CHAPTER 1

Introduction to Signals & Systems

In this chapter we discuss about basic concepts of signals and systems.

Objectives of the chapter

Definition of signals and systems, classification of signals, classification of systems, operations on signals, exponential, complex exponential and sinusoidal signals, exponential and sinusoidal signal concepts of impulse function, unit step function, signum function. etc.

1.1 SIGNAL

A signal is defined as any physical quantity that varies with time, space or any other independent variable. A signal may be also treated as set of data. A signal contains a pattern of variation of some form. The variation may be with respect to time or space.

A signal is a physical variable which is a function of time and associated with a system. Some signals like electric charge is function of space.

Signals are represented as mathematical functions of one or more independent variables. This representation is known as signal modeling.

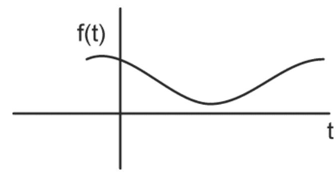
Ex: when a torch light is kept on continuously there is no signal present in it. When it is switched on and off w.r.t time then one will feel there is some signal in it. Hence any variable element constitutes some information in it.

A signal is a variable that carries information. It is a physical quantity which conveys information

Examples:

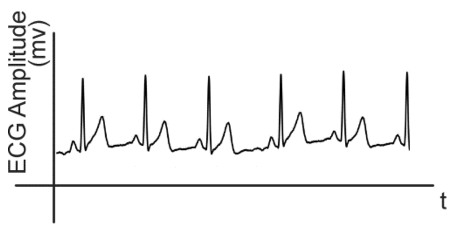
(a) Electrical Signals – Voltages and currents
(b) Acoustic Signals – Acoustic pressure vs time
(c) Mechanical Signals – Velocity of car vs time
(d) Video Signals – Intensity levels vs time
(e) Biological Signals – Sequence of bases in a gene

1.1.1 REPRESENTATION OF SIGNAL

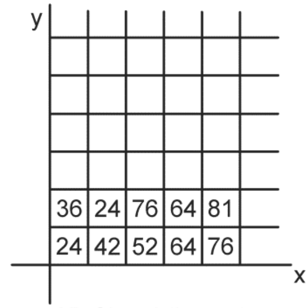


Signals are represented as a function that describes the evolution of one or more independent variables such as time, space etc.

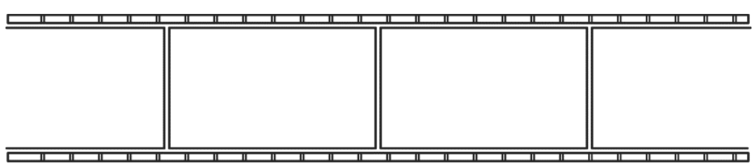
Ex: E C G signal, images and video signals



1D Signal
Independent variable : time



2D Signal (Image)
Independent variable : space



3D Signal
Independent variable : Time & Space

1.1.2 DIFFERENT TYPES OF SIGNALS

There are several classes of signals

1. **Deterministic Signal:** Signals that are modeled or defined completely by specified functions of time are known as deterministic signals.

   Ex: Unit pulse

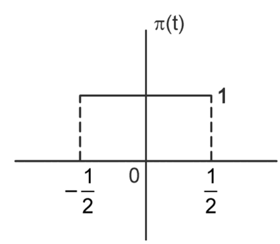


Fig. 1.1

$$\pi(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

A deterministic signal physical description must be known completely in any form.

Even it is not continuous function of time it is deterministic.

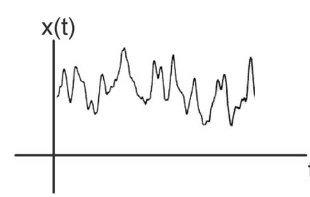


Fig. 1.2

2. **Random signal:** Signals that take random values at any given time and modeled probabilistically are known as random signals.

   Ex: Noise

   These signals are only known by mean, mean square values which are probabilistic.

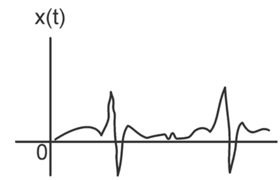


Fig. 1.3

3. **Continuous time Signal:** Signals that are defined for all values of time t are continuous time signals. It is a function of continuous time variable but may not be mathematically continuous function.

   Ex: E C G, AC wave forms

   $$x(t+T) = x(t)$$

4. **Discrete time signal:** Signals that are represented at discrete values of the independent variable such as time are known as discrete time signals.

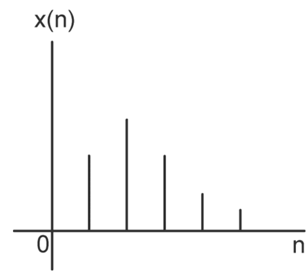


Fig. 1.4

   Ex: $x(nT) = x(t)\big|_{t=nT}$

   Unit pulse

   $$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

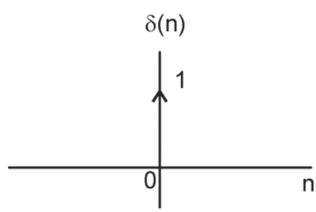


Fig. 1.5

   Unit step

   $$\mu(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

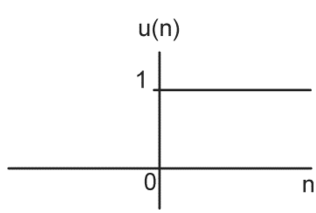


Fig. 1.6

5. **Periodic Signals:** A signal x(t) is periodic if and only if $x(t + T_o) = x(t)$ for $-\alpha < t < \alpha$ where T_o is constant, and its smallest value is known as fundamental period.

   Ex: Sinusoidal

   $$x(t) = \Delta \sin(2\pi f_o t + \theta) \qquad -\alpha < t < \alpha$$

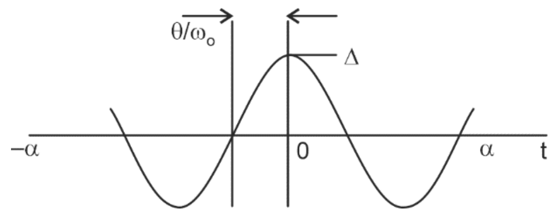


Fig. 1.7

   A periodic signal must start at $t = -\alpha$ because if it starts some finite instant, it may not be same as x(t). A periodic signal can be generated by periodic extension of any segment of f(t).

6. **Aperiodic Signals:** Any deterministic signal that does not satisfy the condition

   $x(t + T_o) = x(t)$ is known as aperiodic signal.

   The above condition is not satisfied for at least one value of t, then it is aperiodic

7. **Causal Signal and Non-Causal Signal:** A continuous time signal x(t) is said to be causal if

   $x(t) = 0$ for $t < 0$, a signal that doesn't start before $t = 0$.

   Other wise it is non-causal.

   Anti Causal Signal is defined as

   $x(t) = 0$ for $t > 0$

8. **Symmetric and anti symmetric Signals:** A signal x(t) is referred as a symmetric (even) if it is identical to its time reversed counterpart.

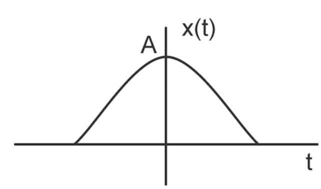


Fig. 1.8

   $x(-t) = x(t)$ – continuous signal

   $x(-n) = x(n)$ – discrete signal

   Ex: $x(t) = A \cos t$

   A signal is said to be anti symmetric (odd) if

   $x(-t) = -x(t)$

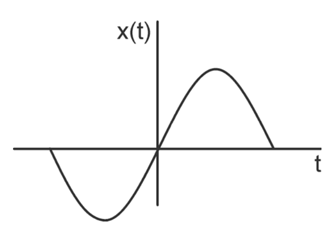


Fig. 1.9

   Any signal x(t) can be represented as sum of even and odd components.

   $$x(t) = x_e(t) + x_o(t)$$

   $$x(-t) = x_e(-t) + x_o(-t)$$

   $$= x_e(t) - x_o(t)$$

$2x_e(t) = x(t) + x(-t)$

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

1.1.3 SIGNAL SIZE

The size of a signal depends on not only amplitude but also on its duration.

Hence the size of a signal is measured in terms of different parameters like energy, power etc. but not by its amplitude.

Signal Energy: Let us consider a signal x(t), its energy is given by

$$E_x = \int_{-\alpha}^{\alpha} \left|x^2(t)\right| dt$$

Signal size is a measure of area under x(t). But for large signals both positive and negative areas could cancel each other and looks like of small size. Hence always signal size is measured using $x^2(t)$.

Signal Power: If the energy is finite. Means signal, amplitude → 0 as time → α

If signal energy is finite then we say measure of signal size is correct. If the amplitude doesn't become zero as time →α, we must measure power P_x, since energy becomes infinite. Power is the time average of energy.

$$P_x = \underset{T\to\alpha}{\text{Lt}} \frac{1}{T} \int_{-T/2}^{T/2} \left|x^2(t)\right| dt$$

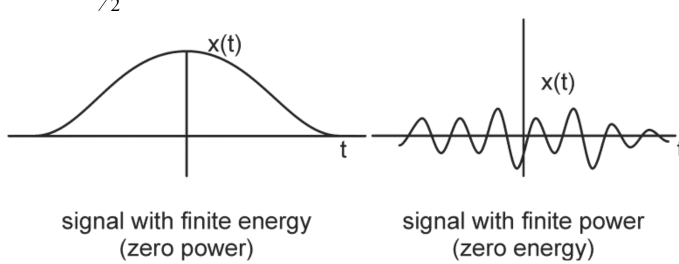


Fig. 1.10

- A signal is an energy signal if $E_\alpha < \alpha$
- A signal is a power signal if $0 < P_\alpha < \alpha$
- Most of the signals are both energy or power signals but not both

Note: Units of energy and power of a signal will not be in conventional manner like jouls and watts, as we are using them to represent size of signal.

1.1.4 OTHER TYPES OF SIGNALS

(a) Classification of signals based on their energy and power:

(i) ***Energy signals*****:** Signals with finite total energy $E_\alpha < \alpha$ are known as energy signals.

These energy signals have zero average power $P_\alpha = 0$. Any signal with finite amplitude and finite duration are called as energy signals.

(ii) ***Power signals*****:** Signal with finite average power $P_\alpha > 0$ are known as power signals

Power signals have finite total energy

$$E_\alpha = \alpha \quad \text{if } P_\alpha > 0$$

A signal with finite energy has zero power, but signal with finite power has infinite energy.

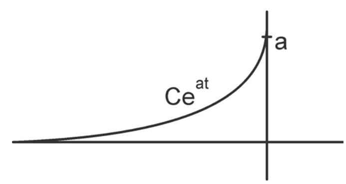


Fig. 1.11

Ex: Periodic signals such as

$x(t) = \cos(t)$ and $x[n] = \sin 5n$

(b) Exponential and sinusoidal signal:

A signal is real exponential if it can be represented by

$$x(t) = C\,e^{at}$$

where C and a are real

If a is imaginary, i.e., $a = j\omega_0$ it is called periodic complex exponential.

(c) Bounded and Unbounded Signals:

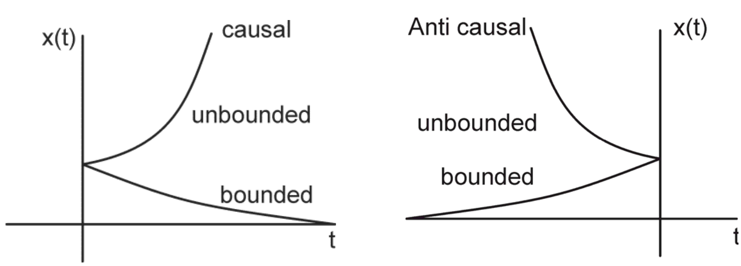


Fig. 1.12

(d) Real and complex exponential signals:

A real exponential signal is defined as

$x(t) = A\,e^{at}$

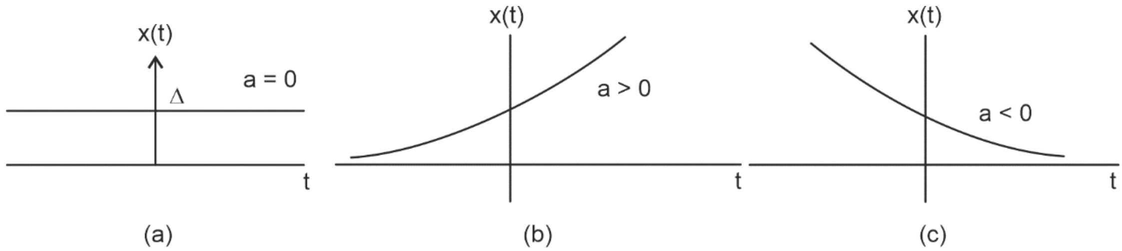


Fig. 1.13

Complex exponential signal is given by

$$x(t) = e^{st} \quad \text{where} \quad s = \sigma + j\omega$$

$$x(t) = e^{(\sigma + j\omega)t} = e^{\sigma t} \, . \, e^{j\omega t}$$

$$= e^{\sigma t}\{\cos \omega t + j \sin \omega t\}$$

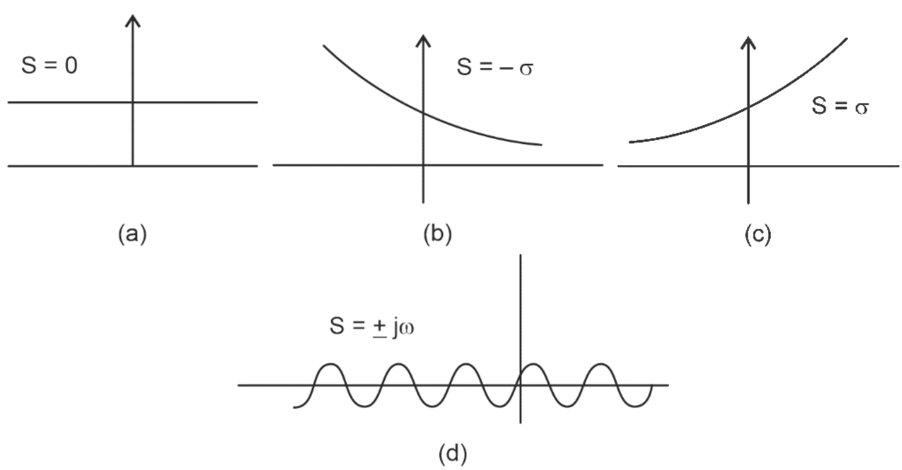


Fig. 1.14

(d) Right and left Sided Signals:

Right Sided Signal**:** If a signal is zero for t > T then it is right sided signal.

Left Sided Signal**:** If a signal is zero for t < T then it is left sided signal.

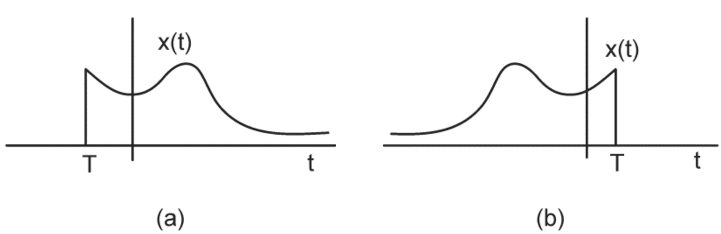


Fig. 1.15

1.2 ELEMENTARY SIGNAL MODELS

Some of the basic signals like unit step, impulse and exponential are needed to build other signals. Unit impulse is a rectangular pulse with a width that has become infinitesimally small, a height that has become infinitely large and overall area is unity.

1. **Unit Step function:** It begins at t = 0 (causal) and is constant up to ∞

   $u(t) = 1 \quad \text{for} \quad t \geq 0$

   $\quad\;\; = 0 \quad \text{for} \quad t < 0$

   $u(t-a) = 1 \quad \text{for} \quad t \geq a$

   $\quad\quad\;\; = 0 \quad \text{for} \quad t < a$

   If we need a signal to start at t = 0, we must multiply signal without x(t).

Fig. 1.16

2. **Impulse function:** It is first defined by P.A.M Dirac as

   $$\int_{-\alpha}^{\alpha} \delta(t)\, dt = 1 \text{ for } \quad t = 0$$

   and

   $$\delta(t) = 0 \qquad \text{for} \quad t \neq 0$$

   It has 0 amplitude everywhere except at t = 0. At = 0 its amplitude is infinite such that area under the curve is equal to 1

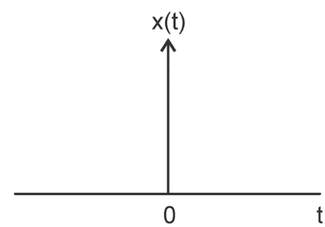


Fig. 1.17

   $$\delta(t) = \lim_{\Delta \to 0} \; x(t) \qquad \text{where } \Delta \text{ is the width of the signal and height is } 1/\Delta.$$

   $$= \lim_{\Delta \to 0} \frac{1}{\Delta}\{u(t) - u(t-\Delta)\}$$

   delayed impulse

   $$\int_{-\alpha}^{\alpha} \delta(t-a) = 1 \quad \text{for} \quad t = a$$

   and

   $$\delta(t-a) = 0 \quad \text{for} \quad t \neq a$$

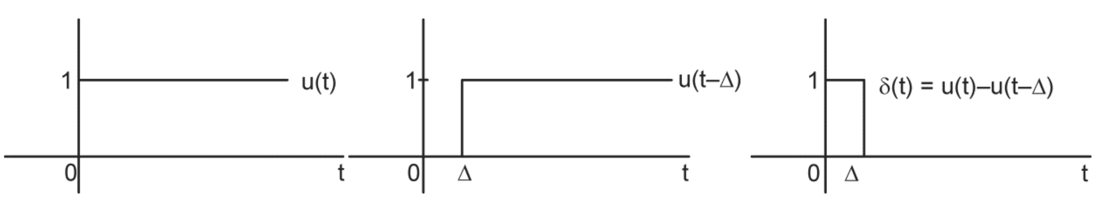


Fig. 1.18

3. **Rectangular pulse function:** A rectangular function can be described by unit step function as shown below.

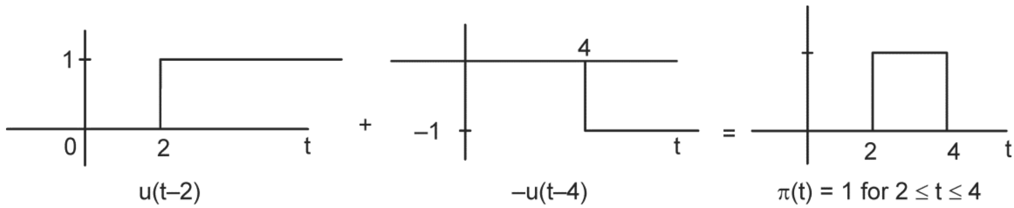


Fig. 1.19

In general a rectangular function is defined by

$$\pi(t) = 1 \quad \text{for} \quad 1t1 \le \frac{1}{2}$$

$$= 0 \quad \text{otherwise}$$

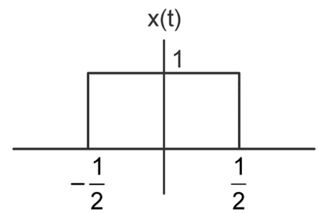


Fig. 1.20

4. **Signum function**

Unit signum function is defined by

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

$$\text{sgn}(t) = -1 + 2u(t)$$

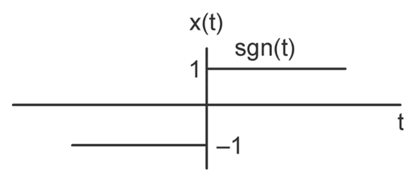


Fig. 1.21

5. **SinC function**

Unit sinC function is given by

$$\text{sinC}(t) = \frac{\sin \pi t}{\pi t} \qquad -\alpha < t < \alpha$$

sinC function oscillates with period 2π and decays with increasing t.

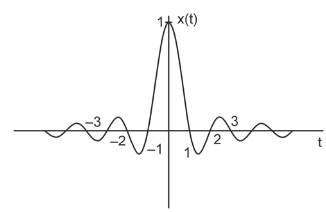


Fig. 1.22

6. **Unit ramp function**

Unit ramp function is defined by

$r(t) = t \quad$ for all t

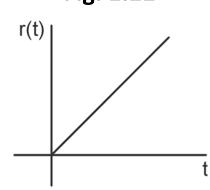


Fig. 1.23

7. **Unit parabolic function**

Unit parabolic function is defined by

$$P(t) = \frac{t^2}{2} \quad \text{for} \quad t \ge 0$$

$$= 0 \quad \text{for} \quad t < 0$$

or $P(t) = \frac{t^2}{2} u(t)$

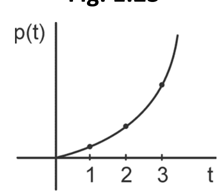


Fig. 1.24

1.3 BASIC OPERATIONS ON SIGNALS

The basic operations on signals are shifting, scaling and inversion.

1. **Time shifting:** Time shifting may delay or advance the signal in time domain.

   $y(t) = x(t - T)$ or $y(n) = x(n - k)$

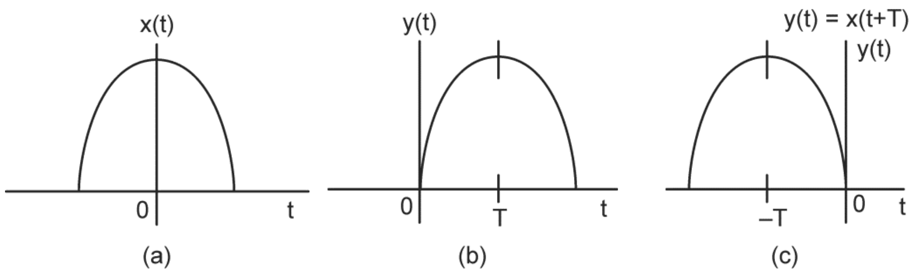


Fig. 1.25

2. **Time reversal:** Time reversal of a signal x(t) can be obtained by folding the signal w.r.t t = 0. Time reversal is obtained by rotating the frame by 180°.

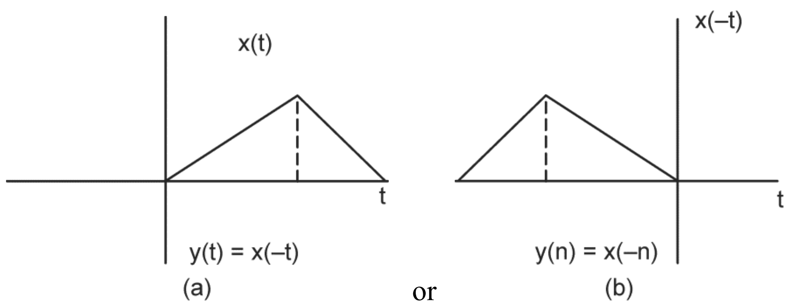
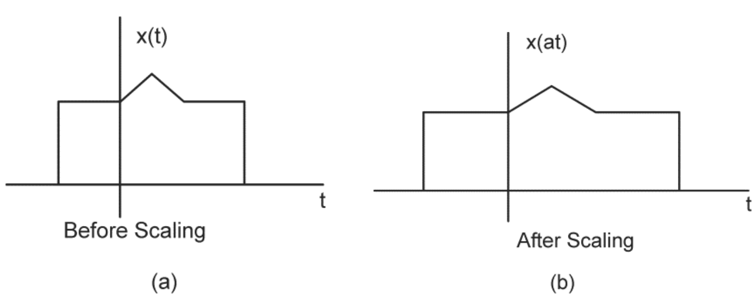


Fig. 1.26

3. **Time scaling:** Time scaling of a signal can be done by replacing t by at compression or expansion of a signal in time is known as time scaling.

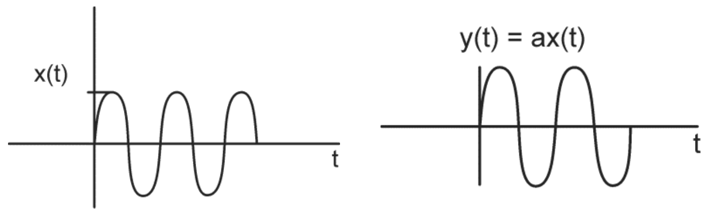


$y(n) = x(an)$

Fig. 1.27

4. **Amplitude scaling:** Amplitude scaling of a signal can be done by multiplying, amplitude by a constant

$$y(t) = ax(t)$$



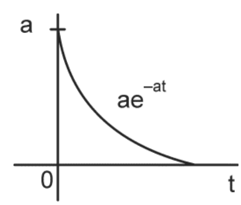
Before scaling (a) after scaling (b)

Fig. 1.28

1.4 PROPERTIES OF UNIT IMPULSE

We can assume impulse as a tall, narrow rectangular pulse of unit area. Exponential pulse, triangular pulse, gaussian pulse etc, can be used for impulse approximation.

Ex: exponential pulse ae^{-at} becomes narrow as a increases $\int_{-\alpha}^{\alpha} ae^{-at} dt = 1$



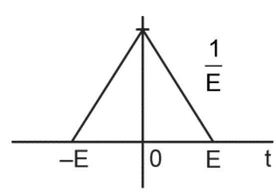
(a)

1. $\int_{-\alpha}^{\alpha} x(t)\,\delta(t)\,dt = x(0)$

   Let us consider the product of x(t) and δ(t) which is x(t) δ(t) impulse exists at t = 0 only thus x(t) exists only at t = 0.

   $\therefore \quad x(t)\,\delta(t) = x(0)\,d(t)$

   $$\int_{-\alpha}^{\alpha} x(t)\,\delta(t)\,dt = x(0)\int \delta(t)\,dt = x(0)$$



(b)

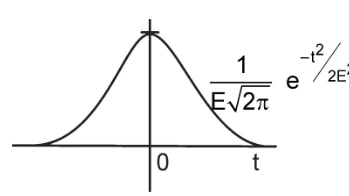
2. $\int_{-\alpha}^{\alpha} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

   Since impulse is existing at t_o, x(t) will exist at t_0 i.e., $x(t_0)$.

3. $\delta(at) = \frac{1}{|a|}\delta(t)$

   $$\int_{-\alpha}^{\alpha} x(t)\,\delta(at)\,dt$$

   Let at $= \lambda$



(c)

$$adt = d\lambda$$

$$dt = \frac{dT}{a}$$

$$= \frac{1}{a}\int_{-\alpha}^{\alpha} x\left(\frac{\lambda}{a}\right)\delta(\lambda)\, d\lambda$$

at $\lambda = 0$

$$= \frac{1}{|a|}x(0) = \frac{1}{a}\int_{-\alpha}^{\alpha} x(t)\,\delta(t)\,dt$$

$$= \frac{1}{a}\delta(t)$$

4. $\int_{-\alpha}^{\alpha} x(T)\,\delta(t-T)\,dt = x(T)$

replace t by T and t_0 by t in below equation

$$\int x(t)\,\delta(t-t_0)dt = \int x(T)\,\delta(T-t)dT$$

as impulse has even property

i.e., $\delta(t-T) = \delta(T-t)$, hence $\int x(t)\delta(t-t_o)dt = x(t_o)$

1.4.1 EXAMPLES USING IMPULSE FUNCTION

1. $\int_{-\alpha}^{\alpha} e^{-at^2}\delta(t-10)dt$

we have $\int x(t)\delta(t-t_0)dt = x(t_0)$

$$\therefore \int e^{-at^2}\delta(t-10)dt = e^{-a(t-10)^2}$$

at $t = 0 = e^{-100a}$

2. $\int_{0}^{\alpha} t^2\delta(t-3)\,dt = t^2|_{t=3} = 9$

3. $\int_{-\alpha}^{\alpha} \delta(t)\cos t + \delta(t-1)\sin t\,dt$

$$= \int_{-\alpha}^{\alpha} \delta(t)\cos t\,dt + \int_{-\alpha}^{\alpha} \delta(t-1)\sin t\,dt$$

$= 1 + \sin 1$

4. $\int_{-\alpha}^{\alpha} \delta(t)e^{-j\omega t}\,dt = e^{-j\omega t}\Big|_{\omega=0} = 1$

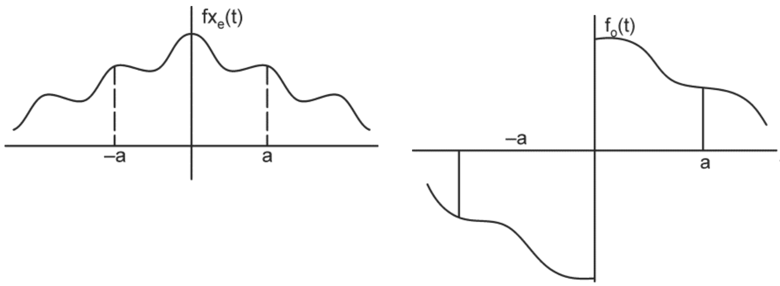
1.5 SOME OTHER PROPERTIES OF SIGNALS

(a) Even and odd functions of signals: Every signal has even and odd components.

Even function × odd function = odd function

Odd function × odd function = even function

Even function × even function = even function



even function (a)

odd function (b)

Fig. 1.29

Even signal is symmetrical about the vertical axis

$$\int_{-a}^{a} x_e(t)dt = 2\int_{0}^{a} x_e(t)dt$$

and for odd signal

$$\int_{-a}^{a} x_0(t)dt = 0$$

Any signal x(t) can be expressed as sum of even and odd components

$$x(t) = \frac{1}{2}\left|x(t) + x(t)\right| + \frac{1}{2} x(t) - x(-t)\right|$$

$$= x_e(t) + x_0(t)$$

$$x_e(t) = \frac{1}{2}\left|x(t) + x(-t)\right|$$

$$x_0(t) = \frac{1}{2}\left|x(t) - x(-t)\right|$$

Examples:

1. Find the even and odd components of e^{jt}

$$e^{jt} = x_e(t) + x_0(t)$$

$$x_e(t) = \frac{1}{2}\left\{e^{jt} + e^{-jt}\right\} = \cos t$$

$$x_0(t) = \frac{1}{2}\left\{e^{jt} - e^{-jt}\right\} = j\sin t$$

SOLVED PROBLEMS

EXERCISE 1

1. Find the following signals are periodic or not.

(a) 2u (t) + 2sin 2t

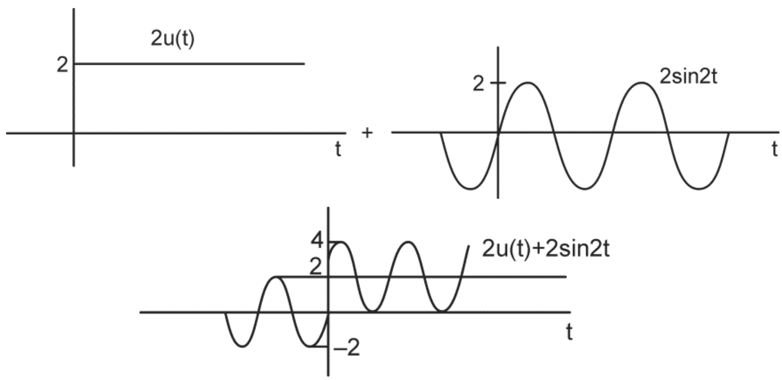


Fig. 1.30

$\therefore$ 2u(t) + 2 sin 2t is aperiodic

(b) 2 cos (10t +1) – sin (4t –1)

Time period of 2 cos (10t + 1) is $\frac{2\pi}{10} = \frac{\pi}{5}$

Time period of 2 sin (4t –1) is $\frac{2\pi}{4} = \frac{\pi}{2}$

T_1/T_2, ratio of two periods = $\frac{2}{5}$ is a rational number, the sum of two signals is periodic

$$T = 2T_2 = 5T_1$$

(c) $x(t) = \cos 60\pi t + \sin 50\pi t$

$\cos 60\pi t$ time period $T_1 = \frac{2\pi}{60\pi} = \frac{1}{30}$

$\sin 50\pi t$ time period $T_2 = \frac{2\pi}{50\pi} = \frac{1}{25}$

$\therefore T_1/T_2 = 5/6$ hence it is periodic

$T = 6T_1 = 5T_2$

2. Find the even and odd components of

(a) $x(t) = \cos t + \sin t + \cos t \sin t$

$x(-t) = \cos t - \sin t - \cos t \sin t$

$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$

$= \frac{1}{2}\{2\cos t\} = \cos t$

$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$

$= \sin t + \text{cost} \sin t$

(b) $x(t) = \cos t + 2 \sin t + 2 \sin^2 t \cos t = \cos t + 2 \sin t + (1 - \cos 2t) \cos t$

$x(-t) = \cos t - 2 \sin t + \cos t - \cos 2t \cos t$

$x_e(t) = \frac{1}{2}\{2\cos t + 2\cos t - 2\cos 2t \cos t\}$

$= 2\cos t - \cos 2t \cos t$

$x_0(t) = \frac{1}{2}\{4\sin t\}$

(c) $x(n) = \{-2, 1, 2, -1, 3\}$

$(n) = \{-2, -1, 0, -1, 2\}$

$x_e(-n) = \frac{1}{2}\{x(n) + x(-n)\}$

$x_e(0) = \frac{1}{2}\{x(0) + x(-0)\} = \frac{1}{2}\times(2+2) = 2$

$x_e(1) = \frac{1}{2}\{x(1) + x(-1)\} = \frac{1}{2}\{-1+1\} = 0$

$x_e(2) = 0.5$

$x_e(n) = \{0.5, 0.2, 0, 0.5\}$

EXERCISE 2

1. Determine the power for the signal

(a) $x(t) = A \cos(\omega_0 t + \theta)$

$$P = \underset{T\to\alpha}{\text{Lt}} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2(\omega_0 t + \theta)dt$$

$$= \underset{T\to\alpha}{\text{Lt}} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \left\{\frac{1+\cos(2\omega_0 t + 2\theta)}{2}\right\}dt$$

$$= \underset{T\to\alpha}{Lt}\frac{A^2}{2T}\left\{\int_{-\frac{T}{2}}^{\frac{T}{2}} 1dt + \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\omega_0 t + 2\theta)dt\right\}$$

$$= \underset{T\to\alpha}{Lt}\frac{A^2}{2T}\cdot T = \frac{A^2}{2}$$

(b) $x(t) = e^{j\omega t}\cos\omega_0 t$

$$P = \underset{T\to\alpha}{Lt}\frac{1}{T}\int_{-T/2}^{T/2}\left(e^{j\omega t}\cos\omega_0 t\right)^2 dt$$

$$= \underset{T\to\alpha}{Lt}\frac{1}{T}\int_{-T/2}^{T/2}\frac{1+\cos 2\omega_0 t}{2}dt$$

$$= \underset{T\to\alpha}{Lt}\frac{1}{2T}\int_{-T/2}^{T/2} dt + \int_{-T/2}^{T/2}\cos 2\omega_0 t\, dt$$

$$= \underset{T\to\alpha}{Lt}\frac{1}{2T}\cdot T$$

$$= \frac{1}{2}$$

(c) $x(t) = A\, e^{j\omega_0 t}$

$$p = \underset{T\to\alpha}{Lt}\frac{1}{T}\int_{-T/2}^{T/2}\left(A\, e^{j\omega_0 t}\right)^2 dt$$

$$= \underset{T\to\alpha}{Lt}\frac{A^2}{T}\int_{-T/2}^{T/2}\left|e^{j2\omega_0 t}\right| dt$$

$$= \frac{A^2}{T}\cdot T = A^2$$

2. Find which of the following are energy signals, power signals

(a) $x(t) = e^{-3t}\, u(t)$

Energy of signal $E = \underset{T\to\alpha}{Lt}\int_{-T}^{T}\left(e^{-3t}\right)^2 u(t)dt$

$$= \underset{T\to\alpha}{Lt}\int_{-T}^{T} e^{-6t}u(t)dt = \underset{T\to\alpha}{Lt}\int_{0}^{T} e^{-6t}\, dt = \frac{1}{6}$$

Power of the signal

$$P = \underset{T\to\alpha}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} e^{-6t}\, dt$$

$$= \underset{T\to\alpha}{Lt} \frac{1}{T} \int_{0}^{T/2} e^{-6t}\, dt$$

$$= \underset{T\to\alpha}{Lt} \frac{1}{-6T} e^{-6t} /_0^{T/2} = 0$$

As energy is finite and power is 0,

It is energy signal

(b) $x(t) = \cos(t)$

$$E = \underset{T\to\alpha}{Lt} \int_{-T}^{T} \cos^2 t\, dt$$

$$= \frac{1}{2} \underset{T\to\alpha}{Lt} \int_{-T}^{T} 1dt + \int_{-T}^{T} \cos 2t\, dt$$

$$= \alpha$$

$$P = \underset{T\to\alpha}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} \cos^2 t\, dt$$

$$= \underset{T\to\alpha}{Lt} \frac{1}{2T} \int_{-T/2}^{T/2} 1 + \cos 2t\, dt$$

$$= \frac{1}{2T} \cdot T = \frac{1}{2}$$

As energy is infinite and power is finite

It is power signal

3. Find which of the signals are causal or non causal?

(a) $x_1(t) = e^{at} u(t)$

for $t < 0$ $x_1(t) = 0$

$\therefore$ it is causal

(b) $x_2(t) = e^{-2t} u(-t)$

for $t < 0$ $x_2(t) \neq 0$

$\therefore$ it is non causal

(c) $x_3(t) = \text{sinc } t$

for $t < 0$, $x_3(t) \neq 0$

$\therefore$ it is non causal

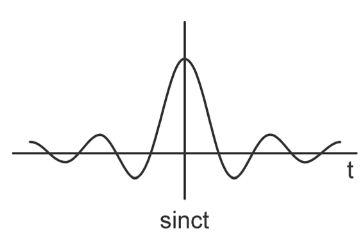


Fig. 1.31

4. Sketch the following signals

(a) $u(-t+1)$ (b) $-2u(t-1)$ (c) $3r(t-1)$ (d) $\pi(t+3)$

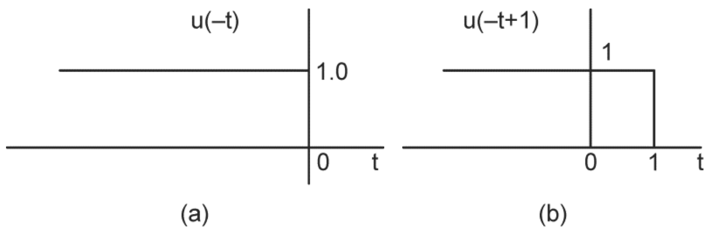


Fig. 1.32

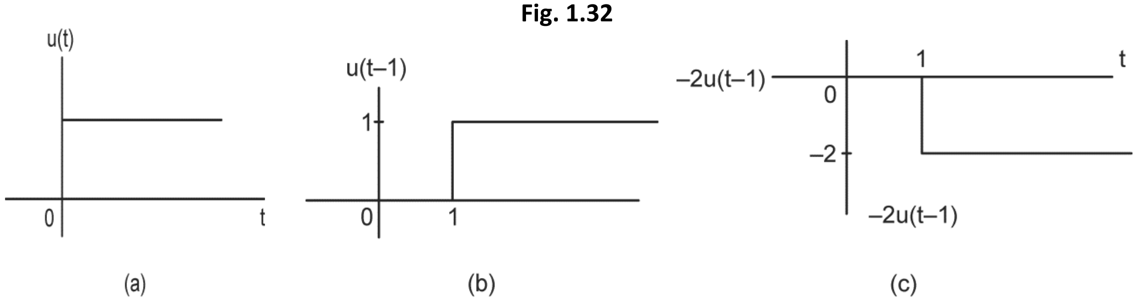


Fig. 1.33

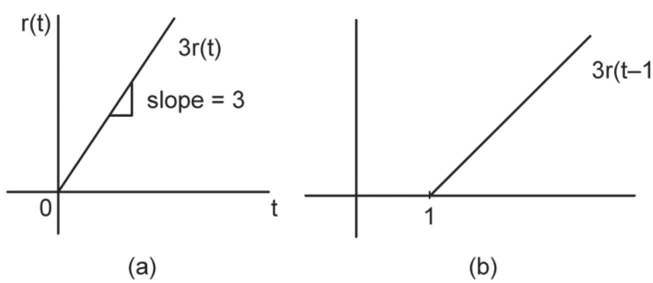


Fig. 1.34

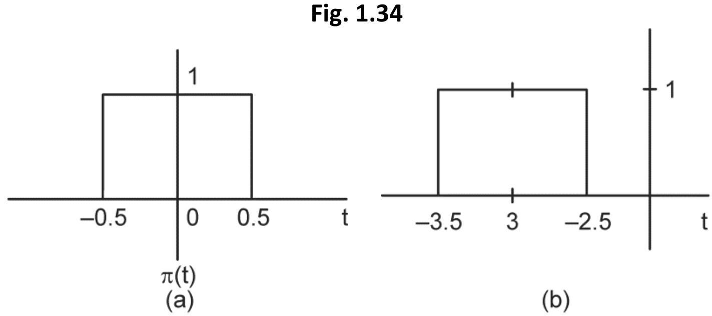


Fig. 1.35

EXERCISE 3

1. Find the even and odd components of below signals

***Ans*:**

(a) $x(t) = \cos t + \sin t + \sin t \cos t$

$x(-t) = \cos t - \sin t - \sin t \cos t$

$\therefore\ x_e(t) = \cos t$

$x_o(t) = \sin t \cos t + \sin t$

(b) $x(t) = 1 + t \cos t + t^2 \sin t + t^3 \sin t \cos t$

$x_e(t) = 1 + t^3 \sin t \cos t$

$x_o(t) = t \cos t + t^2 \sin t$

2. Consider $x(t) = A \cos(\omega t + \phi)$ find average power

***Ans*:** $$P = \frac{1}{T}\int_{-T/2}^{T/2} \left|x^2(t)\right| dt \qquad T = \frac{2\pi}{\omega}$$

$$P = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega} A^2 \cos^2(\omega t + \phi) dt = \frac{A^2}{2}$$

3. Find average power of a discrete signal $x(n) = A\cos(\omega_0 n + \phi)$

$$P = \frac{1}{N}\sum_{n=0}^{N+1} x^2[n]$$

$$N = \frac{2\pi}{\omega_0}$$

$$P = \frac{\omega_0}{2\pi}\sum_{n=0}^{\frac{2\pi}{\omega_0}-1} A^2 \cos^2(\omega_0 n + \phi)$$

$$= \frac{A^2\omega_0}{4\pi}\{1 + \cos 2\phi + 1 + \cos(2\phi - 2\omega_0)\}$$

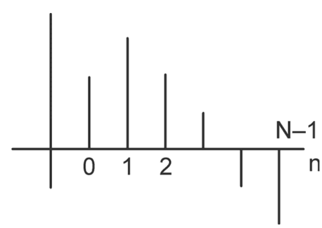


Fig. 1.36

4. Find the energy of signal

$$x(t) = \frac{1}{2}(\cos \omega t + 1) - \frac{\pi}{\omega} \le t \le \pi/\omega$$

$$E = \int_{-\alpha}^{\alpha} x^2(t) dt = \int_{-\pi/\omega}^{\pi/\omega} \frac{1}{2}(\cos \omega t + 1)^2 dt$$

$$= \frac{3\pi}{2\omega}$$

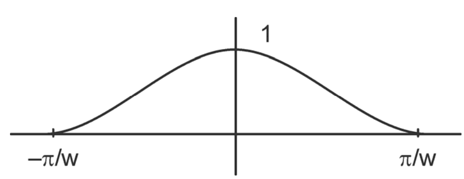


Fig. 1.37

5. Sketch the wave form

(a) $x(t) = u(t+1) - 2u(t) + u(t-1)$

(b) x(t) = r(t + 1) – r(t) + r(t – 2)

(c) x(t) = r(t + 2) – r(t + 1) – r(t – 1) + r(t – 2)

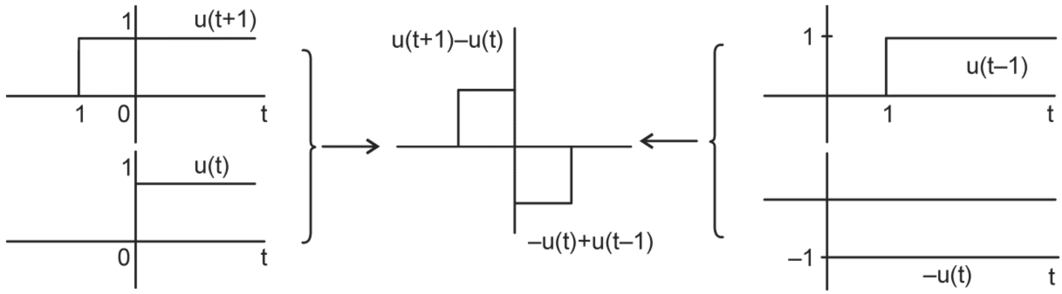


Fig. 1.38

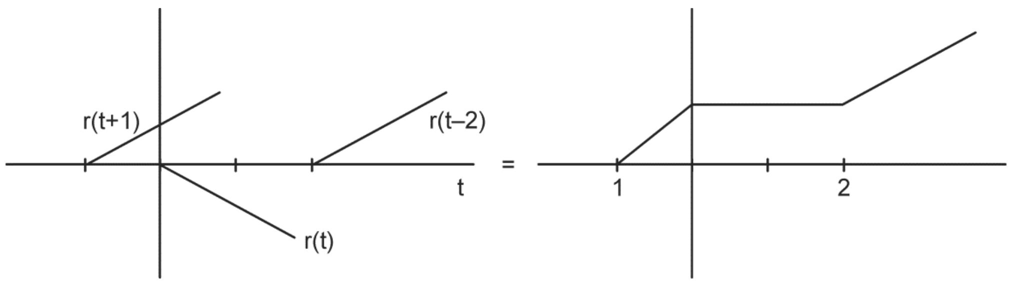


Fig. 1.39

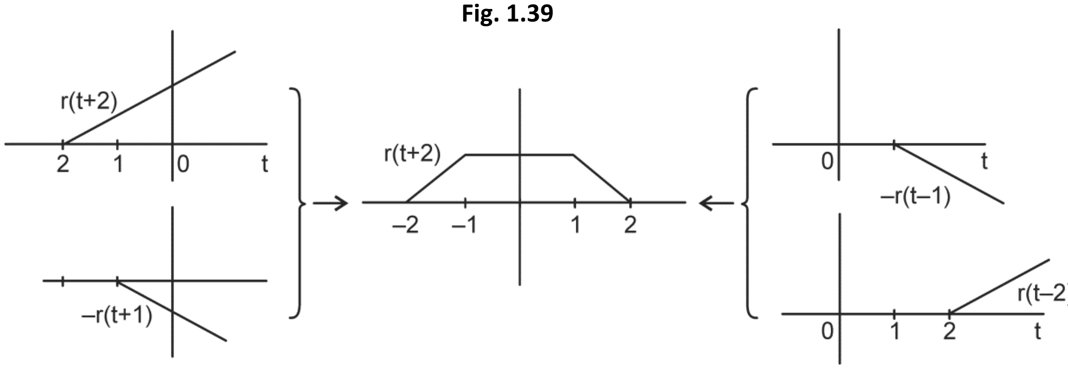


Fig. 1.40

6. Find the below signals are periodic or non periodic

(a) $x[n] = \sum_{k=-\alpha}^{\alpha} \left\{\delta(n-3k)+\delta(n-k^2)\right\}$

(b) $x(t) = \sum_{k=-\alpha}^{\alpha} (-1)^k \, \delta(t-2k)$

(a) It is not periodic, since impulse is not repeating at the same interval

(b) It is periodic, its period is $(-1)^k$

SOLVED PROBLEMS FROM PREVIOUS QUESTION PAPERS

UNIT – 1 NOV - 2008

1. Sketch the following signal

(a) $\pi\left(\frac{t-1}{2}\right)+\pi(t-1)$

We have $\pi(t) = 1$ for $|t| \le \frac{1}{2}$

$= 0$ otherwise

(i) $\pi(t-1)=1$

For $|t-1| \le 1/2$

$\therefore\ t \le 3/2$ and $t \ge 1/2$

(ii) $\pi\left(\frac{t}{2}-\frac{1}{2}\right) = \left|\frac{t-1}{2}\right| \le \frac{1}{2}$

$\therefore\ t \le 2$

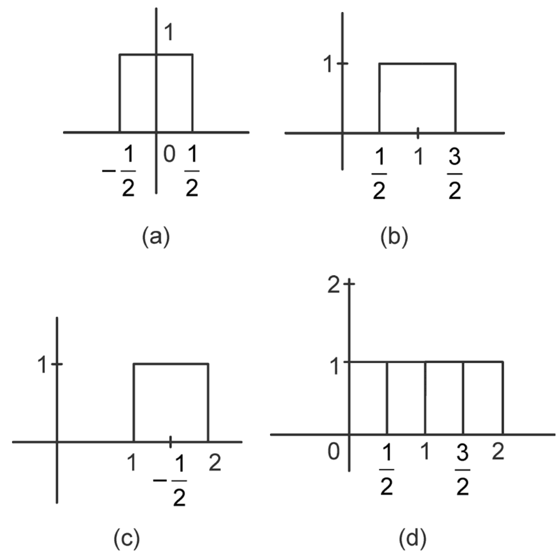
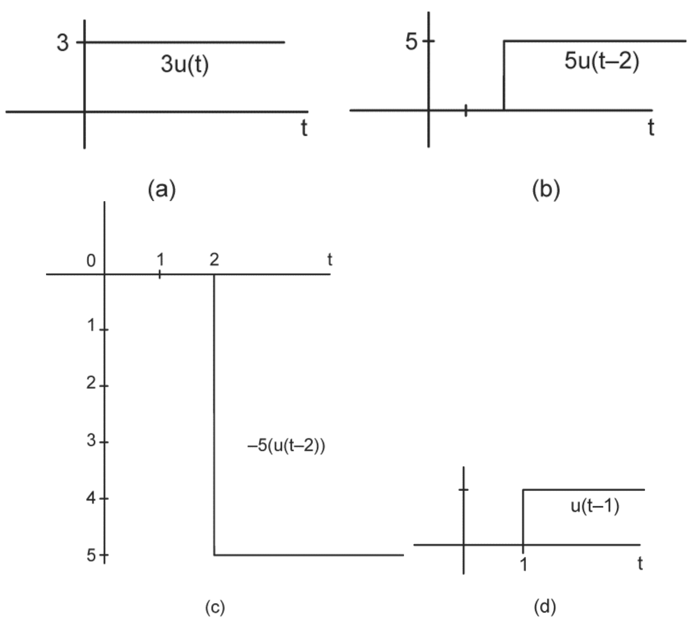


Fig. 1.41

2. $3u(t) - u - (t-1) - 5u(t-2)$

NOV – 2008



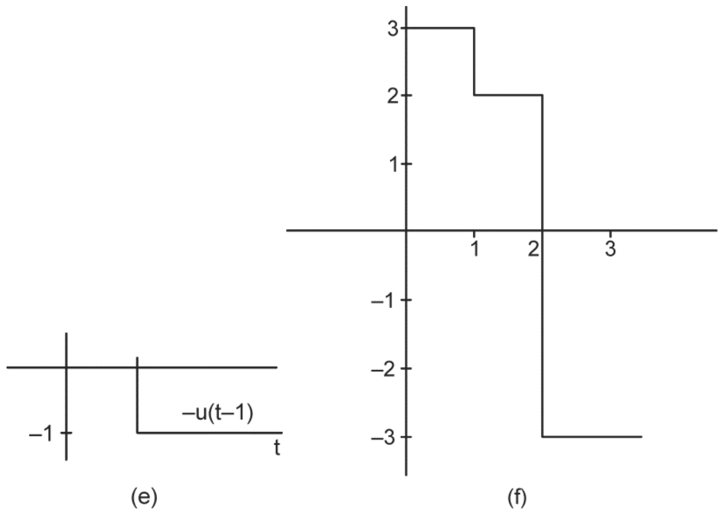


Fig. 1.42

3. Sketch (a) x(t – u) and (b) x(–t) for the signal shown below

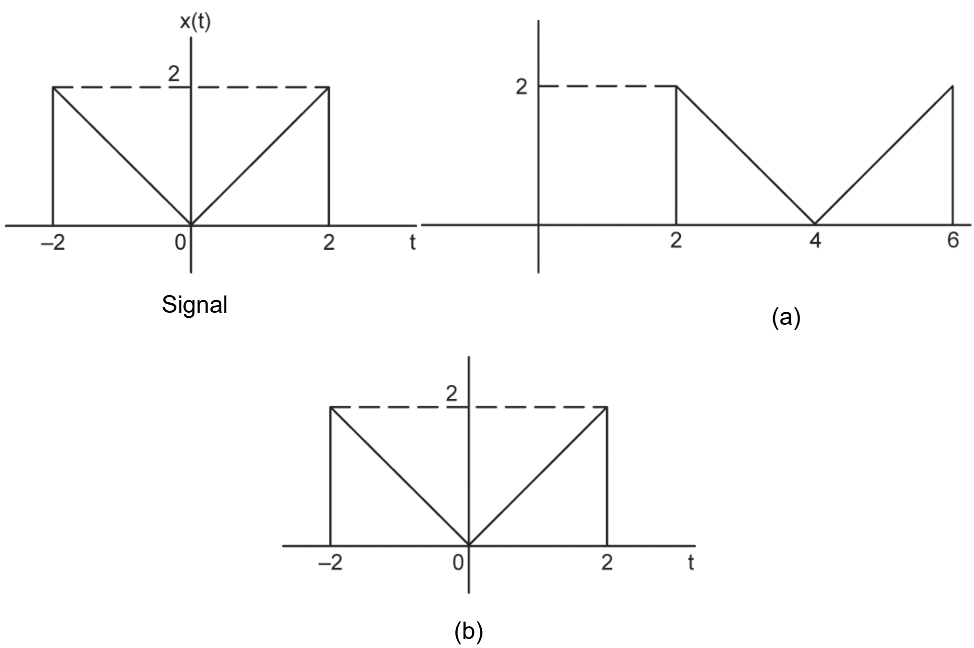


Fig. 1.43

(b) Unit parabolic function

$$P(t) = \frac{t^2}{2} u(t)$$

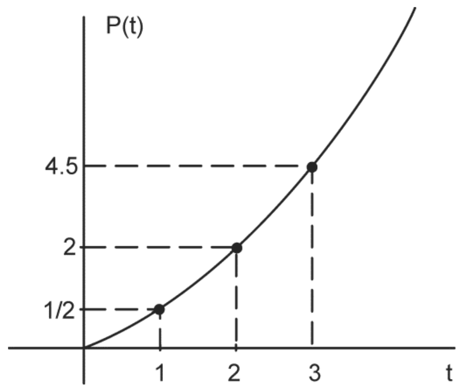


Fig. 1.44

(c) Delayed unit impulse $\delta(t-T)$ **= 1**

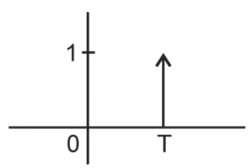


Fig. 1.45

4. Evaluate the integral

(a) $\int_{0}^{\alpha} \delta(t)\sin 2\pi t \, dt = \sin 2\pi t \int_{t=0} \delta(t)dt = 0$

(b) $\int_{-\alpha}^{\alpha} e^{-\alpha t^2} \delta(t-10)dt = e^{-\alpha t^2} \int_{t=10} \delta(t)dt = e^{-100\alpha}$

5. Prove the following Feb 2008

(a) $\delta(n) = u(n) - u(n-1)$

$$u(n) = \sum_{n=0}^{\alpha} \delta(n)$$

$$u(n-1) = \sum_{n=1}^{\alpha} \delta(n)$$

$\therefore$ $u(n) - u(n-1) = \delta(n)$ at $n = 0$

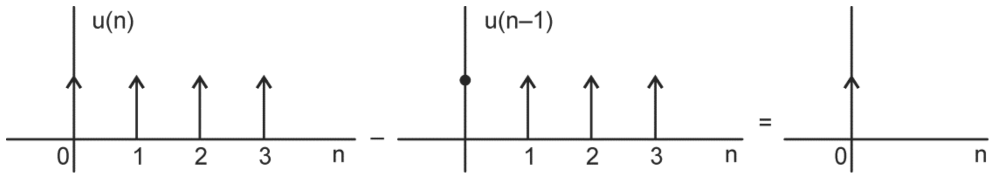


Fig. 1.46

(b) $u(n) = \sum_{n=0}^{\alpha} \delta(k)$ Feb 2008

as $\delta(k)$ exists at $k = 0$

$\sum_{k=0}^{n} \delta(k)$ means $\delta(k)$exists at $K = 0, 1, 2, \ldots\ldots\ldots$

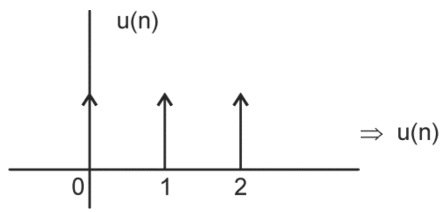
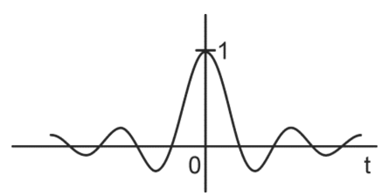


Fig. 1.47

6. Prove that sinc(0) = 1 and plot sin c function Feb 2008

$$\text{sinc}\,(t) = \frac{\sin \pi t}{\pi t} \qquad -\alpha \le t < \alpha$$

$$\text{sinc}\,(0) = \frac{\sin 0}{0} = 1$$



7. Prove that Aug 2007

(a) $$\int_{-\alpha}^{\alpha} x(T)\delta(t-T)\,dt = x(t)$$

impulse function has even property x(t) = (– t)

$\therefore\ \delta(t - T) = \delta(T–t)$

$$\int x(T)\delta(t-T)\,dt = \int x(T)\delta(T-t)\,dt$$
$$= x(t)$$

(b) $$\int_{-\alpha}^{\alpha} \delta(t)x(t)\,dt = -x(0)$$ Aug 2007 where $\delta(t) = \frac{d}{dt}\delta(t)$

let $u = x(t)$ $\quad d\nu = \delta(t)$

$\rightarrow \nu = \delta(t) \quad \int u dv =$

$du = x(t) = u\nu - \int \nu du$

$\int \delta(t)x(t)dt = \int u\ d\nu$

$= x(t)\ \delta(t) - \int x(t)\delta(t)dt$

at $\quad t = 0, \quad x(t) = 0$

$= -\int x(t)\ \delta(t)\ dt = -x(0)$

8. Show that whether x (t) = Ae^{-at} is energy signal or not? Nov 2008

Energy $\quad E = \int_{-\alpha}^{\alpha} |x^2(t)| dt$

$$E = \underset{T\to\alpha}{Lt} \int_{-T}^{T} (Ae^{-at})^2\ dt$$

$$= \underset{T\to\alpha}{Lt}\ A^2 \int_{-T}^{T} e^{-2at} dt = \underset{T\to\alpha}{Lt} \frac{A^2}{-2a} e^{-2at}\Big|_{-T}^{T}$$

$$= \underset{T\to\alpha}{Lt} \frac{A^2}{-2a}\left\{e^{-2aT} - e^{+2aT}\right\} = \propto \quad \text{it is not energy signal}$$

9. Find whether the following signals are periodic or not?

(a) $\quad 3\cos\left(17\pi t + \frac{\pi}{3}\right) + 2\sin\left(19\pi t - \frac{\pi}{3}\right)$

Time period of $3\cos\left(17\pi t + \frac{\pi}{3}\right)$

$$T_1 = \frac{2\cancel{\pi}}{17\cancel{\pi}} = \frac{2}{17}$$

Time period of $2\sin\left(19\pi t - \pi/3\right)$

$$T_2 = \frac{2\cancel{\pi}}{19\cancel{\pi}} = \frac{2}{19}$$

$$\therefore \quad T_1/T_2 = \frac{2/17}{2/19} = \frac{19}{17}$$

$\therefore$ T = 17 T_1=19 T_2

$\frac{19}{17}$ is a rational number. The sum of two signals is periodic

$$T = \cancel{17} \times \frac{2\cancel{\pi}}{\cancel{17}\,\cancel{\pi}} = 2\sec$$

(b) $12e^{j(4t+\pi/2)} + 9e^{j(3\pi t+\pi/3)}$

$T_1 = 2\pi/4$

$T_2 = 2\pi/3\pi$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4} = \pi/2$$

$$T_2 = \frac{2\pi}{3\pi} = 2/3$$

$$\frac{T_1}{T_2} = \frac{3\pi}{4}$$

$4T_1=3\pi T_2$

$\therefore$ it is not periodic

(c) cos (20n)

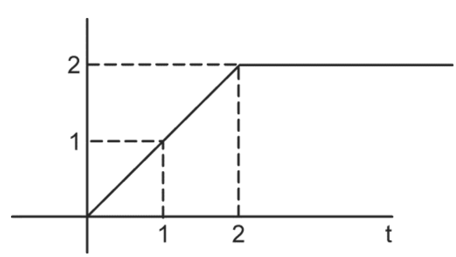
$\omega_0 = 20$

$$N = \frac{2\pi}{20} = \frac{\pi}{10}$$

Since it is an integer, it is aperiodic

10. Which of the following signals are energy signals or power signals?

(a) r(t) – r (t – 2)



$$E = \lim_{T\to\alpha}\left[\int_{-T}^{2}|x(t)|^2\,dt + \int_{2}^{T}|x(t)|^2\,dt\right]$$

$$= \lim_{T\to\alpha}\left[\int_{0}^{2} t^2\,dt + \int_{2}^{T} 4\,dt\right]$$

$$= \lim_{T\to\alpha}\left[\frac{8}{0} + 4(t-2)\right] = \alpha$$

$$P = \lim_{T\to\alpha}\frac{1}{2T}\left[\int_{0}^{2} t^2 dt + \int_{2}^{T} 4dt\right]$$

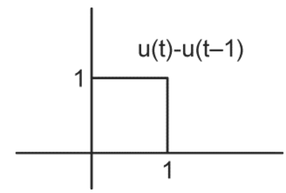
$$= \lim_{T\to\alpha}\frac{1}{2T}\left[\frac{8}{3} + 4(T-2)\right] = 2\omega$$

The energy of the signal is infinite and power is finite. Hence it is power signal.

(b) $u(t) - u(t-1)$

$$E = \lim_{T\to\alpha} \int_{-T}^{T} |x_1(t)|^2 \, dt$$

$$P = \lim_{T\to\alpha} \frac{1}{2T}\int_{0}^{1} 1^2 \, dt = 0$$



Hence it is energy signal

EXERCISE PROBLEMS (SHORT QUESTIONS)

1. Find the even and odd components of the below signals

(a) $x(t) = \cos t + \sin t + \sin t \cos t$

Ans: $\cos t$ is even, $\sin t + \sin t \cos t$ is odd

(b) $x(t) = 1 + t + 3t^2 + 5t^3 + 2t^4$

Ans: $1 + 3t^2 + 2t^4$ is even, $t + 5t^3$ is odd

(c) $x(t) = 1 + t \cos t + t^2 \sin t + t^3 \sin t \cos t$

Ans: $1 + t^3 \sin t \cos t$ is even, $t \cos t + t^2 \sin t$ is odd

(d) $x(t) = (1 + t^3) \cos^3 10t$

Ans: $\cos^3 10t$ is even, $t^3 \cos^3 10t$ is odd

(e) $x(t) = e^{-2t} \cos t = (\cosh 2t - \sinh 2t) \cos t$

Ans: $\cosh 2t \cos t$ is even, $-\sinh 2t \cos t$ is odd

Hint for above problems:

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$x_0(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

2. For the below signals find fundamental period.

(a) $x(t) = \cos^2 2\pi t$ ***Ans:*** Periodic, $T = 0.5s$

(b) $x(t) = \sin^3 2t$ ***Ans:*** Periodic, $T = \frac{1}{\pi}$ s

(c) $x(t) = e^{-2t} \cos 2\pi t$ ***Ans:*** Non periodic

(d) $x(n) = (-1)^n$ ***Ans:*** Periodic, $N = 2$

(e) $x(n) = (-1)n^2$ ***Ans:*** Periodic, $N = 2$

(f) $x(n) = \cos 2n$ ***Ans:*** Non periodic

Hint for above Problems:

If the signal is periodic x(t) = x(t + T)

Fundamental period

$$T = \frac{2\pi}{\omega}$$

$$\text{or} \quad N = \frac{2\pi}{\Omega}$$

3. Classify below signals as energy or power signals

(a) $x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$ ***Ans:*** Energy signal $E = \frac{2}{3}$

(b) $x(n) = \begin{cases} n & 0 \le n \le 5 \\ 10-n & 5 \le t \le 10 \\ 0 & \text{otherwise} \end{cases}$ ***Ans:*** Energy signal $E = 85$

(c) $x(t) = 5\cos t\pi t + \sin 5\pi t, -\alpha \le t \le \alpha$ ***Ans:*** Power signal P = 13

(d) $x(t) \begin{cases} 5\cos t\pi t & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$ ***Ans:*** Energy signal $E = 25$

(e) $x(t) = \begin{cases} 5\cos t\pi t & -0.5 \le t \le 0.5 \\ 0 & \text{otherwise} \end{cases}$ ***Ans:*** Energy signal E = 12.5

(f) $x(n) = \begin{cases} \sin \pi n & -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$ ***Ans:*** Zero signal

(g) $x(t) = \begin{cases} \cos \pi n & n \ge 0 \\ 0 & \text{otherwise} \end{cases}$ ***Ans:***

Hint for above problems:

$$\text{Energy} = \lim_{T\to\alpha} \int_{-T/2}^{T/2} x^2(t)\,dt = \int_{-\alpha}^{\alpha} x^2(t)\,dt$$

$$\text{Power} = \lim_{T\to\alpha} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

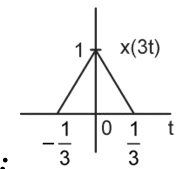
for discrete signal

$$E = \sum_{n=-\alpha}^{\alpha} x^2(n)$$

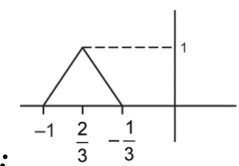
$$P = \lim_{N\to\alpha} \frac{1}{2N} \sum_{n=-N}^{N} x^2(n)$$

4. Sketch the below signals [x (t) is triangular pulse]

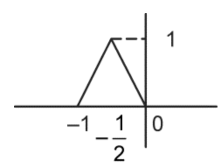
(a) x(3t)

Ans: 

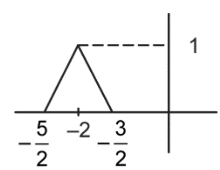
(b) x(3t + 2)

Ans: 

(c) x(–2t –1)

Ans: 

(d) x(2(t + 2)

Ans: 

5. Find the linear, time invariant signals among the below given signals.

(a) $y(t) = t^2\,x(t-1)$ ***Ans:*** Linear but not time invariant

(b) $y(n) = x(n+1) - x(n-1)$ ***Ans:*** Linear, time invariant

(c) $y(n) = x^2(n-2)$ ***Ans:*** Linear, time invariant

6. Determine which of the below properties hold for each signals

(a) Memory less

(b) Time invariant

(c) Linear

(d) Causal

(i) $y(t) = x(t-2) + x(2-t)$ ***Ans:*** Non causal, time invariant

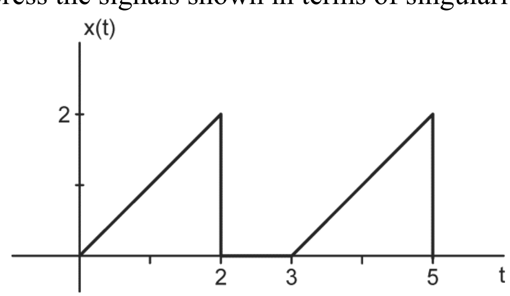
(ii) $y(t) = \frac{dx(t)}{dt} = \frac{x(t) - x(t-h)}{h}$ *Ans:* Non causal, linear

(iii) $y(t) = x(t) \cos 3t$ *Ans:* Memory less, non-causal, linear

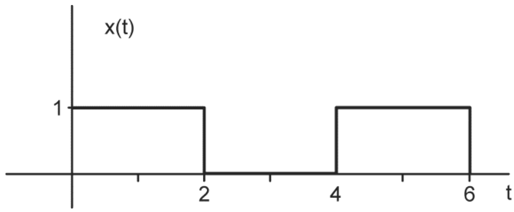
(iv) $y(t) = \int_{-\alpha}^{+\alpha} x(t)\,dt$ *Ans:* Non causal, linear

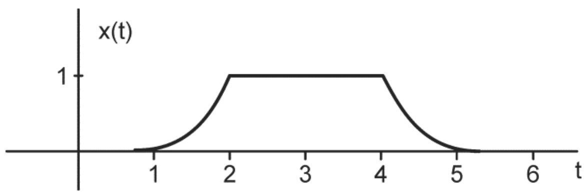
EXERCISE PROBLEMS

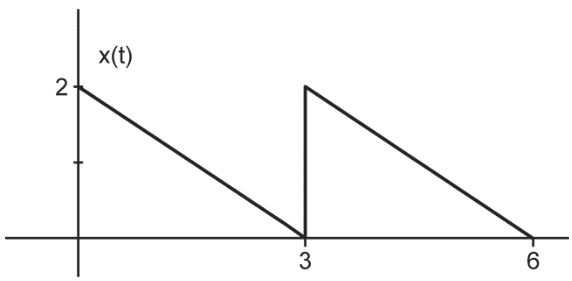
1. Sketch the following signals
   (a) $\pi(t - \frac{1}{2})$
   (b) $\pi((t-1)/2) + \pi(t-1)$
   (c) $r(-2t + 3)$
   (d) $r[(t+1)/3]$
   (e) $u[(t-2)/4]$
   (f) $y(t) = \sum_{n=0}^{\alpha} u(t-2n)u(1+2n-t)$
2. Sketch the following signals
   (a) $r(t+2) - 2r(t) + r(t-2)$
   (b) $u(t)\, u(10 - t)$
   (c) $2u(t) + \delta(t-2)$
   (d) $2\, u(t)\, \delta(t-2)$
   (e) $e^{-10t}u(t)$
   (f) $\cos 10\pi t\, u(t)\, u(2-t)$
3. Express the following signals in terms of singularity functions.
   (a) $r(t)\, u(2-t)$
   (b) $r(t) - r(t-1) - r(t-2) + r(t-3)$
   (c) $\pi[(t-3)/6] + \pi[(t-4)/2]$
4. Find the fundamental periods of signals given below.
   (a) $\sin 50\pi t + \cos 60\pi t$
   (b) $2\cos\left(10\pi t + \pi/6\right)$
   (c) $\sin 2t + \cos \pi t$
   (d) $\cos 60\pi t$
5. Express the signals shown in terms of singularity functions.



(a)

(b) 

(c) 

(d) 

6. Which of the following are energy signals and which are power signals.

   (a) $\left(1-e^{-5t}\right)u(t)$ (b) $t^{-1/4}u(t-3)$

   (c) $t\,e^{-2t}\,u(t)$ (d) $u(t)-\frac{1}{3}u(t-10)$

   (e) cos $10\pi t$ u(t) u(2–t)

7. Evaluate the following integrals

   (a) $\int_{0}^{\alpha}\cos 2\pi t\,\delta(t-2)\,dt$ (b) $\int_{-\alpha}^{\alpha}(t-2)^2\,\delta(t-2)\,dt$

   (c) $\int_{-\alpha}^{\alpha}t^2\delta(t-2)\,dt$ (d) $\int_{-\alpha}^{\alpha}e^{3t}\delta(t-2)\,dt$

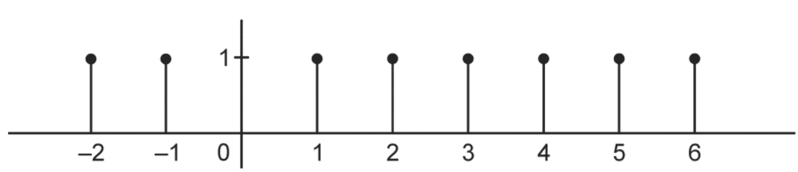
   (e) $\int_{-\alpha}^{\alpha}\left[e^{-3t}+\cos(2\pi t)\right]\delta(t)\,dt$

8. Show that $\delta_{\varepsilon}(t)=\dfrac{e^{-t/\in}}{\in}u(t)$ in the limit $\in\to 0$

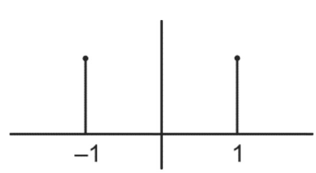
9. Given the signal x(t) = sin2 $(7\pi t - \pi/6) + \cos(3\pi t - \pi/3)$ sketch signal sided amplitude and double sided amplitude.
10. Given the signal x(t) = 16 cos (20πt + π/4) + 6 cos (30πt + π/6) + 4 cos (40πt + π/3) find the power spectral density?

OBJECTIVE QUESTIONS

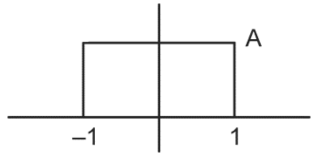
1. The energy E of a discrete signal is given by ________________
2. If the energy E is finite, then x(n) is called as ________________
3. The average power of a discrete signal is ________________
4. The signal x(n) is periodic if and only if ________________
5. The discrete time complex exponential signal is defined as ________________
6. The fundamental frequency of a periodic signal $e^{j\omega_0 n}$ is ________________
7. Odd signal satisfies ________________ condition
8. The smallest value of T, for which x(t) = x(t + T) is known as ________________
9. EEG signal is an example of ________________
10. An energy signal has ________________ power
11. A power signal has ________________ energy
12. If the signal x(t) = e^{at} represent a decay then a is ________________
13. An impulse function δ(n) is defined by ________________
14. δ(t – 1) exists at ________________
15. u(n + 1) = 1 for all ________________
16. u(n) – u(n – 1) = ________________
17. Integral of step function is ________________
18. Fundamental frequency of below signal is ________________



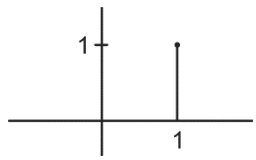
19. x(n) is periodic / aperiodic pulse is ________________



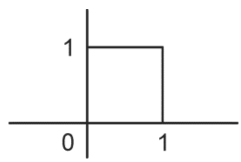
20. Total energy of rectangular pulse is ________________



21. Total energy of x(n) shown is ________________



22. If x(t) is



then x(t – 1) is ________________

23. x(n) = 5sin2n, is x(n) periodic ________________

24. Unit impulse is odd/even function

25. Express following in terms of u(t)

(a) (r(t) = ________________ (b) $\delta(t)$ = ________________

(c) Signum (t) = ________________ (d) $\pi(t)$ = ________________

26. Periodic signal is energy or power signal?

27. What is the period of $2\cos\left(\frac{t}{4}\right)$ ________________

28. Find which of the signals are causal

(a) $\left(\frac{1}{2}\right)^n u(n+3)$ (b) $e^{-2t}u(t-2)$ (c) $u(n+2) - u(n-2)$

29. Find which of the signals are energy or power signals

(a) $e^{-3t}u(t)$ (b) $\left(\frac{1}{3}\right)^n u(n)$ (c) $\cos\left(\frac{\pi}{4}n\right)$

(d) $u(t) - u(t-1)$ (e) $r(t) - r(t-2)$

30. Find the fundamental period of following signals

(a) 2 sin (3t + 1) + 3 sin (4t – 1)

(b) $\sin\left(\frac{7\pi}{3}\right)t$

(c) $e^{j\frac{7\pi}{3}n}$

SUMMARY OF CHAPTER

1. Signals can be classified into continuous and discrete in time domain
2. Continuous signals satisfy condition x(T) = x(t + T) i.e., they are periodic with fundamental period T
3. The signal x(t) is energy signal if $0 < \overline{t} < \alpha$
4. The signal x(t) is power signal if $0 < P < \alpha$
5. The signal x(– t) is obtained by reflecting x(t) about t = 0
6. The signal x(t) is even symmetric if x(t) = x (–t)
7. The signal x(t) is odd symmetric if x(t) = –x (–t)
8. Unit impulse function δ(t) is characterizes by unit area and is connected at a single instant t = 0
9. Unit impulse is given by $u(t) = \int_{-\alpha}^{t} \delta(T)dT$; unit ramp is given by $r(t) = \int_{-\alpha}^{t} u(T)dT$
10. A signal can be either energy or a power signal.
11. A signal whose complete physical description is known in mathematical form, then it is deterministic
12. Two signals $x(T_1)$ and $x(T_2)$ are periodic, then $x(t) = x(T_1) + x(T_2)$ is periodic if T_1/T_2 is a rational number
13. Sampling function $S_a(t) = \frac{\text{sint}}{t}$
14. A signal is said to be causal if x(t) = 0, t < 0
15. Any signal that doesn't contain any singularities (delta function or its donatives) at t = 0 can be written as sum of causal part and anti-causal $x(t) = x^+(t) + x^-(t)$
16. A signal is non causal if it starts before t = 0
17. The area under curve $\int_{-\alpha}^{\alpha} \delta(t)dt$ is unity
18. Singularity is a point at which function doesn't possess a derivative
19. Singularity function are also defined as generalized function
20. A generalized function is defined by its effect on the other functions instead of by its value at every instant of time.