## Chapter 1

## Introduction to Electrical Energy and DC Circuits

### 1.1 Introduction

Electricity is one of the most important blessings that science (so called, almighty) has given to mankind. It has become a part of modern life so much so that one cannot think of a world without it. Electrical energy is the primary source of readily usable energy in the world and it drives almost all present day activities. It has played a vital role in building up of present day civilization. The modern society is so much dependent upon the use of electrical energy that it has become a part and parcel of our life. The following are the reasons which made the Electrical Energy superior than all other forms of energies.

1. Convenient: Electrical energy is a very convenient form of energy. It can be easily converted into other forms of energy like light, heat, mechanical rotation etc.
2. Easily controllable and more flexible: The electrically operated machines provide simple control and ease of operation. For instance, an electric motor can be started or stopped by the flick of a switch
3. Inexpensive: Electric energy is much cheaper than other forms of energy. Thus, it is economical to use this form of energy for domestic, commercial and industrial purposes
4. Clean: Electric energy is not associated with smoke, fumes or poisonous gases and hence provides clean working conditions.
5. Easily Transmittable: The electric energy can be transmitted conveniently and efficiently from one place to other.

### 1.1.1 The Role of Electrical Energy in Modern Life

Electricity is an essential part of modern life and is a major contributor for a global economy. The modern society is so dependent upon the use of electrical energy that it has become a part of our life. Per capita electrical energy consumption is one of the main metrics for assessing economic growth and life quality of anybody. The greater the per capita consumption of electrical energy in a country, the higher is the standard of living of its people. People use electricity for lighting, heating, cooling, and refrigeration and for operating appliances, computers, electronics, machinery, and public transportation systems. The present-day advancement in science and technology has made it possible to convert electrical energy into any desired form. This has given electrical energy a place of pride in the modern world.

Modern means of transportation and communication have been revolutionized by it. Electric trains and battery cars are quick and non-pollutive means of travel. Electricity also provides means of amusement. Radio, television and cinema, which are the most popular forms of entertainment are the result of electricity. Modern equipment like computers and robots have electricity as one of the most important pillars of their development. Electricity plays a pivotal role in the fields of medicines and surgery too, such as X-ray and ECG. The use of electricity is increasing day by day. The residential uses of electrical energy are mainly Heating and Cooling. Computers and office equipment account for largest share of commercial sector electricity consumption. The industrial sector uses electricity to operate machine drives (motors), lights, computers and office equipment, and equipment for facility heating, cooling, and ventilation.

The other major application of electrical energy in modern life is for Electric Vehicles (EVs). These vehicles have brought a remarkable change in the present mobility system. The EVs are gaining popularity as these vehicles are environmental friendly and do not produce any pollutants. The Electric Vehicles are cheap and economical for a given drive range.

The use of Renewable Energy Systems is growing at rapid pace in recent days. The Electrical Energy which got generated from Renewable Energy Sources like solar, wind, hydel is most acceptable as the corresponding conversion mechanisms do not cause for environmental degradation. The Renewable Energy systems are forming the skeleton of future energy scenario.

### 1.1.2 Electrical Energy vs Computer Science Engineering

The Computer Science and Engineering is one sub field among Electronics, Communication and Instrumentation of Electrical Engineering. The engineering era has begun with Civil Engineering which mostly deals with the static systems like bridges and buildings. The rotational energy has made the Mechanical Engineering revolutionary and it has given birth to Electrical Engineering with the invention of generator. The Electrical Engineering in association with other engineering disciplines formed the basis for the growth and promotion of Computer Science and Engineering. It is the primary responsibility of any computer science techy to appreciate the electrical energy which makes a computer system functional. The study of electrical engineering greatly helps for the inculcation of necessary logical ability in computer science engineers and it is useful in delivering optimal codes/programs.

### 1.2 Over View of Electrical Energy Generation, Transmission and Utilization

As we aware that energy is available in nature in different forms like heat, wind, solar, light, chemical, mechanical etc. But, according to the law of energy conservation, energy can be neither generated nor destroyed. But it can be converted from one form to other. Generation of electrical power means converting all forms of existing energies into electrical form. This process has to go on continuously to meet the consumer demand at all times. Hence, the input can be any form of energies, being the output always electrical energy. This conceptualized in Figure 1.1.


Figure 1.1 Electrical energy generation
An efficient, convenient and conventional way of generating electrical energy is by conversion of mechanical energy into electrical energy. The device which supplies the required mechanical energy is called Prime-mover. Since the steam turbines and hydraulic turbines are most generally used prime-movers, the location of generating units is always constrained to the availability of resources like coal, water at an elevated point etc. Unfortunately, this is not the area where the electrical power is needed. Hence, the electrical power generated must be transmitted to the load centers using a conductor system which is called Transmission System. Upon receiving the power at load centers, it is distributed to various consumers and utilized. The entire system which consists of Electrical Power Generation, Transmission, Distribution and finally Utilization is called an Electrical Power System.


Figure 1.2 Electrical power system
As it is cleared from the figure 1.2 that the Power System starts with mechanical power and it gets converted into electrical form. The electrical power thus generated is transmitted to load center using transmission network and then it is utilized for different purposes. It is also cleared from above figure 1.2 that other than lighting and heating loads, all loads involve motion or mechanical power at their output and hence, they convert electrical power into mechanical power. These loads are called Motive Loads and these loads constitute $80 \%$ of total existing loads. So, an electrical power system mostly ends with mechanical energy. Hence, the process of 'Electromechanical energy conversion' is very essential in a power system. Electrical Machines are responsible for this conversion process in an electric power system. If a machine converts Mechanical Energy into Electrical Energy, that machine is called 'Electric Generator' and if a machine converts Electrical Energy into Mechanical Energy, that machine is called 'Electric Motor'.

### 1.3 Electric Circuit

The electric circuit is a simple arrangement that consists of Source or Electromotive Force (battery or generator), Load (like lamp, fan, heater, motor etc which consumes electrical energy), controlling switch and connecting wires. The purpose of a circuit is to transfer electrical energy to the load. The schematic of a circuit is shown in figure 1.3. The electric current is circulated in the circuit by EMF (source) against the Resistance (R). The current (I) is always proportional to the voltage (V).


Figure 1.3 Basic circuit schematic
The interconnection of two or more circuit elements is called Electric Network. In circuits, the current should pass through all the elements whereas in networks, it need not be so. A Circuit can be a Network. But, a Network need not be a Circuit.

An electric circuit has got lot of analogy with a magnetic circuit. In magnetic circuit, an Magneto Motive Force (MMF) is used to pass the magnetic flux (Ø) through an iron core which has got finite value of Reluctance(S) which opposes the flux. Similarly, in a mechanical system, the Driving Torque (T) is responsible to keep the system at a speed (N) against the friction. Hence, the EMF, MMF and Torque are analogous. And the current, flux and speed are similar. Also, the resistance, reluctance and friction are of same kind.


Magnetic Circuit


Mechanical System

Figure 1.4 Analogy between electrical, magnetic and mechanical systems

### 1.3.1 Circuit Elements

A Circuit Element can be represented by any or combination of three basic parameters 1 . Resistance 2. Inductance and 3. Capacitance.

## (a) Resistance

Electrical Resistance is the property of a material by which it opposes the flow of electrons or current through it. When the electrons flow through a conducting material, these electrons collide with the other atoms and lose their energy and hence the potential. The voltage drop (V) in a conducting material is proportional to the flow of current (I) at given temperature.
or

$$
\text { Voltage drop }(\mathrm{V}) \propto I
$$

Where the constant ' R ' is called Resistance. The units for Resistance is Ohms. As the current through a Resistance increases, the voltage drop also increases linearly as shown in figure 1.5. Hence, the Resistance is a Linear Element.

The resistance of a material is proportional to its length (l) and inversely proportional to its crosssectional area (A).

$$
R \propto \frac{l}{A} \quad \text { or } \quad R \propto \rho \frac{l}{A}
$$

Where $\rho$ is its Specific Resistance or Resistivity


Figure 1.5 V-I Characteristics of a Resistance


Figure 1.6 Circular Conductor whose units are Ohm-meters. The resistivity of metals increases as the temperature increases and hence, the resistance changes with temperature.

The power absorbed by the resistance $P=V \times I$ or $P=I^{2} \times R$ or $P=\frac{V^{2}}{R}$
The corresponding energy consumed in the resistance will be appeared as heat and it is expressed as

$$
W=\int_{0}^{t} P . d t=P \cdot T \text { joules }
$$

Hence, Resistance is a Passive element which consumes energy and can't store or deliver energy. And also, a resistor should be properly designed to dissipate heat produced, otherwise it melts. This is the reason the resistors used for electronic circuits of low power ratings are less in size than that used for high power electrical circuits.

## (b) Inductance

Inductance is the property of a circuit element that is related to the magnetic flux. It is evident that a current carrying conductor produces magnetic flux in concentric form around it. The direction of flux is given by Right Hand Thumb Rule which is illustrated below in figure 1.7.


Figure 1.7 Flux lines around a current carrying conductor
When the conductor of negligible resistance is made into a coil form, all the flux lines get gathered and produces a resultant flux whose axis is called the Coil Axis. When this flux (Ø) links with the coil having ' $N$ ' number of turns, the coil get the flux linkages ( $\Psi$ ). The flux linkages are given by $\varphi=N \emptyset$. The units are Wb -Turns.


Figure 1.8 Representation of a Coil or Winding Axis
According to the Faraday's Laws of electromagnetic induction, whenever flux linkages with the coil change with respect to time, an emf is induced in it. This emf is given by

$$
e \propto \frac{d \varphi}{d t}
$$

Where $\quad \varphi \propto \emptyset$ and $\varnothing \propto i$
Then, $\quad e \propto \frac{d \varphi}{d t}$ or $e=L \frac{d \varphi}{d t}$
The constant ' $L$ ' is called Inductance whose units are 'Henry $(\mathrm{H})$ '. One Henry is the inductance of a coil when it produces one volt in it if the current passing through it changes at the rate of one Ampere per Second. As per Lenz's Law, this emf opposes the cause of its generation. Here, the cause is the change in current. Hence, the changes in currents are opposed by the emf produced in the coil. This property exhibited by the coil is called its Inductance. When the current passing through the coil is steady or constant, $\frac{d i}{d t}=0$ and hence no emf is generated.

Hence, Inductance is the property of a coil by which the changes in currents are opposed.


Figure 1.9 Emf induced in a coil opposes changes in current

Like a flywheel having huge inertia opposes the changes in speeds, an Inductor opposes the changes in currents. Hence, Inductance is called 'Electrical Inertia'.

The electrical energy absorbed by an inductor is given by

$$
W=\int_{0}^{t} P . d t=\int_{0}^{t} e . i . d t=\int_{0}^{t} L \times \frac{d i}{d t} \times d t=\int_{0}^{t} L \times d i=\frac{1}{2} \times L I^{2}
$$

Since, the coil is made with a conductor having negligible resistance and assumed to be ideal. It does not consume any energy. It stores the electrical energy absorbed by it in the form of magnetic field. Hence, inductor is an Active Element.

Therefore, the energy stored by an inductor $=W=\frac{1}{2} \times L I^{2}$ Joules
Hence, the important points related to an inductor are summarized as

1. The induced emf across an inductor is zero if $\frac{d i}{d t}=0$. Hence, a pure inductor behaves as a short circuit to direct current (DC)
2. The current in an inductor cannot change instantaneously as the emf induced become infinite which does not allow the current to change
3. An inductor stores the energy only when the current passing through it changes
4. A pure inductor does not consume any power or energy. It stores energy.

## (c) Capacitance

When a conductor is formed into a coil, it exhibits the property of inductance. Similarly, when a conductor is made into two plates and these plates are separated by a thin insulating material called Dielectric, a Capacitor is formed and it is shown in figure 1.10.

When a capacitor is connected across a source (battery), it gets charged ie., the charge is of opposite polarities get transferred to the plates. Thus, an electrostatic field is established between the plates. The charge accumulated the plates is proportional to the voltage ' V '.


Figure 1.10 Capacitor

$$
\begin{array}{ll} 
& Q \propto V \\
\text { or } & Q=C V
\end{array}
$$

Where the proportionality constant ' C ' is called Capacitance. Capacitance is the capability of an element to store electric charge on it. For a given voltage, as the capacitance increases, the charge on capacitor plates also increases. The units for capacitance are 'Farads'. The capacitance is said to be one farad provided one coulomb of charge can be stored with one volt across the capacitor.

The charging current 'I' is given by $i=\frac{d q}{d t}=\frac{d(C V)}{d t}=C \times \frac{d V}{d t}$
The energy absorbed by the capacitor

$$
W=\int_{0}^{t} P . d t=\int_{0}^{t} V . i . d t=\int_{0}^{t} V \times C \times \frac{d V}{d t} \times d t=\frac{1}{2} C V^{2}
$$

Neglecting the resistance of connecting wires and electrodes and assuming no leakage currents through dielectric, the energy input to the capacitor will be stored in capacitor in the form of electrostatic field.

Energy stored in the capacitor $=\frac{1}{2} C V^{2}$ joules
As the capacitor stores energy and does not consume energy, it is called an Active Element. The important points related to a capacitor are summarized as follows

1. The current in a capacitor is zero if the voltage across it is a constant $\left(\frac{d V}{d t}=0\right)$. Hence, it acts as an open circuit for Direct Currents (DC)
2. The voltage across a capacitor can't change instantaneously as infinite current is produced
3. Capacitor stores energy only when the voltage across it changes
4. A pure capacitor does not consume energy. It stores energy only.

### 1.3.2 Electrical Energy Sources

The purpose of an Electrical Energy source is to supply electrical energy or power to a load. These sources are broadly classified into Voltage Sources and Current Sources. The Voltage Sources supply different values of currents at given voltage whereas the Current Sources supply different values of voltages at a constant current.

The other classification of electrical energy sources is 1 . Independent Sources and 2. Dependent Sources.

## 1. Independent Sources

An Independent Source can be an Independent Voltage Source or an Independent Current Sources.

## (a) Independent Voltage Source

An Independent Voltage source is characterized by a terminal voltage which is completely independent of current supplied by it. The magnitude of voltage does not change by any variation in connected load network.



V-I Characteristics

Figure 1.11 Independent Voltage source-symbol and V-I Characteristics

## (b) Independent Current Source

In this source, the current supplied is completely independent of voltage across it. The strength of current does not change by the variation in the load network.


Symbol


V-I Characteristics

Figure 1.12 Independent Current Source-Symbol and V-I Characteristics

## 2. Dependent Sources

In Dependent Sources, the voltage of a voltage source and the current of a current source are controlled by a voltage or a current that exist at some other location/point of the circuit. These sources are also called as Controlled Sources. These are represented as


Dependent Voltage Source


Dependent Current Source

Figure 1.13 Symbols of Dependent Voltage and Current Sources
Sine, the control of Dependent Sources is achieved by a voltage or current, there are four possible dependent sources.
(a) Voltage Controlled Voltage Source (VCVS)
(b) Current Controlled Voltage Source (CCVS)
(c) Voltage Controlled Current Source (VCCS)
(d) Current Controlled Current Source (CCCS)
(a) Voltage Controlled Voltage Source

In this case, the source voltage $V_{2}$ is set by a control voltage $V_{1}$.

$$
V_{2}=\mu V_{1}
$$

Where $\mu$ is a constant having no units. For such controlled source, the voltage- current characteristics become a set of curves as shown in figure 1.14.


Figure 1.14 Equivalent Circuit and V-I Characteristics of Voltage controlled Voltage source

## (b) Current Controlled Voltage Source

Here, the control signal which sets voltage is current $\mathrm{i}_{1}$. The source voltage is given by $V_{2}=$ $R_{m} i_{1}$. Where $\mathrm{R}_{\mathrm{m}}$ is a constant and its units are Ohms or Volt per Amp. The volt-current characteristics of this source are shown in figure 1.15.

equivalent circuit


V-I Characteristics

Figure 1.15 Equivalent Circuit and V-I Characteristics of Current controlled Voltage source

## (c) Voltage Controlled Current Source

Here, the control signal which controls source current is a voltage $\mathrm{V}_{1}$. The source current is given by $i_{2}=G_{m} \times V_{1}$. where $\mathrm{G}_{\mathrm{m}}$ is a constant whose units are mhos or Amp per Volt. The following figure 1.16 shows the V-I characteristics of this source.

equivalent circuit


V-I Characteristics

Figure 1.16 Equivalent Circuit and V-I Characteristics of Voltage controlled Current source

## (d) Current Controlled Current Source

The current signal $i_{1}$ controls the current of the current source. The source current $i_{2}$ is given by $i_{2}=\alpha \times i_{1}$. Where $\alpha$ is a constant having no units. The volt-current characteristics of this source are shown below in figure 1.17. Control Sources are used in many models of devices like transistors.


Figure 1.17 Equivalent Circuit and V-I Characteristics of Current controlled Current source

## Practical Voltage Sources

In practice, no voltage source is ideal. As the current supplied by the source increases, its terminal voltage decreases. This is because its internal resistance $\mathrm{R}_{\mathrm{s}}$.

The terminal Voltage $\mathrm{V}_{\mathrm{L}}$ is given by $V_{L}=V_{S}-I R_{S}$
As I becomes zero under no load conditions, $\mathrm{V}_{\mathrm{L}}$ becomes equal to $\mathrm{V}_{\mathrm{S}}$. As the load current increases, $\mathrm{IR}_{\mathrm{S}}$ drop increases and the terminal voltage $\mathrm{V}_{\mathrm{L}}$ decreases as shown in figure 1.18. Hence, a practical voltage source is represented by an ideal voltage source connected in series with an internal resistance $\mathrm{R}_{\mathrm{S}}$.


Figure 1.18 V-I Characteristics of a Practical Voltage Source

## Practical Current Sources

Ideal current sources are not found in practice. As the demanded voltage by load increases by increase of load resistance, the source current starts reducing. Hence, a practical current source is represented by an ideal current source in parallel with an internal resistance $\mathrm{R}_{\mathrm{P}}$.

As the load resistance increases, the current through $\mathrm{R}_{\mathrm{P}}$ increases and the load current slightly decreases as shown in figure 1.19.


Figure 1.19 V-I Characteristics of a Practical Current Source

### 1.4 Source Transformation

A Practical Voltage Source can be converted into an equivalent practical current source and a practical current source can be converted into its equivalent practical voltage source so that identical values of load voltage and load currents are maintained.

Consider a practical voltage source as shown in figure 1.20 feeding a load current $\mathrm{I}_{\mathrm{L}}$ at voltage $\mathrm{V}_{\mathrm{L}}$. The load current $\mathrm{I}_{\mathrm{L}}$ is calculated as follows.


Figure 1.20 Practical Voltage Source

$$
\begin{aligned}
I_{L} & =\frac{V_{S}}{R_{S}+R_{L}} \\
V_{L} & =I_{L} R_{L}=\frac{V_{S}}{R_{S}+R_{L}} \times R_{L}
\end{aligned}
$$

And also, the open circuit voltage across the load terminals that would drive the load current is

$$
V_{0}=V_{S}-R_{S} \times 0=V_{S}
$$

Similarly consider a practical current source as shown in figure 1.21 that drives the same load current $\mathrm{I}_{\mathrm{L}}$ at same load voltage $\mathrm{V}_{\mathrm{L}}$.


Figure 1.21 Practical Current Source
The load current $\mathrm{I}_{\mathrm{L}}$ is given by

$$
I_{L}=I_{S} \times \frac{R_{P}}{R_{P}+R_{L}}
$$

The load voltage $V_{L}=I_{L} R_{L}=I_{S} \times \frac{R_{P}}{R_{P}+R_{L}} \times R_{L}$
The open circuit voltage across the load terminals $V_{0}=I_{S} R_{P}$
By equating the open circuit voltages in both cases, $V_{0}=I_{S} R_{S}=V_{S}$
Similarly, by equating the load voltages in both cases,

$$
\begin{aligned}
& V_{L}=\frac{V_{S}}{R_{S}+R_{L}} \times R_{L}=I_{S} \times \frac{R_{P}}{R_{P}+R_{L}} \times R_{L} \\
& \frac{V_{S}}{R_{S}+R_{L}}=I_{S} \times \frac{R_{P}}{R_{P}+R_{L}}
\end{aligned}
$$

Also writing $I_{S} R_{S}=V_{S}$ in above equation, we get

$$
R_{S}=R_{P}
$$

and

$$
V_{S}=I_{S} R_{P}
$$

Hence, a practical Voltage source of voltage $\mathrm{V}_{\mathrm{S}}$ and internal resistance $\mathrm{R}_{\mathrm{S}}$ can be equated to a Practical Current Source of current $I_{S}\left(=\frac{V_{S}}{R_{S}}\right)$ with an internal resistance $R_{P}\left(=R_{S}\right)$ connected in parallel and it is shown in figure 1.22


Figure 1.22 Conversion of a Voltage Source into a Current Source
Similarly, a current source of current $\mathrm{I}_{\mathrm{S}}$ and internal resistance $\mathrm{R}_{\mathrm{P}}$ can be replaced by a practical voltage source of voltage $V_{S}\left(=I_{S} R_{P}\right)$ and series connected internal resistance $R_{S}\left(=R_{P}\right)$. It is shown in figure 1.23.


Figure 1.23 Conversion of a Current Source into a Voltage Source
Exercise 1.4.1: Determine the equivalent Voltage Source for a given Current Source.


Figure 1.24 Conversion of a Current Source into a Voltage Source

Exercise 1.4.2: Find the equivalent Current Source for the given Voltage Source.


Figure 1.25 Conversion of a Voltage Source into a Current Source

### 1.5 Kirchhoff's Laws

The Kirchhoff's laws in association with the Ohm's Law become a powerful tool to solve and to analyze any complex electric network. These laws are
(a) Kirchhoff's Current Law (KCL)
(b) Kirchhoff's Voltage Law (KVL)

## (a) Kirchhoff's Current Law (KCL)

The Kirchhoff's Current Law (KCL) states that the algebraic sum of the currents at a given node is zero.

Mathematically, KCL is represented as $\sum_{n=1}^{n=N} i_{n}=0$
Where ' N ' is the total number of branches connected to the node and $i_{n}$ is current passing through the node ' $n$ '. (Node is the point where two or many branches are connected together). The currents entering the node are generally regarded as positive currents and the currents leaving the node are treated as negative currents. Consider the node ' $a$ ' in the figure 1.26 . It is associated with 5 branches carrying currents $i_{1}, i_{2}, i_{3}, i_{4}$ and $i_{5}$.

It is evident that the currents $i_{1}$ and $i_{2}$ are the currents


Figure 1.26 Representation of KCL entering and hence these are considered as positive currents and the currents $\mathrm{i}_{3}, \mathrm{i}_{4}$ and $\mathrm{i}_{5}$ are leaving and hence these are negative currents.

The algebraic sum of the currents at node ' a ' $=i_{1}+i_{2}-i_{3}-i_{4}-i_{5}=0$
Or $i_{1}+i_{2}=i_{3}+i_{4}+i_{5}$
The sum of the currents entering the node is equal to the sum of the currents leaving the node.

As the charge transferred towards a node in a given time is equal to the charge carried away from the node, no charge gets accumulated a node or a node does not store any net charge.

## (b) Kirchhoff's Voltage Law

The KVL states that the algebraic sum of all voltages around a closed path (loop) is zero.
Mathematically. $\sum_{m=1}^{m=N} V_{m}=0$
Where N is the number of branches in a loop and $\mathrm{V}_{\mathrm{m}}$ is the voltage across $\mathrm{m}^{\text {th }}$ branch.
Consider the circuit shown in figure 1.27 to illustrate the KVL.


Figure 1.27 Representation of KVL
As we move from the node 'a' towards node 'b', the potential rises by $\mathrm{V}_{1}$ (-ve of battery to + ve of battery). Hence, the voltage $V_{1}$ should be treated as $+v e$ in the direction of a to $b$. From point ' $b$ ' to ' $c$ ', the potential reduces by $V_{3}$ as the potential decreases in the direction of current. Hence, $\mathrm{V}_{3}$ is negative. Likewise, the polarities of the voltages $\mathrm{V}_{2}, \mathrm{~V}_{4}$ and $\mathrm{V}_{5}$ are found. Then, the algebraic sum of the voltages is $V_{1}-V_{3}-V_{4}+V_{2}-V_{5}=0$

Or

$$
V_{1}+V_{2}=V_{3}+V_{4}+V_{5}
$$

Sum of the voltage rises or sources $=$ Sum of the voltage drops

## Exercise 1.5.1 :

Find the values of $V_{1}$ and $V_{2}$ in the network given in figure 1.28.


Figure 1.28

## Solution:



Figure 1.28 (a)
Assume the current ' $I$ ' in a particular direction. Let $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ be the voltage drops in the 4 ohm and 2 ohm resistances respectively for a given current 'I' direction.

Applying KVL,

$$
\begin{aligned}
& 10-4 i+8-2 i=0 \\
& 18-6 i=0 \\
& 6 i=18 \\
& i=\frac{18}{6}=3 A
\end{aligned}
$$

Or

$$
\begin{aligned}
& V_{1}=4 \times i=4 \times 3=12 \mathrm{~V} \\
& V_{3}=2 \times i=2 \times 3=6 \mathrm{~V} \\
& V_{2}=-V_{3}=-6 \mathrm{~V}
\end{aligned}
$$

## Exercise 1.5.2:

Find the values of $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{0}$ in the circuit shown in figure 1.29.


Vo


Figure 1.29 (a)

## Solution:

Assume the current direction in the circuit as shown.
Let $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{\mathrm{y}}$ be the voltage drops in the resistances 10 ohms and 5 ohms respectively for a given current direction.

Applying KVL,

$$
35-10 i-2 V_{x}-V_{y}=0
$$

( $2 \mathrm{~V}_{\mathrm{x}}$ is a dependent voltage source whose voltage is a function of voltage drop across 10 ohms resistance)

$$
\begin{aligned}
& 35-10 i-2(10 \times i)-V_{y}=0 \\
& 35-10 i-20 i-V_{y}=0 \\
& 35-30 i-5 i=0 \\
& 35-35 i=0 \\
& i=\frac{35}{35}=1
\end{aligned}
$$

Then,

$$
V_{x}=10 i=10 \times 1=10 \mathrm{~V}
$$

$$
V_{0}=-V_{y}=-(5 \times 1)=-5 \mathrm{~V}
$$

## Exercise 1.5.3:

Find $\mathrm{V}_{0}$ and $\mathrm{I}_{0}$ in the given circuit (figure1.30).


Figure 1.30

## Solution:

Assuming the current ' I ' through 8 ohm resistance in a particular direction to satisfy the polarity of given voltage $\mathrm{V}_{0}$.


Figure 1.30 (a)
Since, 8 ohms and 2 ohms are connected in parallel,

$$
\begin{equation*}
2 i_{0}=V_{0} \quad \text { or } \quad i_{0}=\frac{V_{0}}{2} \tag{1.1}
\end{equation*}
$$

Applying KCL at node ' $a$ '

$$
\begin{aligned}
& 6-i_{0}-\frac{i_{0}}{4}-i=0 \\
& 6-i_{0}\left[1+\frac{1}{4}\right]-\frac{V_{0}}{8}=0 \\
& 6-1.25 i_{0}-\frac{V_{0}}{8}=0
\end{aligned}
$$

Writing $i_{0}=\frac{V_{0}}{2}$ in above equation,

$$
\begin{aligned}
& 6-1.25\left[\frac{V_{0}}{2}\right]-\frac{V_{0}}{8}=0 \\
& 6-1.375 \times \frac{V_{0}}{2}=0
\end{aligned}
$$

or $\quad V_{0}=\frac{6 \times 2}{1.375}=8 \mathrm{~V}$
And $\quad i_{0}=\frac{V_{0}}{2}=\frac{8}{2}=4 \mathrm{~A}$

## Exercise 1.5.4:

Obtain the currents $i_{1}, i_{2}$ and $i_{3}$ using KCL for the network shown in figure 1.31
Solution:
Applying KCL at node ' a ', $8-i_{1}-12=0$
Or

$$
i_{1}=8-12=-4 \mathrm{~mA}
$$

Applying KCl at node ' c ', $\quad 12+i_{3}-9=0$
Or

$$
i_{3}=9-12=-3 \mathrm{~mA}
$$

Applying KCL at node 'b',

$$
\begin{aligned}
& i_{1}+i_{2}=i_{3} \\
& -4+i_{2}=-3
\end{aligned}
$$



Figure 1.31

Or

$$
i_{2}=-3+4=1 \mathrm{~mA}
$$

## Exercise 1.5.5:

Find the values of $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ in the network given in figure 1.32.


Figure 1.32

## Solution:

In the loop abcda, $V_{1}-1-5=0$ or $V_{1}=6 \mathrm{~V}$
In the loop adefa, $5-2-V_{2}=0$ or $V_{2}=3 \mathrm{~V}$

## Exercise 1.5.6:

Calculate the values of V and $\mathrm{i}_{\mathrm{x}}$ in the circuit shown in figure 1.33.


Figure 1.33

## Solution:

Assuming the currents $i_{1}$ and $i_{2}$. Also identify the different nodes $a, b, c$ and $d$.


Figure 1.33 (a)
In loop abca, $12-12 i_{1}-2=0$

$$
\begin{aligned}
& 10-12 i_{1}=0 \\
& i_{1}=\frac{10}{12}=\frac{5}{6} \mathrm{~A}
\end{aligned}
$$

and

$$
V=i_{1} \times 12=\frac{5}{6} \times 12=10 \mathrm{~V}
$$

In the loop acda,

$$
\begin{aligned}
& 2-8-3 i_{x}=0 \\
& 3 i_{x}=-6 \\
& i_{x}=\frac{-6}{3}=-2 \mathrm{~A}
\end{aligned}
$$

## Exercise 1.5.7:

Find the voltages $\mathrm{V}_{1}, \mathrm{~V}_{2}$ and $\mathrm{V}_{3}$ using KVL for the circuit shown in figure 1.34


Figure 1.34

## Solution:

Identify the nodes $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$, e and f .
In the loop abcdefa, $24-V_{1}-10-12=0$
$24-V_{1}-22=0$ or $V_{1}=2 \mathrm{~V}$
In the loop abca, $24-2+V_{2}=0$ or $V_{2}=-22 \mathrm{~V}$
In the loop fcdef, $\quad V_{3}-10=0$ or $V_{3}=10$

## Exercise 1.5.8:

Find the values of $\mathrm{V}_{\mathrm{ab}}$ and I in the given circuit shown in figure 1.35 .


Figure 1.35

## Solution:

Applying KVL,

$$
\begin{aligned}
& 30-3 i+10-5 i-8=0 \\
& 32-8 i=0 \text { or } \quad i=\frac{32}{8}=4 \mathrm{~A}
\end{aligned}
$$

Voltage drop in 5 ohm $=4 \times 5=20 \mathrm{~V}$
$V_{a b}=$ Potential of point ' $a$ ' with respect to point ' $b$ '.
Then, $V_{a b}($ starting from $a)=+8+20=28 \mathrm{~V}$

## Exercise 1.5.9:

Determine the voltage drop across 10 ohm resistance for the circuit shown in figure 1.36.


Figure 1.36

## Solution:

Let us identify the points ' $A$ ' and ' $B$ ' across 10A current source initially. These points get extended to points of same potential as shown. The above circuit gets modified as shown in figure 1.36(a).


Figure 1.36 (a)

Let 'V' be the voltage across 10 ohm resistance.

$$
V=i_{2} \times 10
$$

Applying KCL at node 'A',

$$
\begin{aligned}
& 10-\frac{V}{20}-\frac{V}{10}-\frac{V}{5}+15=0 \\
& 25-V\left[\frac{1}{20}+\frac{1}{10}+\frac{1}{5}\right]=0 \\
& 25-V\left[\frac{1+2+4}{20}\right]=0
\end{aligned}
$$

Or $\quad V=71.4 \mathrm{~V}$

## Exercise : 1.5.10

Find the current flowing through different resistances using Kirchhoff's Laws for the circuit shown in figure 1.37.


Figure 1.37

## Solution:

Identify the different nodes $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d and assume the current directions through all branches.
Applying KVL for the loop abca,

$$
\begin{aligned}
& -2 i_{2}-1 \times i_{3}+1\left[i_{1}-i_{2}\right]=0 \\
& -2 i_{2}-i_{3}+i_{1}-i_{2}=0
\end{aligned}
$$

$$
\begin{equation*}
i_{1}-3 i_{2}-i_{3}=0 \tag{1}
\end{equation*}
$$

Applying KVL for the loop cbdc,

$$
\begin{align*}
& 1 \times i_{3}-4\left(i_{2}-i_{3}\right)+3\left(i_{1}-i_{2}+i_{3}\right)=0 \\
& i_{3}-4 i_{2}+4 i_{3}+3 i_{1}-3 i_{2}+3 i_{3}=0 \\
& 3 i_{1}-7 i_{2}+8 i_{3}=0 \tag{2}
\end{align*}
$$

Applying KVL for the loop acda through 12 V source,


Figure 1.37(a)
$-\left(i_{1}-i_{2}\right) \times 1-\left(i_{1}-i_{2}+i_{3}\right) \times 3+12=0$
$-i_{1}+i_{2}-3 i_{1}+3 i_{2}-3 i_{3}+12=0$
$-4 i_{1}+4 i_{2}-3 i_{3}=-12$
$4 i_{1}-4 i_{2}+3 i_{3}=12$
Solving the equations (1) and (2),
(1) $\mathrm{x} 33 i_{1}-9 i_{2}-3 i_{3}=0$
-(2), $\quad-3 i_{1}+7 i_{2}-8 i_{3}=0$

$$
\begin{equation*}
-2 i_{2}+11 i_{3}=0 \tag{4}
\end{equation*}
$$

Solving equations (2) and (3),
(2) $\mathrm{x} 4, \quad 12 i_{1}-28 i_{2}+32 i_{3}=0$
$-(3) \times 3, \quad-12 i_{1}+12 i_{2}-9 i_{3}=-36$

$$
\begin{equation*}
-16 i_{2}+23 i_{3}=-36 \tag{5}
\end{equation*}
$$

Solving equations (4) and (5),
-(4) $\mathrm{x} 8, \quad 16 i_{2}-88 i_{3}=0$
(5), $\quad-16 i_{2}+23 i_{3}=-36$

$$
0-65 i_{3}=-36 i_{3}=\frac{36}{65}=0.553 \mathrm{~A}
$$

From equation (4), $-2 \times i_{2}+11 \times \frac{36}{65}=0$ or $i_{2}=\frac{396}{130} \mathrm{~A}$
From equation (1), $i_{1}-3 \times \frac{396}{130}-\frac{36}{65}=0$ or $i_{1}=9.69 \mathrm{~A}$
The different branch currents are $\mathrm{i}_{1}=9.69 \mathrm{~A}, i_{2}=\frac{396}{130} \mathrm{~A}$ and $\mathrm{i}_{3}=0.553 \mathrm{~A}$

## Exercise: 1.5.11

Find the voltage $\mathrm{V}_{\mathrm{x}}$ in the circuit shown in figure 1.38.


Figure 1.38

## Solution:

Let us assume the current ' $I$ ' in the circuit and applying KVL,

$$
\begin{aligned}
& 15-1 \times i-2 V_{x}-5 i-2 i=0 \\
& 15-8 i-2 V_{x}=0 \\
& 15-8 \times \frac{V_{x}}{5}-2 V_{x}=0 \\
& 15=1.6 V_{x}+2 V_{x} \\
& V_{x}=\frac{15}{3.6}=4.16 V
\end{aligned}
$$

## Exercise : 1.5.12

Find the voltage across all the resistances and branch currents using Kirchhoff's Laws for the circuit shown in figure 1.39.


Figure 1.39

## Solution:

Identify different nodes a,b,c and d. Assume the currents in different branches as shown.
Applying KCL at node c,

$$
i_{1}=i_{2}+i_{3} \text { or } i_{3}=i_{1}-i_{2}
$$

Applying KVL for the loop abca

$$
\begin{equation*}
5-2 i_{1}-8 i_{2}=0 \tag{1}
\end{equation*}
$$

Or $\quad 2 i_{1}+8 i_{2}=5$
Applying KVL for the loop acda,

$$
\begin{equation*}
8 i_{2}-4\left(i_{1}-i_{2}\right)+3=0 \tag{2}
\end{equation*}
$$

Or $\quad-4 i_{1}+12 i_{2}=-3$

Solving the equations (1) and (2),
$2 \mathrm{x}(1), \quad 4 i_{1}+16 i_{2}=10$
(2), $-4 i_{1}+12 i_{2}=-3$
$28 i_{2}=7$ or $i_{2}=\frac{7}{28}=0.25 \mathrm{~A}$
From equation (1), $2 i_{1}+8 \times 0.25=5$ or $i_{1}=\frac{3}{2}=1.5 \mathrm{~A}$
And also, $i_{3}=i_{1}-i_{2}=1.5-0.25=1.25 \mathrm{~A}$
Then, the voltage drops across different resistances are

$$
\begin{aligned}
& V_{1}=2 \times i_{1}=2 \times 1.5=3 V \\
& V_{2}=8 \times i_{2}=8 \times 0.25=2 \mathrm{~V} \\
& V_{3}=4 \times i_{3}=4 \times 1.25=5 \mathrm{~V}
\end{aligned}
$$

## Exercise 1.5.13:

What is the voltage between the points A and B in the circuit shown in figure 1.40 ?


Figure 1.40

## Solution:

Assume the current directions $i_{1}$ and $i_{2}$. No current flows through 12 V source as there is no closed path or return path for the current which enters this branch.

Identify the polarity of voltage drops across different resistances for a given directions of the currents.

$$
i_{1}=\frac{6}{6+4}=0.6 \mathrm{~A} \quad \text { and } \quad i_{2}=\frac{12}{4+10}=\frac{6}{7} \mathrm{~A}
$$

To find the voltage between the points A and B , start considering the potential from point B .
From B to D, the voltage rises by $4 \times i_{2}=4 \times \frac{6}{7}=3.4 \mathrm{~V}$
From D to C, the voltage rises by 12 V
From C to A , voltage drops by $4 i_{1}=4 \times 0.6=2.4 \mathrm{~V}$
Therefore, the voltage between points A and $\mathrm{B}=V_{A B}=3.4+12-2.4=13 \mathrm{~V}$
Or the potential of point B with respect to point $\mathrm{A}=V_{B A}=-13 \mathrm{~V}$
Or the potential difference between A and $\mathrm{B}=\mp 13 \mathrm{~V}$

## Exercise 1.5.14:

Find the power absorbed by each element in circuit shown in figure 1.41.


Figure 1.41

## Solution:

Identify the nodes ' $x$ ' and ' $y$ '. By keen observation, all the elements got connected in parallel between x and y . The resultant circuit becomes,


Figure 1.41 (a)

$$
1000 \times i_{1}=4000 \times i_{2}
$$

Or $\quad i_{2}=\frac{-1000}{4000} \times i_{1}=-0.25 i_{1}$
Applying KCL at the node ' $x$ ',

$$
\begin{aligned}
& 3 i_{1}+20 \times 10^{-3}-i_{2}+i_{1}=0 \\
& 3 i_{1}+20 \times 10^{-3}-\left(-0.25 i_{1}\right)+i_{1}=0 \\
& 4.25 i_{1}+20 \times 10^{-3}=0 \\
& 4.25 i_{1}=-20 \times 10^{-3} \text { or } i_{1}=-4.7 \mathrm{~mA} \\
& i_{2}=-(0.25) \times\left(-4.7 \times 10^{-3}\right)=1.175 \mathrm{~A}
\end{aligned}
$$

Power absorbed by 4 K resistance $=i_{2}^{2} \times 4000=\left(1.175 \times 10^{-3}\right)^{2} \times 4000=5.5 \mathrm{~mW}$
Power absorbed by 1 K resistance $=i_{1}^{2} \times 1000=\left(4.7 \times 10^{-3}\right)^{2} \times 1000=22 \mathrm{~mW}$

### 1.6 Problems on Series-Parallel Connections

## Exercise 1.6.1:

Find the current delivered by the source for the circuit shown in figure 1.42.


Figure 1.42

## Solution:

Identify the nodes $\mathrm{A}, \mathrm{B}$ and C . Between A and $\mathrm{B}, 2$ ohms and 2 ohms are in series.
Between the points B and C, 2 ohms and 1 ohm are connected in series. Then, the circuit gets simplified to:


Figure 1.42 (a)
Between A and B, 4 ohms and 4 ohms are connected in parallel.
Between B and C, 2 ohms and 3 ohms are in parallel.


Figure 1.42 (b)
Between A and C, 2 ohms and 1.2 ohms are in series.
Then, $R_{A C}=\frac{(2+1.2) \times 2}{2+(2+1.2)}=1.23 \mathrm{ohms}$


Figure 1.42 (c)


Figure 1.42 (d)


Figure 1.42 (e)

The current supplied by the source $i=\frac{30}{1.05}=28.57 \mathrm{~A}$

## Exercise : 1.6.2

Find the resistance between the points ' $x$ ' and ' $y$ 'in the circuit shown in figure 1.43. Also find the total current supplied by 6 V sources when it is connected between ' $x$ ' and ' $y$ '.


Figure 1.43

## Solution:

Identify the nodes $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E as shown.


Figure 1.43 (a)

Between E and D, 5 K and 4.7 K resistances are connected in parallel.


Figure 1.43 (b)
Between C and $\mathrm{E}, 4 \mathrm{~K}$ and 2.42 K resistances are connected in series and it is in parallel with 3 K resistance.


Figure 1.43 (c)
Resistance between ' $x$ ' and ' $y$ ' is obtained as follows.


Figure 1.43 (d)

$$
R_{x y}=\frac{10 \times 2.04}{10+2.04}=1.698 \mathrm{~K} \mathrm{Ohms}
$$

When 6 V battery is connected between x and y ,
The current driven $=\frac{6}{1.698 \times 10^{3}}=3.53 \mathrm{~mA}$

## Exercise : 1.6.3

Find the resistance between ' $a$ ' and ' $b$ ' for the circuit shown in figure 1.44

## Solution:

The nodes A and B are of same potential. Hence, these two nodes are one and same. Then, the above circuit gets reduced to:



Figure 1.44

Figure 1.44 (a)

### 1.7 Star/Delta Transformations

Star- Delta transformations are useful to solve the networks which cannot be easily solved by regular series parallel circuit reduction technics.


Figure 1.45 (a) Star Network (b) Delta Network

## For Star Connection:

The resistance between the points ' a ' and ' b ' $=R_{a-b}=R_{a}+R_{b}$
The resistance between points ' b ' and ' c ' $=R_{b-c}=R_{b}+R_{c}$
Similarly, the resistance between ' c ' and ' a ' $=R_{c-a}=R_{a}+R_{c}$

## For Delta Connection:

The resistance between the points ' a ' and ' b ' $R_{a-b}=R_{a b} \|\left(R_{b c}+R_{c a}\right)$

$$
\begin{aligned}
& =\frac{R_{a b}\left(R_{b c}+R_{c a}\right)}{R_{a b}+R_{b c}+R_{c a}} \\
& =\frac{R_{a b} R_{b c}+R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}
\end{aligned}
$$

The resistance between the points 'b' and 'c' $R_{b-c}=R_{b c} \|\left(R_{a b}+R_{c a}\right)$

$$
\begin{aligned}
& =\frac{R_{b c}\left(R_{a b}+R_{c a}\right)}{R_{a b}+R_{b c}+R_{c a}} \\
& =\frac{R_{b c} R_{a b}+R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}
\end{aligned}
$$

The resistance between the points ' c ' and ' a ' $R_{c-a}=R_{c a} \|\left(R_{a b}+R_{b c}\right)$

$$
\begin{aligned}
& =\frac{R_{c a}\left(R_{a b}+R_{b c}\right)}{R_{a b}+R_{b c}+R_{c a}} \\
& =\frac{R_{c a} R_{a b}+R_{c a} R_{b c}}{R_{a b}+R_{b c}+R_{c a}}
\end{aligned}
$$

## To convert the given Delta Network into equivalent Star Network:

By equating the values of $\mathrm{R}_{\mathrm{a}-\mathrm{b}}, \mathrm{R}_{\mathrm{b}-\mathrm{c}}$ and $\mathrm{R}_{\mathrm{c}-\mathrm{a}}$ of both star and delta connections, we get

$$
\begin{align*}
& R_{a}+R_{b}=\frac{R_{a b} R_{b c}+R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}  \tag{1}\\
& R_{b}+R_{c}=\frac{R_{b c} R_{a b}+R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}  \tag{2}\\
& R_{c}+R_{a}=\frac{R_{c a} R_{a b}+R_{c a} R_{b c}}{R_{a b}+R_{b c}+R_{c a}} \tag{3}
\end{align*}
$$

Equations (1) -(2),

$$
\begin{align*}
& R_{a}+R_{b}-R_{b}-R_{c}=R_{a}-R_{c} \\
& \frac{R_{a b} R_{b c}+R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}-\frac{R_{b c} R_{a b}+R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}=\frac{R_{a b} R_{b c}+R_{a b} R_{c a}-R_{b c} R_{a b}-R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \\
& R_{a}-R_{C}=\frac{R_{a b} R_{c a}-R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \tag{4}
\end{align*}
$$

Equation (3) + (4),

$$
\begin{aligned}
& R_{a}+R_{c}+R_{a}-R_{c}=2 R_{a} \\
& =\frac{R_{c a} R_{a b}+R_{c a} R_{b c}}{R_{a b}+R_{b c}+R_{c a}}+\frac{R_{a b} R_{c a}-R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \\
& =\frac{2 R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}
\end{aligned}
$$

Or

$$
\begin{equation*}
R_{a}=\frac{R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \tag{5}
\end{equation*}
$$

Similarly

$$
\begin{equation*}
R_{b}=\frac{R_{a b} R_{b c}}{R_{a b}+R_{b c}+R_{c a}} \tag{6}
\end{equation*}
$$

Similarly $\quad R_{c}=\frac{R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}$
To convert the given Star Network into equivalent Delta Network:
Multiplying equations (5) and (6)

$$
\begin{aligned}
R_{a} \times R_{b} & =\frac{R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \times \frac{R_{a b} R_{b c}}{R_{a b}+R_{b c}+R_{c a}} \\
& =\frac{R_{a b}^{2} R_{b c} R_{c a}}{\left(R_{a b}+R_{b c}+R_{c a}\right)^{2}}
\end{aligned}
$$

Multiplying equations (6) and (7)

$$
\begin{aligned}
R_{b} \times R_{c} & =\frac{R_{a b} R_{b c}}{R_{a b}+R_{b c}+R_{c a}} \times \frac{R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \\
& =\frac{R_{a b} R_{b c}^{2} R_{c a}}{\left(R_{a b}+R_{b c}+R_{c a}\right)^{2}}
\end{aligned}
$$

Multiplying equations (7) and (5)

$$
\begin{align*}
& R_{c} \times R_{a}=\frac{R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \times \frac{R_{a b} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \\
& \begin{aligned}
=\frac{R_{a b} R_{b c} R_{c a}^{2}}{\left(R_{a b}+R_{b c}+R_{c a}\right)^{2}}
\end{aligned} \\
& \begin{aligned}
R_{a} R_{b}+R_{b} R_{c}+R_{c} R_{a} & =\frac{R_{a b}^{2} R_{b c} R_{c a}+R_{a b} R_{b c}^{2} R_{c a}+R_{a b} R_{b c} R_{c a}^{2}}{\left(R_{a b}+R_{b c}+R_{c a}\right)^{2}} \\
& =\frac{R_{a b} R_{b c} R_{c a}\left(R_{a b}+R_{b c}+R_{c a}\right)}{\left(R_{a b}+R_{b c}+R_{c a}\right)^{2}} \\
& =\frac{R_{a b} R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}}
\end{aligned}
\end{align*}
$$

By equation (8)/(5),

$$
\begin{aligned}
& \frac{R_{a} R_{b}+R_{b} R_{c}+R_{c} R_{a}}{R_{a}}=\frac{R_{a b} R_{b c} R_{c a}}{R_{a b}+R_{b c}+R_{c a}} \times \frac{R_{a b}+R_{b c}+R_{c a}}{R_{a b} R_{c a}} \\
& R_{b c}=\frac{R_{a} R_{b}+R_{b} R_{c}+R_{c} R_{a}}{R_{a}}
\end{aligned}
$$

Similarly, by performing the operations, (8)/(6) and (8)/(7), we get

$$
\begin{aligned}
& R_{c a}=\frac{R_{a} R_{b}+R_{b} R_{c}+R_{c} R_{a}}{R_{b}} \\
& R_{a b}=\frac{R_{a} R_{b}+R_{b} R_{c}+R_{c} R_{a}}{R_{c}}
\end{aligned}
$$

If all the branches have equal values of resistances $\left(R_{\Delta}\right)$ in a Delta network, the equivalent resistance value in each branch of Star network is

$$
R_{a}=R_{b}=R_{c}=R_{s t a r}=\frac{R_{\Delta}}{3}
$$

Similarly, If all the branches have equal values of resistances $\left(R_{\text {Star }}\right)$ in a Star network, the equivalent resistance value in each branch of Delta network is

$$
R_{a b}=R_{b c}=R_{c a}=R_{\Delta}=3 \times R_{s t a r}
$$

As the connection is transformed from Delta to Star, the resistance in each leg or branch decreases. Similarly, as the transformation is from Star to Delta, the resistance in each branch increases.

## Exercise 1.7.1:

Convert the given Delta Network shown in figure 1.46 into equivalent Star Network.


Figure 1.46

## Solution:

$$
\begin{aligned}
& R_{a}=\frac{25 \times 10}{25+15+10}=\frac{250}{50}=5 \mathrm{ohms} \\
& R_{b}=\frac{25 \times 15}{25+15+10}=\frac{375}{50}=7.5 \mathrm{ohms} \\
& R_{c}=\frac{10 \times 15}{25+15+10}=\frac{150}{50}=3 \mathrm{ohms}
\end{aligned}
$$

## Exercise 1.7.2:

Find the current supplied by 120 V Battery shown in figure 1.47.


Figure 1.47

## Solution:

The Star network between the points $a, b, c$ is converted into Delta network as follows.


Figure 1.47 (a)

$$
\begin{aligned}
& R_{a c}=\frac{5 \times 10+10 \times 20+20 \times 5}{20}=\frac{50+200+100}{20}=\frac{350}{20}=17.5 \mathrm{ohms} \\
& R_{a b}=\frac{5 \times 10+10 \times 20+20 \times 5}{5}=\frac{50+200+100}{5}=\frac{350}{5}=70 \mathrm{ohms}
\end{aligned}
$$

$$
R_{b c}=\frac{5 \times 10+10 \times 20+20 \times 5}{10}=\frac{50+200+100}{10}=\frac{350}{10}=35 \mathrm{ohms}
$$



Figure 1.47 (b)

The above circuit gets reduced to


Figure 1.47 (c)
It is further reduced to


Figure 1.47 (d)
and


Figure 1.47 (e)

The current supplied $i=\frac{120}{9.63}=12.46 \mathrm{~A}$

## Exercise: 1.7.3:

Find the current supplied by the source for the circuit shown in figure 1.48


Figure 1.48

## Solution:

The Delta Network between the points a,b,c is converted into equivalent Star Network.


Figure 1.48 (a)

$$
\begin{aligned}
& R_{a}=\frac{3 \times 2}{3+2+1}=\frac{6}{6}=1 \mathrm{ohms} \\
& R_{b}=\frac{3 \times 1}{3+2+1}=\frac{3}{6}=0.5 \mathrm{ohms} \\
& R_{c}=\frac{1 \times 2}{3+2+1}=\frac{2}{6}=0.33 \mathrm{ohms}
\end{aligned}
$$

By substituting these values in above network, we get


Figure 1.48 (b)
It is simplified to


Figure 1.48 (c)

It is again further reduced to


Figure 1.48 (d)
The current delivered by the source $i=\frac{12}{3.44}=3.49 \mathrm{~A}$
Exercise 1.7.4 : Find the equivalent resistance and the current ' $I$ ' for the circuit shown in figure 1.49.


Figure 1.49

## Solution:

The resistances 2 ohms and 1 ohm are connected in series. Also, 3 ohms and 5 ohms are in series. Then, the circuit gets changed to


Figure 1.49 (a)
The delta connections between the points $a, b, c$ and $d, e, f$ are changed to equivalent star networks as follows.


Figure 1.49 (b)

It is further reduced to


Figure 1.49 (c)
Then, it is simplified to


Figure 1.49 (d)

The equivalent resistance is given by $R_{e q}=12.211 \mathrm{ohm}$
The current ' I ' is given by $i=\frac{20}{12.211}=1.638 \mathrm{~A}$

### 1.8 Superposition Theorem

The Superposition Theorem states that in any linear, bilateral network containing two or more sources, the response in any element is equal to the algebraic sum of responses caused by individual responses acting alone, while the other sources are non-operative. When considering the responses of individual sources, other ideal voltage sources and ideal current sources are replaced by short circuit and open circuits respectively. This theorem is valid only for linear systems.

A linear network is a network whose parameters do not change with voltage and current. That means, V-I characteristic is linear. Bilateral Network is network whose characteristics or behavior is same irrespective of the direction of current through various elements. Simple resistor is an example of a Bilateral network and Diode is an Unilateral element.

The Superposition Theorem is well illustrated through the following example.
Exercise 1.8.1: Find the voltage across the 2 ohms of the circuit shown in figure 1.50 using Superposition Theorem.


Figure 1.50

## Solution:

To find the voltage across 2 ohm resistor, the current passing through this resistance should be known. The individual responses or currents in 2 ohm resistance are determined as follows.

## To find the response caused by 10 V Source alone:

For this, 20 V source is replaced by an open circuit and 2 A current source is replaced by an open circuit as shown below.


Figure 1.50 (a)
The 2 ohms and 5 ohms are connected in series and this combination is in parallel with 20 ohms resistance. Then, the resultant of this combination is given by

$$
R_{1}=\frac{20 \times(2+5)}{20+2+5}=5.18 \mathrm{ohm}
$$

The total resistance $R=10+5.18=15.18 \mathrm{ohm}$
The total current supplied by 10 V source is $I=\frac{10}{15.18}=0.658 \mathrm{~A}$
The current passing through the 2 ohms from A to $\mathrm{B} i_{1}=0.658 \times \frac{20}{20+7}=0.4879 \mathrm{~A}$

## To find the response caused by 2 A Source alone:

For this, the 10 V and 20 V voltage resources are replaced by short circuits as shown.


Figure 1.50 (b)

The 10 ohm resistance gets connected in shunt with 20 ohm resistance and this combination is in series with the 2 ohm resistance. Then, the resultant resistance of this branch is given by

$$
R_{e q}=2+\frac{20 \times 10}{20+10}=8.66 \mathrm{ohm}
$$

Current passing through the 2 ohms resistance from B to A is given by

$$
i_{2}=2 \times \frac{5}{5+8.66}=0.732 \mathrm{~A}
$$

Since, the current flows from B to $\mathrm{A}, i_{2}=-0.732 \mathrm{~A}$

## To find the response caused by 20V Source alone:

For this, the 10 V voltage source and 2 Acurrent resources are replaced by open circuit and short circuits respectively as shown.


Figure 1.50 (c)
The total resistance of the circuit is given by

$$
R_{e q}=\frac{10 \times 20}{10+20}+2+5=13.66 \mathrm{ohm}
$$

The current supplied by 20 V battery is $I=\frac{20}{13.66}=1.464 \mathrm{~A}$
The same current passes through 2 ohms resistance also from B to A. Hence, $i_{3}=-1.434 \mathrm{~A}$
The sum of the individual responses $=0.4879-0.732-1.434=-1.708 \mathrm{~A}$
Then, the voltage across 2 ohm resistance $=1.708 \times 2=3.416 \mathrm{~V}$
Or $V_{A B}=-3.416 \mathrm{~V}$ or $V_{B A}=3.416 \mathrm{~V}$ (Since, the potential of B is higher than A . the net current flows from point B to A )

### 1.9 Thevenin's Theorem

It is quite common to find the current in a particular resistance (Generally called as Load Resistance $R_{L}$ ) which changes with remaining circuit leftovers same. To avoid the labor in simplifying the circuit for every value of $R_{L}$, Thevenin's Theorem is useful. This theorem replaces the leftover circuit with an equivalent circuit called Thevenin's Equivalent circuit.

Thevenin's Theorem states that a linear, bilateral two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $V_{t h}$ in series with a resistance $R_{t h}$. Where $V_{t h}$ is the open circuit voltage between the load terminals and $\mathrm{R}_{\mathrm{th}}$ is the resistance between the load terminals.

The figure 1.51 depicts how a given circuit is represented by its equivalent Thevenin's equivalent circuit.


Figure 1.51 Thevenin's equivalent Circuit
The steps involved for applying Thevenin's Theorem to solve a given circuit are:

1. Finding Thevenin's Resistance $R_{T h}$ : Remove the load resistance between the terminals ' $a$ ' and ' $b$ '. Replace the voltage sources with the short circuits and current sources with open circuits. Find the resultant resistance between the terminals 'a and 'b. this resistance is called 'Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$ '. The procedure for the above circuit is illustrated in figure 1.52.


Figure 1.52 Thevenin's equivalent resistance

The Thevenin's Equivalent Resistance $R_{T h}=\frac{R_{1} \times R_{2}}{R_{1}+R_{2}}+R_{2}$
2. Finding Thevenin's Voltage $\mathrm{V}_{\mathrm{Th}}$ : Remove the load resistance between the terminals 'a' and ' $b$ '. Obtain the open circuit voltage between the terminals ' $a$ ' and ' $b$ '. This open circuit voltage $\mathrm{V}_{\mathrm{ab}}$ is known as 'Thevenin's Voltage $\mathrm{V}_{\mathrm{Th}}$ '. For the above circuit, the Thevenin's Voltage is given by


Figure 1.53 Finding Thevenin's voltage

$$
\begin{aligned}
& I_{1}=\frac{V_{S}}{R_{1}+R_{2}} \\
& \begin{aligned}
V_{0} & =V_{a b}=I_{1} \times R_{3} \\
& =\frac{V_{S}}{R_{1}+R_{2}} \times R_{3} \\
& =V_{T h}
\end{aligned}
\end{aligned}
$$

3. Obtain the Thevenin's equivalent circuit as shown in figure 1.54. This circuit remains same irrespective of changes in load resistance.


Figure 1.54 Thevenin's Equivalent Circuit
4. Reconnect the load resistance $R_{L}$ and find the load current $I_{L}$.


Figure 1.55 Thevenin's Equivalent Circuit including load

$$
I_{L}=\frac{V_{T h}}{R_{T h}+R_{L}}
$$

Example 1.9.1: Find the current passing through 4 ohms resistance using Thevenin's Theorem.


Figure 1.56

## Solution:

To find the Thevenin's resistance, the resistance 4 ohm is removed from the circuit and the 20 V voltage source is short circuited. The resistance between the load terminals ' $a$ ' and ' $b$ ' is


Figure 1.57(a)

$$
R_{T h}=R_{a b}=\frac{5 \times 15}{5+15}+2+3=8.75 \mathrm{ohms}
$$

To find the Thevenin's resistance, the resistance 4 ohm is removed from the circuit and the open circuit voltage is found between the load terminals ' $a$ ' and ' $b$ '.


Figure 1.57(b)
$V_{T h}=V_{a b}=V_{x y}$ since no current flows through 2 and 3 ohms resistances.

$$
I=\frac{20}{5+15}=1 \mathrm{~A}
$$

The voltage across the 15 ohm resistance $V_{x y}=V_{a b}=V_{T h}=I \times 15=1 \times 15=15 \mathrm{~V}$
The Thevenin's equivalent circuit is given by


Figure 1.57(c)
The current passing through 4 ohms resistance is $I_{L}=\frac{V_{T h}}{R_{T h}+R_{L}}=\frac{15}{8.75+4}=1.17 \mathrm{~A}$
Example 1.9.2: Find the Thevenin's equivalent circuit for the circuit shown in figure . Find the value of 'I' for i) $R=3$ ohms and ii) $R=8$ ohms.


Figure 1.58

## Solution:

To find the Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$, the voltage sources are short circuited and the load resistance $R$ is removed. The resistance between the terminals ' $a$ ' and ' $b$ is called Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$.


Figure 1.59(a)

$$
R_{T h}=R_{a b}=\frac{(8+2) \times 20}{(8+2)+20}=\frac{200}{30}=6.66 \mathrm{ohm}
$$

To find the Thevenin's Voltage $\mathrm{V}_{\mathrm{Th}}$, the load resistance R is removed and the open circuit voltage between the terminals ' $a$ ' and ' $b$ ' is found.


Figure 1.59(b)

$$
\begin{aligned}
& V_{T h}=V_{a b}=V_{x y} \\
& I_{1}=\frac{8-32}{(8+2+20)}=-0.8 \mathrm{~A}
\end{aligned}
$$

$V_{T h}=V_{a b}=V_{x y}=+32+(-0.8 \times 20)=16 \mathrm{~V}$ (The potential of point 'a' with respect to point 'b')
(i) When $\mathrm{R}=3$ ohms


Figure 1.59(c)

$$
I_{L}=\frac{V_{T h}}{R_{T h}+R}=\frac{16}{6.66+3}=1.65 \mathrm{~A}
$$

(ii) When $\mathrm{R}=8$ ohms


Figure 1.59(d)

$$
I_{L}=\frac{V_{T h}}{R_{T h}+R}=\frac{16}{6.66+8}=1.09 \mathrm{~A}
$$

Example 1.9.3: Find the current flowing through 2 ohms resistor connected between the points 'a' and 'b' using Thevenin's Theorem.


Figure 1.60

## Solution:

To find the Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$, the voltage sources are short circuited and the load resistance 2 ohms is removed. The resistance between the terminals ' $a$ ' and ' $b$ is called Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$.


Figure 1.61(a)
The above circuit gets reduced to


Figure 1.61(b)

It further reduces to


Figure 1.61(c)
The Thevenin's Resistance $\mathrm{R}_{\text {Th }}$ is given by $R_{T h}=R_{a b}=2.4+1.875=4.275 \mathrm{ohms}$
To find the Thevenin's Voltage $\mathrm{V}_{\mathrm{Th}}$, the load resistance R is removed and the open circuit voltage between the terminals ' $a$ ' and ' $b$ ' is found.


Figure 1.61(d)

$$
\begin{aligned}
& x=\frac{12}{(4+6)}=1.2 \mathrm{~A} \\
& y=\frac{12}{(3+5)}=1.5 \mathrm{~A}
\end{aligned}
$$

The voltage drop across 4 ohms resistance $=1.2 \times 4=4.8 \mathrm{~V}$

The voltage drop across 4 ohms resistance $=1.5 \times 3=4.5 \mathrm{~V}$
The potential of point ' $a$ ' with respect to point ' $b$ ' is $+4.5 \mathrm{~V}-4.8 \mathrm{~V}=-0.3 \mathrm{~V}$ or the potential difference between the points ' a ' and ' b ' is $V_{T h}=V_{a b}=0.3 \mathrm{~V}$.

The Thevenin's equivalent circuit is shown in figure.


Figure 1.61(e)
The current flowing through 2 ohms resistance

$$
I=\frac{V_{T h}}{R_{T h}+R}=\frac{0.3}{4.275+2}=\frac{0.3}{6.275}=0.0478 \mathrm{~A}
$$

Example 1.9.4: Find the current passing through the load resistance $\mathrm{R}_{\mathrm{L}}$ using the Thevenin's Theorem for $\mathrm{R}_{\mathrm{L}}$ values 6 ohms and 16 ohms.


Figure 1.62

## Solution:

In above circuit, the given current source I converted into its equivalent voltage source to simplify the circuit as follows.


Figure 1.63(a)
By including this voltage source in above circuit in its appropriate place,


Figure 1.63(b)
To find the Thevenin's Voltage $\mathrm{V}_{\mathrm{Th}}$, the load resistance $\mathrm{R}_{\mathrm{L}}$ is removed and the open circuit voltage between the terminals ' $a$ ' and ' $b$ ' is found.


Figure 1.63(c)

$$
i=\frac{(32-24)}{(4+12)}=\frac{8}{16}=0.5 \mathrm{~A}
$$

Voltage drop across 12 ohms resistance $=12 \times 0.5=6 \mathrm{~V}$

$$
V_{T h}=V_{a b}=24+6=30 \mathrm{~V}
$$

To find the Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$, the voltage sources are short circuited and the load resistance $R_{L}$ is removed. The resistance between the terminals ' $a$ ' and ' $b$ is called Thevenin's Resistance $\mathrm{R}_{\mathrm{Th}}$.


Figure 1.63(d)
Thevenin's Resistance $R_{T h}=1+\frac{(4 \times 12)}{(4+12)}=1+3=4 \mathrm{ohms}$
The Thevenin's equivalent circuit is shown in figure 1.63(e).


Figure 1.63(e)
(i) When $\mathrm{R}_{\mathrm{L}}=6$ ohms,

$$
I_{L}=\frac{V_{T h}}{R_{T h}+R_{L}}=\frac{30}{4+6}=3 \mathrm{~A}
$$

(ii) When $\mathrm{R}_{\mathrm{L}}=16$ ohms,

$$
I_{L}=\frac{V_{T h}}{R_{T h}+R_{L}}=\frac{30}{4+16}=1.5 \mathrm{~A}
$$

### 1.10 Time Response of RL and RC Circuits

### 1.10.1 Introduction to Transients

A network subjected to constant energy sources is said to be in steady state if the branch currents and voltages do not change with time. The networks with the sinusoidal voltage and currents (which are periodic functions of time) are also assumed to be in steady state condition. That means that the amplitude and frequency of sinusoidal quantity do not change in steady state.

Before a circuit reaches its steady state, which is different from its previous state, it passes through a transition period. This period is required for the currents and voltages to adjust themselves to their steady state modes of variation is called 'Transient State' or 'Transient Period'. The behavioral analysis of voltages and currents in this state is called 'Transient Analysis'. In general, any switching operation within the circuit itself causes transient conditions to exist in the circuit. Although transient periods are of very short spans, they result for some serious operating problems.

The application of Kirchhoff's Laws is extended to analyze the circuits under transient conditions too. Such application results for differential equations whose solution consists of two parts; Complementary Function and Particular Solution. The Complementary Function always becomes zero in relatively short time which is referred to as Transient Response or Source Free response. The particular Solution is the Steady State response or Forced Response. The study of transient analysis is started with a network having passive elements and sources having known initial voltage and currents values. At the reference time, $\mathrm{t}=0$, the system state is altered by closing/opening the switch.

### 1.10.2 DC Response of an R-L Circuit

Consider a circuit consisting of a series R-L parameters connected to a DC source (battery) through a controlling switch ' S '. as shown in figure 1.64.

When the switch ' S ' is closed at $\mathrm{t}=0$, the circuit gets subjected to a transient state. By applying KVL to the circuit in this state,


Figure 1.64 Series R-L Circuit

$$
V=i R+L \frac{d i}{d t}
$$

Or $\frac{d i}{d t}+i \frac{R}{L}=\frac{V}{L}$ which is first order linear differential equation. The current ' i ' is the solution to be found.

By comparing the above equation with a non-homogeneous differential equation

$$
\frac{d x}{d t}+P x=K
$$

Whose solution is

$$
x=e^{-P t} \int K e^{P t} d t+C e^{-P t}
$$

Where ' C ' is an arbitrary constant.
Also by comparison, $x=i, P=\frac{R}{L}$ and $K=\frac{V}{L}$
By substituting these values in above equation,

$$
\begin{align*}
i & =e^{-\left(\frac{R}{L}\right) t} \int \frac{V}{L} e^{\left(\frac{R}{L}\right) t} d t+C e^{-\left(\frac{R}{L}\right) t} \\
& =e^{-\left(\frac{R}{L}\right) t} \times \frac{V}{L} \times e^{\left(\frac{R}{L}\right) t} \times \frac{L}{R}+C e^{\left(\frac{-R}{L}\right) t} \\
i & =\frac{V}{R}+C e^{-\left(\frac{R}{L}\right) t} \tag{1}
\end{align*}
$$

To determine the value of ' C ', initial conditions are used. Initial current in the circuit at $\mathrm{t}=0$ is zero. ie., $\mathrm{i}=0$ at $\mathrm{t}=0$.

Writing these values in above equation (1),

$$
\begin{aligned}
0 & =\frac{V}{R}+C e^{-\left(\frac{R}{L}\right) \times 0} \\
\text { Or } \quad C & =\frac{V}{R}
\end{aligned}
$$

Again writing the value of ' $C$ ' in above equation (1),

$$
i=\frac{V}{R}-\frac{V}{R} e^{-\left(\frac{R}{L}\right) t}
$$

$$
\begin{equation*}
i=\frac{V}{R}\left[1-\exp \left[-\frac{R}{L} t\right]\right] \tag{2}
\end{equation*}
$$

The time response of the current for DC excitation in a series RL circuit is shown in the figure 1.65 .


Figure 1.65 Time response of current in Series R-L Circuit
The equation (2) consists of two parts. The steady state part $\frac{V}{R}$ and
The transient state part $\frac{V}{R} e^{-\left(\frac{R}{L}\right) t}$
After the switch is closed, the response (i) takes some time to reach a steady state as shown. This transition period taken for the current to reach its final or steady state value from its initial value is called Transient period.

The time constant of the transient part of the solution $\frac{V}{R} e^{-\left(\frac{R}{L}\right) t}$ is the time at which the exponent of 'e' becomes unity. Where 'e' is the base of natural logarithms.

Therefore, $\left(\frac{R}{L}\right) t=1 \quad$ or $\quad t=\frac{L}{R}=\tau \quad$ which is called 'Time Constant' and has the units 'seconds'.

At one Time Constant (TC), the value of ' t ' is $\frac{L}{R}$ seconds. The transient part of the solution at one TC is

$$
i(\tau)=-\frac{V}{R} e^{-\left(\frac{t}{\tau}\right)}=-\frac{V}{R} e^{-1}=-0.368 \frac{V}{R}
$$

That means, at one Time Constant (TC), the transient term of current reaches $36.8 \%$ of its initial value $\frac{V}{R}$. The total current at this one TC becomes $i=\frac{V}{R}-0.368 \frac{V}{R}=0.632 \frac{V}{R}$

Hence, in one Time Constant (TC), the current in the circuit increases to $63.2 \%$ of its final steady state value from its initial value as shown in figure 1.66.


Figure 1.66 Representation of Time Constant in Series R-L Circuit
The voltage across the resistor and inductor change as follows during transient state.

$$
\begin{aligned}
& V_{R}=i R=R \times \frac{V}{R}\left[1-e^{-\left(\frac{R}{L}\right) t}\right] \\
& =V\left[1-e^{-\left(\frac{R}{L}\right) t}\right]
\end{aligned}
$$

and,

$$
V_{L}=L \frac{d i}{d t}=L \times \frac{V}{R} \times \frac{R}{L} \times e^{-\left(\frac{R}{L}\right) t}=V e^{-\left(\frac{R}{L}\right) t}
$$



Figure 1.67 Variation of voltage and current in a Series R-L Circuit

The variations in voltages across resistance and inductor are shown in figure 1.67.
Similarly, the power consumed in the resistance during the transient state is

$$
\begin{aligned}
& P_{R}=V_{R} i=V\left[1-e^{-\left(\frac{R}{L}\right) t}\right]\left[1-e^{-\left(\frac{R}{L}\right) t}\right] \frac{V}{R} \\
& =\frac{V^{2}}{R}\left[1-2 e^{-\left(\frac{R}{L}\right) t}+e^{-\left(2 \frac{R}{L}\right) t}\right]
\end{aligned}
$$

And the power consumed in the inductor during transient state is

$$
\begin{aligned}
& P_{L}=V_{L} i=V e^{-\left(\frac{R}{L}\right) t} \times \frac{V}{R}\left[1-e^{-\left(\frac{R}{L}\right) t}\right] \\
& =\frac{V^{2}}{R}\left[e^{-\left(\frac{R}{L}\right) t}-e^{-\left(\frac{2 R}{L}\right) t}\right]
\end{aligned}
$$

The responses of powers are shown below in the figure 1.68.


Figure 1.68 Time response of Power in Series R-L Circuit

### 1.10.3 DC Response of an R-C Circuit

Consider a circuit having Series Resistance- Capacitance connected across a DC source (battery) through a control switch ' $S$ ' as shown in figure 1.69. Initially, the capacitor is assumed to be uncharged.


Figure 1.69 Series R-C Circuit
The switch ' $S$ ' is closed at $\mathrm{t}=0$. By applying the Kirchhoff's Laws to the circuit during the transient state,

$$
V=i R+\frac{1}{C} \int i . d t
$$

By differentiating the above equation, we get

$$
0=\frac{d i}{d t} R+\frac{1}{C} i
$$

Or $\quad \frac{d i}{d t}+\frac{1}{R C} i=0$
It is a linear differential equation and it is compared with a non-homogeneous differential equation

$$
\frac{d x}{d t}+P x=K
$$

Whose solution is

$$
x=e^{-P t} \int K e^{P t} d t+C e^{-P t}
$$

Where ' C ' is an arbitrary constant.
Also by comparison, $x=i, P=\frac{1}{R C}$ and $K=0$
The solution is

$$
\begin{aligned}
& i=e^{\frac{1}{R C} t} \int 0 \cdot e^{\frac{1}{R C} t} \cdot d t+C e^{\frac{1}{R C}} \\
& =C e^{-\frac{t}{R C}}
\end{aligned}
$$

This linear differential equation has only the Complementary Function (only transient response). The particular solution (steady state response) for above equation is zero.

Initial conditions have been used to find the value of ' C '.
At $\mathrm{t}=0$, the switch ' S ' is closed. Since, the capacitor does not allow the sudden changes in voltages, it acts as short circuit $(\mathrm{V}=0)$.

Then, at $t=0, i=\frac{V}{R}$
By writing the value of ' $I$ ' in above equation,

$$
\frac{V}{R}=C e^{-\frac{0}{R C}} \quad \text { or } \quad C=\frac{V}{R}
$$

Then, the current equation becomes,

$$
i=\frac{V}{R} e^{-\frac{t}{R C}}
$$

When the switch ' $S$ ' is closed, the response (current) decays with time as shown in figure1.70.


Figure 1.70 Time response of current in Series R-C Circuit
The time constant of transient is the time at which the exponent of e becomes unity.
That is, $\frac{t}{R C}=1$ or $t=R C=\tau=$ Time Constant (TC)
The voltage across the resistance is

$$
\begin{aligned}
& V_{R}=R \times i=R \times \frac{V}{R} e^{-\frac{t}{R C}} \\
& =V e^{-\frac{t}{R C}}
\end{aligned}
$$

Similarly, the voltage across the capacitor is

$$
\begin{aligned}
V_{C} & =\frac{1}{C} \int i . d t \\
& =\frac{1}{C} \int \frac{V}{R} e^{-\frac{-t}{R C}} d t \\
& =-\left[\frac{V}{R C} \times R C e^{-\frac{t}{R C}}\right]+C
\end{aligned}
$$

To find the value of C :
At $t=0$, voltage across capacitor is zero.

$$
0=-V e^{-0}+C \quad \text { or } \quad C=V
$$

Therefore, $V_{C}=-V e^{-\frac{t}{R C}}+V$

$$
=V\left[1-e^{-\frac{t}{R C}}\right]
$$

The variations in voltages are shown in figure 1.71.


Figure 1.71 Variation of voltages across R and C
The power consumed in resistor during transient period is

$$
\begin{aligned}
P_{R} & =V_{R} \times i=V e^{-\frac{t}{R C}} \times \frac{V}{R} e^{-\frac{t}{R C}} \\
& =\frac{V^{2}}{R} e^{-\frac{2 t}{R C}}
\end{aligned}
$$

Power in capacitor is

$$
\begin{aligned}
P_{C} & =V_{C} \times i=V\left(1-e^{-\frac{t}{R C}}\right) \frac{V}{R} e^{-\frac{t}{R C}} \\
& =\frac{V^{2}}{R}\left[e^{-\frac{t}{R C}}-e^{-\frac{2 t}{R C}}\right]
\end{aligned}
$$

The variations in powers with respect to time are shown in the figure 1.72.


Figure 1.72 Time response of Powers in Series R-C Circuit

## Review Questions

## Short Answer (Two marks) Questions

1. List the advantages of electrical energy over other forms of energies
2. Define 'Electric Generator' and 'Electric Motor'
3. What is an Electric Circuit003F
4. Compare Electric and Magnetic Circuits
5. Name the basis circuit parameters
6. Differentiate between Resistance and Inductance
7. How does an inductor behave in a DC circuit?
8. Write down the expression for energy stored in a Capacitor
9. How does a capacitor behave in a DC circuit?
10. Differentiate between Independent and Dependent Voltage Sources
11. Show the V-I characteristics of ideal voltage and current sources
12. Show the equivalent circuit of practical voltage and current sources
13. Find the equivalent current source for the voltage source given in figure 1.73. (Ans: 1.92A, 25 Ohms)


Figure 1.73
14. State KCL and KVL
15. Define Transient State
16. Write down the expression for time constant of a RC network

## Essay (Six marks) Questions

1. Explain the role of Electrical Energy in modern life
2. With the help of a single line diagram, briefly explain the concept of Electrical Power Generation, Transmission and Utilization
3. Briefly discuss on the behavior of a) Inductance and b) Capacitance when placed in electrical circuits
4. What are the different types of dependent sources? Show their equivalent circuits and V-I characteristics
5. Explain the procedure for converting a given Current Source into its equivalent Voltage Source
6. Explain briefly about the Kirchhoff's Laws with examples
7. Find the current flowing through 3 ohms resistor of figure1.61 using Kirchhoff's laws (Ans: 1.736A)


Figure 1.74
8. Find the source current using Star-Delta Transformation techniques for the circuit shown in figure 1.62. (Ans: 2.81 A )


Figure 1.75
9. State and explain the Superposition theorem
10. Discuss on DC response of a RL circuit. And write down the expression for current transient with respect to time.

