CHAPTER

1

Mathematics, the Science of Quantity

1.1 Introduction

Mathematics has been and will continue to be behind many developments in human capabilities, in the understanding of the real world and the place of humans in it. It helps us to understand what we see around us and to live well with it. It enables us to have some quantitative measure of the entities we come across in the real world, and thereby helps us in making us conscious of our scale and place in the universe, and the way the things happen around us.

To know how many, we are in our families, how old each of the members in them is, how large our dwellings are, how much money we have, how much food and water we have, how hot or cold it is outside, how long we have to wait to get the fruits of a crop, etc. we need quantitative expressions. Many more examples from our daily lives that are connected with quantities and measures can be cited. We get an understanding of the physical world, from the '*values*' of the entities in terms of which we conceive it. The physical world is fitted in a framework of dimensions- space and time, and all these should have a '*measure'* for us to be able to understand. Our needs have 'measures' although our greed may not have any!

According to Aristotle, "*mathematics is the science of quantity*". Mathematics is the study of measures, relationships, properties of quantities and sets using numbers and symbols. Mathematics is also a universal language with great clarity, definitiveness, brevity and exactitude. Symbols in mathematics are like alphabet, number forms as words having special rules of syntax, expressions and statements bound by logic, and facts stated in theorems which are like poems. Mathematics is a language common to other sciences. '*Number*' is central to mathematics. Its evolution into various forms, its properties, and its role in enabling us to create models – mathematical objects to represent real world situations on our study desks, is what makes it the key element in mathematics.

Like the words in different world languages, numbers had seen a diversity of expressions in the ancient world in different regions. Finally, they found an expression in a system that became unique and universal and the language of mathematics has been standardized. It is the ubiquitous *Hindu-Arabic Number System* or *Indian Number System* (INS). This book is about the number concept in general, and the impact of the INS on the world of mathematics and thereby on other sciences and all aspects of our day-to-day lives in particular. Its focus is on the mathematical 'REPRESENTATION' and the process of 'EVALUATION' of a multitude of entities which are applied in various modern fields of science and technology. Einstein asserted that no worthwhile scientific discoveries could have been made without the INS. In this book, the author presents his own findings which fully justify this sweeping remark from the greatest scientist of all time.

1.1.1 Positive Integers

Small positive integer numbers are recognized even by some animals. A bird may recognize the absence of one egg from its original set of two or three. An animal may have a sense of the number of its offspring. Beyond the small limits, even humans will not be sure unless they keep some record and compare the count with the number of real objects. This trend in record keeping by marks was seen in writing by humans on the Ishango bone. This bone with cut marks was found at the Ishango settlement of fishermen in what is at present the Democratic Republic of Congo in Africa. This is called the 'Ishango bone' and is understood to belong to the Upper Paleolithic era. It is about 10 cm long bone dark brown in color with a piece of quartz attached to it, seemingly for engraving on the bone. The bone has been scratched and polished leaving no clues of the animal to which it belonged. It was probably of a mammal. It is estimated to be 22000 years old. There is also another bone found in Southern Africa which is estimated to be 42000 year old. The cut marks in three sets along its length of the Ishango bone gave rise to different speculations. It could have been a tally stick, or a stick with the marks to facilitate grip or a counting tool. The cuts are in sets of 60s also suggesting some astronomical context in which the stick was used as a counting and recording tool.

Human capability to name numbers developed from humble beginnings. Some communities for example, the Aboriginals in Australia, name numbers as one, two, up to three or four only. Beyond this limit, for them, any number was *many*. The Piraha Amazonian people had no words in consistent use to refer to numbers, even for a single entity. Names for natural numbers were very limited even until many millennia later.

Counting up to ten and twenty for humans became possible probably due to the use of fingers and toes. As names for greater numbers were created, the nomenclature began to follow a systematic approach. The names of numbers literally followed certain pattern of groups such as tens, hundreds, thousands, and so on.

1.1.2 The Symbol for ONE

In the process of recording numbers, a short vertical line, suggesting a cut mark or a single raised finger, became the symbol for ONE in many communities and it remains the same as our modern symbol for ONE. The Babylonians made cuneiform impressions with a wedge like tool on soft clay tablets which were then dried or burned for permanence. They maintained the wedge symbol even for ONE because it lasts better than a vertical line which may be closed or lost in the process of handling soft clay tablets due to its thinness. The same wedge was used for all impressions; only the pressure varied to change the size of the impression. The symbols and the tools used in the ancient communities together with their numeration practices are described an impressive detail by Gullberg (2007) -a medical man with love for mathematics. Terry Jones' popular documentary on 'The Story of 1' (https://vimeo.com/56113926) is an interesting documentary on the history of numbers, in particular of ONE.

1.1.3 Names and Symbols for a Few Initial Natural Numbers

Names for a few initial natural numbers probably came up in a natural way. For example, ONE stands for the moon in the Indian Synonyms due to its uniqueness and regular appearance in the sky. TWO is due to pairs of eyes, arms, ears, etc. In this way, familiarity with objects and organs may have been responsible for the creation of names for TWO. Going further, the fingers of one and both hands were probably behind the creation of numbers FIVE and TEN. Due to the familiarity of these sets of objects, these numbers acquired names. Extensions beyond were mere combinations of the available names. Examples are ONE OVER TEN, TWO OVER TEN, … and so on.

1.1.4 Repeated ONES as Symbols for a FEW Initial Natural Numbers

Having a symbol for ONE suggested repetition of this symbol to denote a few initial numbers. In the Roman system which has been in use until a few centuries ago numbers up to THREE were denoted by sets of repeated ONES. The ancient Egyptian system of numeration had such a direct representation of numbers up to NINE. Such a simple collection of symbols merely becomes a *set* but not a *system*. Larger numbers as repeated ONE symbols become cumbersome to handle occupying increasingly large amount of space for writing. When numbers are written as a sequence of digits in a line, grouping the symbols into the individual digits is a confusing process. Ideally, each of the few initial natural numbers should have individually only one symbol. To have a single symbol for every number indefinitely is another problem especially while writing large numbers. Humans did know this and made some improvements.

Beyond a chosen limit, they began denoting numbers by strings of combinations of basic symbols following certain rules. Some created new symbols for larger numbers. Thus, number systems came into existence.

Table 1.1 Egyptian numerals up to NINE as sets of ONES

ONE	I	SIX	III III
TWO	II	SEVEN	IIII III
THREE	III	EIGHT	IIII IIII
FOUR	IIII	NINE	III III III
FIVE	IIIII		

1.2 Counting in Bunches: The Sense of a Base in Counting

Counting entities one by one is a cumbersome process especially when the total number is large. Suppose we have a bag of grains of rice or wheat. We do not count the number of grains in our taking stock or making trade with them; instead, we measure by volume or weight which combines a number of grains into units of measure used such as liter or kilogram and all our transactions are in these measures and never in the number of grains.

If we have items which are larger than grains, and countable as in the case of medium sized fruits, we may still count them individually. However, in the event of handling large quantities of such items, we resort to making bunches containing a known number of items. They may be bunches of TEN each. Again, these bunches can be counted in larger groups of TEN- each such group contain a TEN of TENS and this process may continue. In this process we get a quicker count of the total number.

Therefore, counting to a familiar limit and then forming a group to be reckoned separately reduces the burden of counting items one-by-one. That familiar number, say TEN in the present case, becomes the basis or base in the method of counting. The orders of the respective groups increase in steps of ONE. In the case groups of TEN, the number of single entities in the first group is TEN, in the next size HUNDRED, THOUSAND in the next and so on. Those left over after the bunching process that are less than TEN and they are termed as UNITS. In a positional system the numbers of the single entities or subgroups from UNITS through TENS, HUNDREDS, .. are written from right to left. A problem arises when some groups vanish in this process of successive bunching. That is, the groups of missing order do not have a way to be shown that they are absent unless we have a symbol denoting ZERO.

Table 1.2 and 1.3 show the ways in which some communities wrote numbers in the ancient times. In most cases, one finds repeated symbols as sets except in Ionian and Hindu systems.

Table 1.2 Twelve written by different communities

People	**Expression**	**Remarks**
Babylonians	∢∇∇	Two symbols for ONE used to denote TWO
Romans	XII	
Egyptians	∩II	
Mayans	●● ═	
Greeks (Attic)	ΔII	
Greeks (Ionian)	*i*β	Alphabetic numeration system
Hindus	12	Each number has a single unique symbol

Table 1.3 The systems of numeration by different communities

Baby- lonian	Y	YY	YYY	YYYY	YYYYY	YYY YYY	YYYY YYY	YYYY YYYY	YYYY YYYYY	<	
Egyptian	I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII	∩	
Greek	A	B	Γ	Δ	E	F	Z	H	θ	I	
Roman	I	II	III	IV	V	VI	VII	VIII	IX	X	
Chinese	一	二	三	四	五	六	七	八	九	十	〇
Mayan	•	••	•••	••••	—	• —	•• —	••• —	•••• —	— —	∩ ∪
Hindu	1	2	3	4	5	6	7	8	9	10	0

Only the INS has ten distinct symbols 0, 1, 2, ….9 without repetitions.

Roman system was in wide use in the west until about 800 years ago. New symbols were created for FIVE (V), TEN (X), FIFTY (L), HUNDRED (C), THOUSAND (M) and so on. The number of symbols increases indefinitely which is a major problem

I II IIIIV V VI VII VIII IX X …… L…C…. M….

Combined symbols are used for numbers around a new symbol with additive and subtractive features in the Roman system: I before the new symbol V denotes subtraction by one, that is, IV denoting FOUR. After the new symbols up to THREE I's are added to denote SIXas VI, SEVEN as VII, and EIGHT as VIII. This rule is followed to denote NINE as IX and so on. This additive and subtractive rule associated with indefinitely increasing burden of new symbols is subjected to conditions which make the Roman numeration quite cumbersome. The Hindu decimal number system appeared to limit the burden to the minimum. The decimal system has been introduced with only ten symbols to represent a number of any size. Hindus also had literal names for many large numbers although they did not use any special symbols for them.

This subtractive rule is applied as in the following

IV = 4

IX = 9

XL = 40

XC = 90

CD = 400

CM = 900

To us this system appears to be quite difficult to understand but the Roman System only was in use till the 13th century CE in the West and an abacus was used as a tool to facilitate calculations. As the numbers grow larger and larger the number of symbols increases and after a limit the Roman alphabet will not be able to provide further symbols; new and weird symbols have to be introduced.

Most number systems of the ancient times carried symbols to denote the sizes of various sets. For all practical purposes, the communities using this system of writing numbers were able to carry on with their use. The Egyptians built great pyramids, the Chinese built the Wall and humans were able to travel great distances and more recently the Romans spread the use of their number system in all walks of life in their domain. The Egyptians used hieroglyph symbols.

1.2.1 The Babylonian System of Numeration

The Babylonian system of numeration (Figure 1.1) is one of the earliest with positional feature. It had *two bases* TEN and SIXTY mixed. Further, there was initially no symbol to identify a blank space until a double wedge symbol  came into use. It has no numerical status.

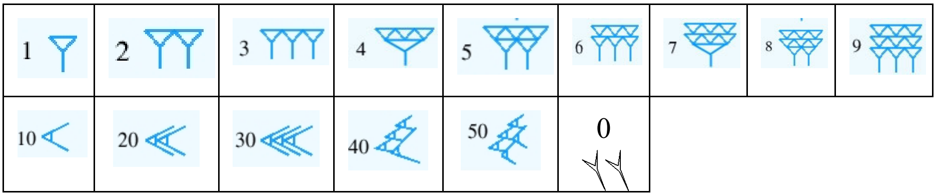

Figure 1.1 The Babylonian system of numeration

This mixture of bases TEN and SIXTY being somewhat difficult for all people to understand and only the scribes could correctly apply this system. The basis of SIXTY is very special in the system and there can be many reasons behind the use of this basis.

1.2.2 The Egyptian Numbers and Symbols

The Egyptians denoted larger numbers pictorially as shown in Figure 1.2.

Stroke	Heelbone	Coiled rope	Lotus flower	Pointing finger	Tadpole	Scribe
1	**10**	**100**	**1000**	**10000**	**100000**	**1000000**

Figure 1.2 Ancient Egyptian number representation

To represent a number less than the basis, there were no special symbols. For instance, FOUR units are denoted by four ONES as **IIII** and FORTY as ∩∩∩∩ and so on. .In the Roman system the same can be noticed with three as the largest before the next symbol appears as **IV** as THREE units are denoted by **III,** and THIRTY is denoted as **XXX.**

1.2.3 Important Features of the INS

Symbols for numbers from ONE to NINE. (1,2,3,4,5,6,7,8.9)

No new symbols for higher numbers; this is the greatest advantage in the writing and understanding numbers. However, large numbers over an extremely large range were given names. However, these names are used in day-to-day activities only up to a limit. But the numbers can be literally read on the basis of the sequence in which the number string is written.

The way we read numbers follows the decimal system and facilitated by the INS.

A symbol for ZERO (0) with the status of a number associated with special rules of arithmetic.

1.2.4 Names of Large Numbers in Sanskrit

The Hindu decimal number system eliminated the burden of additional new symbols to the minimum. It has been introduced with only ten symbols to represent a number of any size. Hindus also had literal names for many large numbers although they did not use any special symbols for them.

Human ability to name numbers progressed. After TEN the names sound as TEN and ONE, TWENTY -ONE, and so on. Indians have been able to name numbers to a great extent. The list of names is given in Table 3. A *crore* which is 10 million is common. Names for further powers of TEN are extensive (Ifrah, 2000). However, names of numbers larger than 10^7 are not used in day-to-life.

Table 1.4 Names of numbers in Sanskrit over a wide range

k	**Name of the number of the form 10^k**
1	*DaSa*
2	*Sata*
3	*sahasra* (सहस्त्र)
5	*laksha* (लक्ष)
7	*kOTi* (कोटि)
9	*ayuta* (अयुत)
13	*niyuta* (नियुत)
14	*pakOTi* (पकोटि)
15	*vivAraa* (विवारा)
17	*kshObhyaa* (क्षोभ्या)
19	*vivAhaa*(विवाहा)
21	*kOTippakOTi* (कोटिपकोटी)

Contd…

23	*bahuLa* (बहुल)
25	*naagabaala* (नागाबाला)
28	*naahUTa* (नाहूटा)
29	*teetalambha* (तीतलम्भा)
31	*vyavasthaanaapajnaapati* (व्यवस्थानापज्ञापति)
33	*hetuhilaa* (हेतुहीला)
35	*ninnaahutaa* (निन्नाहुता)
37	*hEtvindriya* (हेत्विन्द्रिय)
39	*samAptalambha* (समाप्तलम्भ)
41	*ganAnAgatee* (गनानागती)
42	*akkhObini* (अक्खोबिनि)
43	*niraavaadya* (निरावाद्य)
45	*mudraabaalaa* (मुद्राबाला)
47	*sarvabaalaa* (सर्वबाला)
49	*bindu* (बिंदु or बिन्दु)
51	*sarvajna* (सर्वज्ञ)
53	*vibhutangamaa* (विभुतन्गामा)
56	*abbuda* (अब्बुद)
63	*nirbuddha* (निर्बुद्ध)
70	*ahaahaa* (अहाहा)
77	*abaabaa* (अबाबा).
84	*aTaaTaa* (अटाटा)
91	*sogaangheeka* (सोगान्धीक)
98	*uppala* (उप्पल)
105	*kumuda* (कुमुद)
112	*pundareeka* (पुन्डरीक)
119	*padma* (पद्म)
126	*kathana* (कथन)
133	*mahakathana* (महाकथन)
140	*asankhyEya* (असंख्येय)
421	*dhvajaagraniSamanee* (ध्वजाग्रनिशमनी)