triangular Element	32
4.3.3	Volume Coordinates for a 4-node Tetrahedral Element	34
4.4	Construction of interpolation functions	37
4.4.1	Interpolation Functions for Bar Element	37
4.4.2	Natural Coordinates for Beam Element	39
4.4.3	Interpolation Functions for a 3-node Triangular Element	42
4.4.4	Interpolation Functions for a 4-node Rectangular Element	45
4.4.5	Interpolation Functions for a 6-Node Triangular Element	48
4.4.6	Interpolation Functions for a 4-node Tetrahedral Element	52
4.5	Lagrangian Interpolation functions	53
4.5.1	Two Node One Dimensional Element	54
4.5.2	Three Node One Dimensional Element	55
4.5.3	Four Node Rectangular Element	57
4.5.4	Nine Node Two Dimensional Lagrangian Element	58
4.6	Serendipity Elements	59

CHAPTER 5**Formalisation of Finite Element Method**

5.1	Principle of virtual displacements	63
5.2	Strain Energy Stored in a Deformable Body	66
5.3	Obtaining Stiffness Matrix and Force Vector	67
5.4	Steps involved in Finite Element Analysis	68
5.4.1	Pre-Processing	68
5.4.2	Processing – Solution of Linear Matrix Equations	69
5.4.3	Post-Processing of Solution	69

CHAPTER 6**One Dimensional Elements**

6.1	Bar element	70
6.1.1	Stiffness Matrix for a Bar Element	71
6.1.2	Force Vector for a Bar Element	71
6.1.3	Numerical Examples	72
6.2	Plane truss element	78
6.2.1	Stiffness Matrix and Force Vector for Plane Truss Element	81
6.2.2	Numerical Examples	83
6.3	Space truss element	92
6.3.1	Stiffness Matrix for Space Truss Element	95
6.3.2	Force Vector for Space Truss Element	95
6.3.3	Numerical Examples	96
6.4	Beam element	100
6.4.1	Stiffness Matrix for Beam Element	101
6.4.2	Force Vector for Beam Element	102
6.4.3	Numerical Examples	108
6.5	Plane frame element	112
6.5.1	Stiffness Matrix for a Plane Frame Element	113

6.5.2 Force Vector for Plane Frame Element	115
6.5.3 Numerical Examples	117
6.6 Space Frame Element	121
6.6.1 Stiffness Matrix for the Space Frame Element	122
6.6.2 Rotation Transformation Matrix for the Space Frame Element	122
6.6.3 Stiffness Matrix of Space Frame Element with w.r.t Global Axes	133
6.6.4 Numerical Examples	134
6.7 <i>Exercise Problems</i>	136

CHAPTER 7**Two Dimensional Elements**

7.1 Simple two-dimensional elements	142
7.2 Elements of arbitrary shape	142
7.3 Constant strain triangular (CST) element	144
7.3.1 Numerical Examples	149
7.4 Six-node (Linear strain) triangular element (LST)	162
7.4.1 Numerical Example	167
7.4.2 Nodal Stress Recovery	171
7.5 Four-node quadrilateral element	176
7.5.1 Numerical Example	179
7.5.2 Nodal Stress Recovery	182
7.6 Eight-Node quadrilateral element	186
7.6.1 Nodal Stress Recovery	190
7.6.2 Numerical Example	191
7.7 <i>Exercise Problems</i>	196

CHAPTER 8**Numerical Integration**

8.1	Numerical Integration using Points with Equal Spacing	199
8.2	Numerical Integration using Points with Unequal Spacing - Gauss Quadrature	199
8.2.1	Computing Abscissa Symbols and Weights Symbols	202
8.2.2	Numerical Example 8.1	203
8.2.3	Numerical Example 8.2	203
8.3	Evaluation of double and triple integrals using Gaussian quadrature	204
8.3.1	Numerical Example – Double Integral	205
8.4	Hammer Integral on Triangles	206
8.5	Exercise problems	209

CHAPTER 9**Axi-Symmetric Elements**

9.1	Stress analysis of axisymmetric elements	210
9.2	Strain displacement and stress-strain relationships	211
9.3	Finite Element Model of Axisymmetric Element	212
9.4	Three-node triangular axisymmetric element	214
9.5	Example problem	217
9.6	Exercise Problems	229

CHAPTER 10**Three Dimensional Elements**

10.1	Four node tetrahedral element	231
10.2	Eight node brick elements	233
10.2.1	Stiffness Matrix of Eight Node Tetrahedral Element	235
10.2.2	Incompatible Modes	238
10.2.3	Numerical Example	239
10.3	Exercise Problems	240

CHAPTER 11**Analysis of Plates**

11.1	Introduction	241
11.2	Bending of thin plates – Kirchhoff's plate theory	242
11.2.1	Assumptions made in Thin Plate Theory	242
11.2.2	Differential Equations in Thin Plate Theory	243
11.2.3	Governing Equations for Bending of Thin Plates	247
11.3	Finite Element Formulation of Thin Plates	247
11.3.1	Three Noded Triangular Plate Elements	248
11.3.2	Four Noded Rectangular Plate Elements	250
11.3.3	Boundary Conditions	254
11.3.4	Description of the Program ACM Plate.m	254
11.3.5	Numerical Examples	255
11.4	Theory of thick plates – Reissner-Mindlin theory	260
11.4.1	Finite Element Formulation of Thick Plates	265
11.4.2	Boundary Conditions	267
11.5	Description of MATLAB program S4plate.m (Appendix A.13)	268
11.5.1	Example Problems	269
11.6	Description of MATLAB code S8 plate C.m (Appendix A.14)	280
11.6.1	Example Problems	282
11.7	<i>Exercise Problems</i>	293

CHAPTER 12**Free Vibration Analysis of Structures**

12.1	Free Vibration Analysis of structures	294
12.2	Free Vibration Analysis of beams	295
12.2.1	Lumped Mass Matrix	295
12.2.2	Consistent Mass Matrix	296

12.2.3 Numerical Example 1	297
12.2.4 Numerical Example 2	299
12.3 Free Vibration Analysis of bars and trusses	302
12.3.1 Numerical Example 3	302
12.3.2 Numerical Example 4	303
12.4 Free Vibration Analysis of Plane Frames	305
12.4.1 Numerical Example 5	305
12.5 <i>Exercise Problems</i>	308
Appendix A Matlab Code, Sample Input and Output for each Element	311
Appendix B Step-by-Step Procedure for Solving Problems on ABAQUS	525
Bibliography	569
Index	573