CHAPTER

1

Transmission Line Analysis

1.1 INTRODUCTION

This chapter deals with the analysis of a transmission line. The factors analyzing the performance of a transmission line are dealt. Further, short, medium and long transmissions are analyzed. Further the analysis is performed with respect to ABCD constants, open circuit and short circuit conditions. The concept of travelling waves in power systems has been dealt.

1.2 CONCEPTS OF A TRANSMISSION LINE

An electric transmission line can be represented by a series combination of resistance, inductance and shunt combination of conductance and capacitance. These parameters are symbolized as R, L, G and C respectively. Among these R and G are least important as they do not affect much the total equivalent impedance of the line and hence the transmission capacity.

1.3 PERFORMANCE OF TRANSMISSION LINES

Figure 1.1 Transmission line to determine % Efficiency

The performance of a transmission line can be calculated using: (i) % Efficiency, (ii) % Regulation

(i) **% Efficiency:** Consider a transmission line as shown in Figure 1.1.

P_s is sending end power and P_R is the receiving end power

% Efficiency of a transmission line is

$$\% \ \eta \frac{P_R}{P_S} \times 100 = \frac{P_R}{P_R + P_{Losses}} \times 100$$

P_{losses} : For a single phase system, power loss = I^2 R

For a three phase system, power loss = $3I^2$ R

where ‘I’ is the phase current and ‘R’ is the resistance per phase.

(ii) **%Regulation:** As the transmission line is a stationary device, % voltage regulation is calculated.

Consider a transmission line as shown in Figure 1.2.

V_S is sending end voltage and V_R is the receiving end voltage.

Voltage drop in transmission line = $V_S - V_R$

$$\% \in = \frac{V_S - V_R}{V_R} \times 100$$

Figure 1.2 Transmission line to determine % voltage regulation

Case: Assume that the receiving end of transmission line is operating under no–load condition,

As the receiving end is operating at no-load condition, I = 0

Voltage drop in transmission line = 0

Sending end voltage, V_S = No load-receiving end voltage, V_{R_o}

$$\% \in = \frac{V_S - V_R}{V_R} \times 100$$

The percentage regulation of a transmission line is defined as the change in receiving end voltage from no load to full load, expressed as % full load receiving end voltage.

Note: The percentage efficiency of a transmission line must be high and percentage regulation of a transmission line must be low for better performance of transmission lines.

1.4 CLASSIFICATION OF TRANSMISSION LINES

(a) Transmission lines are primarily classified based on wavelength.

(b) The term power transmission means travel of voltage wave and current wave from sending end to receiving end of the transmission line.

(c) Voltage wave & current wave travels at velocity of light from sending end to receiving end of the transmission line.

Consider a voltage wave travelling from sending end to the receiving end of transmission line as shown in figure 1.3.

$$\text{Wavelength}(\lambda) = \frac{\text{Velocity of travelling wave}}{\text{Frequency}}$$

$$= \frac{3 \times 10^8 \dfrac{\text{m}}{\text{sec}}}{50 \dfrac{\text{cycles}}{\text{sec}}}$$

$$\lambda = 6000 \frac{\text{km}}{\text{cycle}}$$

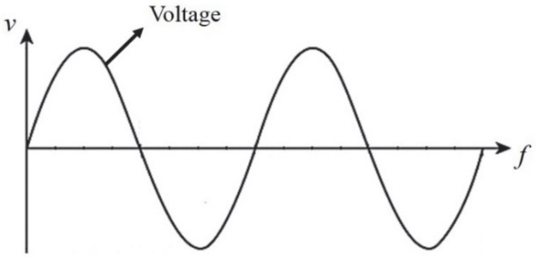


Figure 1.3 Voltage travelling wave

i.e., voltage (or) current wave travels an angular distance of 6000 km to complete one cycle, but due to complexity of power system network, transmission lines are further classified based on

1. Physical length of the line
2. Operating voltage
3. Effect of capacitance

and is tabulated as follows.

Transmission Line	Physical Length	Operating Voltage	Effect of Capacitance
Short line	0–80km	0–20kV	Neglected
Medium–line	80–120km	20–100kV	Lumped or concentrated
Long–line	> 120 km	>100kV	uniformly distributed

1.5 ANALYSIS OF SHORT TRANSMISSION LINES

1. **Equivalent Circuit:** A short transmission line consists of resistance and inductance connected in series.

   R is the resistance and jX_L is the inductive reactance.

   Consider the equivalent circuit as shown in the Figure 1.4.

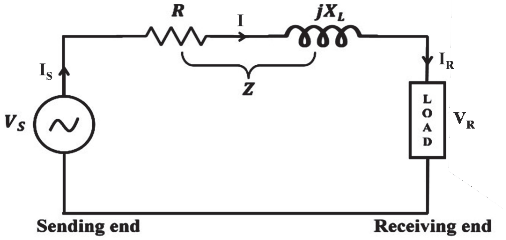


Figure 1.4 Short Transmission Line

2. **Mathematical Relations:**

$$I_S = I_R = I \text{ (say)}$$

   Resistive voltage drop = IR

   Reactive voltage drop $= j\,IX_L$

   Total voltage drop $= IR + I(jX_L) = I(R + jX_L) = IZ$

   Sending end voltage, $V_S = V_R + I_R + I\,(jX_L)$

$$V_S = V_R + I(R + jX_L)$$

$$V_S = V_R + IZ$$

3. **Vector Diagram:** Consider receiving end voltage, V_R as the reference vector.

   Assume R-L Load

   Receiving end current, I_R lags receiving end voltage, V_R by ϕ_R

   where $0 < \phi_R \leq 90, \text{lag}$

   The vector diagram is shown in Figure 1.5.

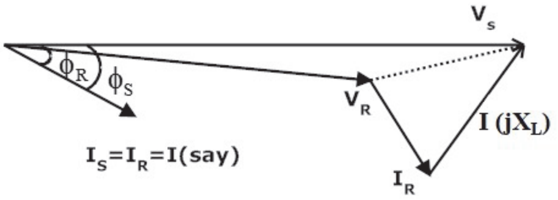


Figure 1.5 Vector diagram of short transmission line

For lagging power factor loads: $\phi_R < \phi_S$

Note: For leading power factor loads: $\phi_R > \phi_S$

4. **ABCD Constants:** For a two-port network,

$$V_S = AV_R + BI_R \quad .....(1.1)$$

$$I_S = CV_R + DI_R \quad .....(1.2)$$

$$V_S = V_R + Z\,I_R \quad .....(1.3)$$

Compare (1.1) & (1.3)

$A = 1$ and $B = Z$

$I_S = I_R$

$$I_S = 0.\,V_R + 1.I_R \quad .....(1.4)$$

Compare (1.2) & (1.4)

$C = 0$ and $D = 1$

$A = D = 1$, i.e., short line is symmetrical

$AD - BC = 1$, i.e., short line is reciprocal

5. **Conclusion:**
   1. $A = 1$, $D = 1$ either for series branch (or) shunt branch
   2. Constant 'B' determines impedance (Z) of series branch
   3. Constant 'C' determines admittance (Y) of shunt branch

1.6 ANALYSIS OF MEDIUM TRANSMISSION LINES

Medium lines are classified based on the location of the capacitance in the equivalent circuit into four methods.

- **(a) Load condenser method:** Capacitor is connected across load
- **(b) Source condenser method:** Capacitor is connected across source
- **(c) Nominal-π method:** Capacitance is equally split across source and load
- **(d) Nominal-T method:** Capacitor is at the middle of the line

1.6.1 Load Condenser Method

1. **Equivalent Circuit:** Consider the equivalent circuit as shown in the Figure 1.6. R is the resistance and jX_L is the inductive reactance.

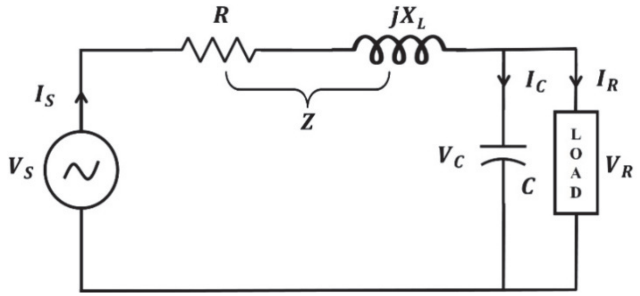


Figure 1.6 Medium Transmission Line by Load condenser method

2. **Mathematical Relations:** Sending end current, $I_S = I_R + I_C$

   Resistive voltage drop = $I_S R$

   Reactive voltage drop = $I_S(jX_L)$

   Total voltage drop = $I_S R + I_S(jX_L)$

   $= I_S(R + jX_L)$

   $= I_S Z$

   Sending end voltage, $V_S = V_R + I_S R + I_S(jX_L)$

   $V_S = V_R + I_S(R + jX_L)$

   $V_S = V_R + I_S Z$

3. **Vector Diagram:** Let V_R be the Reference vector

   Assume R-L Load.

   Receiving end current, I_R lags receiving end voltage, V_R by ϕ_R

   where $0 < \phi_R \leq 90$, lag

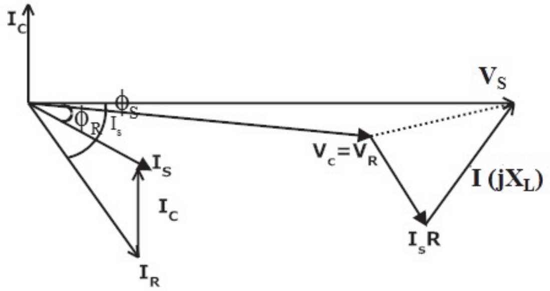


Figure 1.7 Vector diagram of Medium Transmission Line by Load condenser method

4. **ABCD Constants:**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}$$

$A \neq D$ i.e., Load condenser method is unsymmetrical

$AD - BC = 1$ i.e., Load condenser method is reciprocal

Note: As capacitor is unsymmetrically located in the equivalent circuit, load condenser method is unsymmetrical.

1.6.2 Source Condenser Method

1. **Equivalent Circuit:** Consider the equivalent circuit as shown in the figure 1.8.

   R is the resistance and jX_L is the inductive reactance.

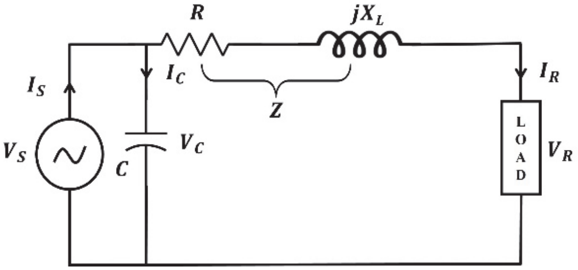


Figure 1.8 Medium Transmission Line by Source condenser method

2. **Mathematical Relations:** Sending end current, $I_S = I_R + I_C$

   Resistive voltage drop = $I_R R$

   Reactive voltage drop = $I_R(jX_L)$

   Total voltage drop = $I_R R + I_R(jX_L) = I_R (R + jX_L) = I_R Z$

   Sending end voltage, $V_S = V_R + I_R R + I_R(jX_L)$

   $= V_R + I_R(R + jX_L)$

   $V_S = V_R + I_R Z$

3. **Vector Diagram:** Let V_R be the reference vector

   Assume R-L Load.

   Receiving end current, I_R lags receiving end voltage, V_R by ϕ_R

   where $0° < \phi_R \leq 90°$, lag

   The vector diagram is shown in Figure 1.9.

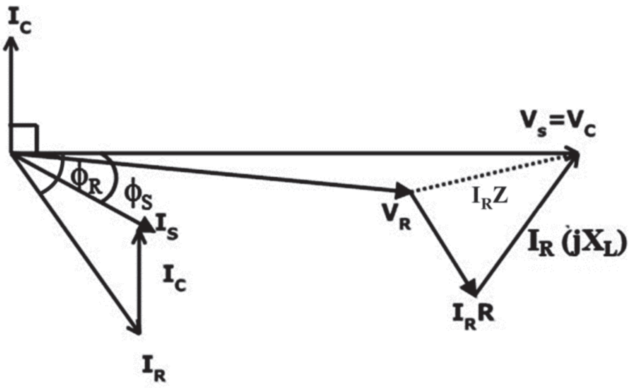


Figure 1.9 Vector diagram of Medium Transmission Line by Source condenser method

4. **ABCD–Constants:**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & I \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}$$

i.e., $A \neq D$ i.e., source condenser method is unsymmetrical

$$AD - BC = 1 + YZ - YZ = 1$$

$AD - BC = 1$ i.e., source condenser method is reciprocal

Note: As capacitor is unsymmetrically located in the equivalent circuit, source condenser method is unsymmetrical.

1.6.3 Nominal T–Method

As capacitor is located exactly at the middle of the line, Nominal T–method is also known as middle condenser method.

The term 'nominal' represents 'rated voltage'

1. **Equivalent Circuit:** Consider the equivalent circuit as shown in the Figure 1.10.

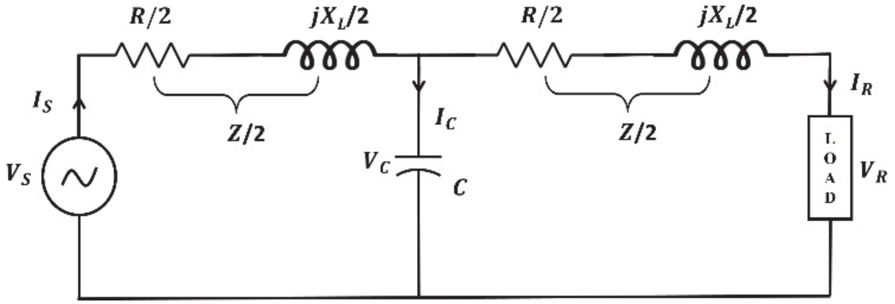


Figure 1.10 Medium Transmission Line by Nominal–T method

2. **Mathematical Relation:** Sending end current, $I_S = I_R + I_C$

   Voltage across capacitor, $V_C = V_R + I_R \frac{R}{2} + I_R\left(\frac{jX_L}{2}\right)$

   Sending end voltage, $V_S = V_C + I_S \frac{R}{2} + I_S\left(\frac{jX_L}{2}\right)$

3. **Vector Diagram:** Let V_R be the reference vector

   Assume R-L Load.

   Receiving end current, I_R lags receiving end voltage, V_R by ϕ_R

   where $0° < \phi_R \le 90°$, lag

   The vector diagram is shown in Figure 1.11.

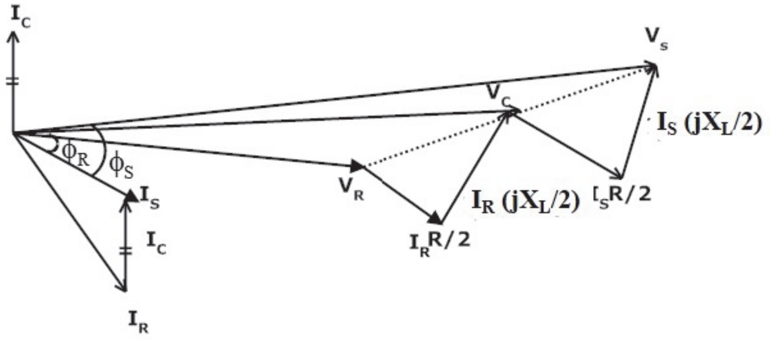


Figure 1.11 Vector diagram of Medium Transmission Line by Nominal–T method

4. **ABCD – Constants**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & \frac{Z}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{Z}{2} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z\left(1+\frac{YZ}{4}\right) \\ 0 & 1+\frac{YZ}{2} \end{bmatrix}$$

   i.e., A = D, i.e., Nominal T–method is symmetrical

   AD – BC = 1, i.e., Nominal T–method is reciprocal

Note: As capacitor is located symmetrically in the equivalent circuit Nominal–T method is symmetrical.

Case - With receiving end of transmission line operating under no–load condition:

1. **Equivalent Circuit:** Consider the equivalent circuit as shown in Figure 1.12.

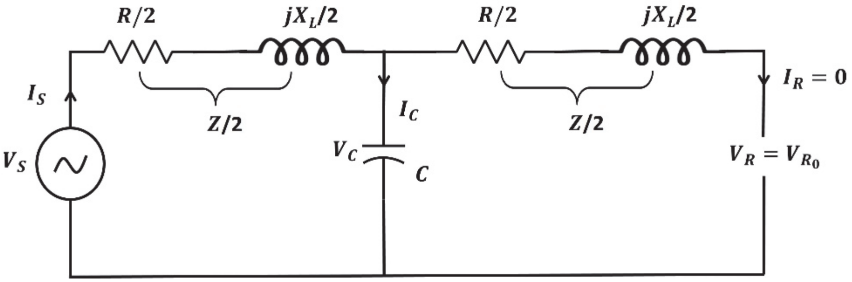


Figure 1.12 Medium Transmission Line by Nominal–T method with open circuited receiving end

2. **Mathematical Relations:** As $I_R = 0$, voltage drop due to $I_R = 0$

$$\therefore \quad V_C = V_R = V_{R_0}$$

$$V_S = V_C + I_C \frac{R}{2} + I_S \left(\frac{jX_L}{2}\right)$$

$$I_S = I_C$$

$$V_S = V_C + I_C \frac{R}{2} + I_C \left(\frac{jX_L}{2}\right)$$

3. **Vector Diagram:** Let V_R be the Reference vector

   The vector diagram is shown in Figure 1.13.

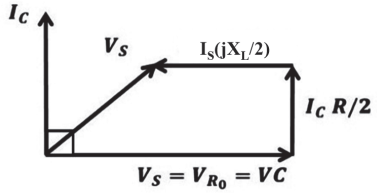


Figure 1.13 Vector diagram of Ferranti Effect

Ferranti Effect

- The magnitude of sending end voltage is less than the magnitude of receiving end voltage at no–load condition
- Ferranti effect is due to capacitive current (or) capacitance in the equivalent circuit.
- As short line do not have capacitance, Ferranti effect does not occur in short line.
- Therefore, Ferranti effect occurs in medium line and long line at no load conditions.
- % Voltage rise in transmission line due to Ferranti effect

$$= \frac{\omega^2 l^2}{18} \times 10^{-8} \text{V}$$

where 'l' is the length of transmission line in Km.

- Receiving end voltage at no–load condition is

$$V_{RO} = V_C = I_C\,(-jX_C)$$

$$= \left[\frac{V_S}{\frac{R}{2} + j\frac{X_L}{2} + (-jX_C)}\right](-jX_C) = \left[\frac{V_S}{\frac{R}{2} + j\frac{\omega_L}{2} + (-\frac{j}{\omega C})}\right](-j\frac{1}{\omega C})$$

1.6.4 Nominal–π Method

*As the capacitor is split into two equal parts across source and load, nominal–π method is also referred as "split condenser method".

1. **Equivalent Circuit:** Consider the equivalent circuit as shown in the Figure 1.14.

   R is the resistance and jX_L is the inductive reactance.

   Vc_s = Voltage across capacitor at the sending end.

   Vc_R = Voltage across capacitor at the receiving end.

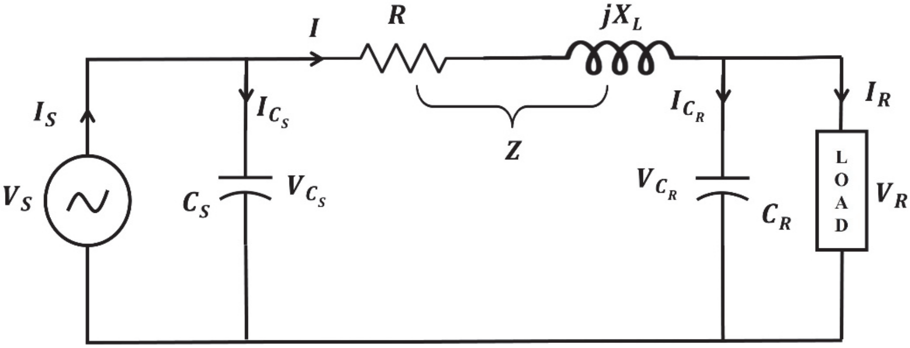


Figure 1.14 Medium Transmission Line by Nominal–π method

2. **Mathematical Relations:**

$$C_s = C_R = \frac{C}{2}$$

$$V_{C_R} = V_R$$

   Resistive voltage drop = IR

   Reactive voltage drop = IjX_L

   Total voltage drop = I R + I jX_L

   = I (R + jX_L)

   = IZ

   Sending end voltage, $V_S = V_R + I_R + I\ (jX_L)$

$$V_S = V_R + I\ (R + jX_L)$$

$$V_S = V_R + IZ$$

3. **Vector Diagram:** Let V_R be the reference vector.

   Assume R-L Load

   Receiving end current, I_R lags receiving end voltage, V_R by ϕ_R

$$0 < \phi_R \le 90,\ \text{lag}$$

The vector diagram is shown in Figure 1.15.

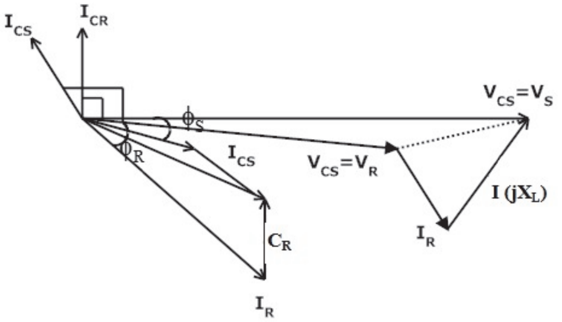


Figure 1.15 Vector diagram of Medium Transmission Line by Nominal– π method

4. **ABCD – Constants:**

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{Y}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{Y}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y\left[1+\frac{YZ}{4}\right] & 1+\frac{YZ}{2} \end{bmatrix}$$

i.e., A = D i.e., Nominal π– method is symmetrical

AD – BC = 1 i.e., Nominal π– method is reciprocal

Note: As Equal capacitance $\left(\frac{c}{2}\right)$ is across source and load, nominal π method is symmetrical.

Case - With receiving end of transmission line operating at no–load condition

1. **Equivalent Circuit:** Consider the equivalent circuit as shown in figure 1.16.

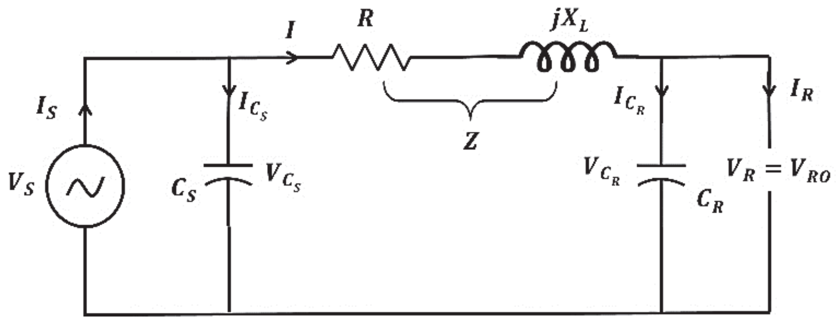


Figure 1.16 Medium Transmission Line by Nominal– π method with open circuited receiving end

2. **Mathematical Relations:** As $I_R = 0$

$$V_{C_R} = V_R = V_{R_0}$$

$$I = I_{C_R} \, (\because I_R = 0)$$

$$V_S = V_{R_0} + IR + I(jX_L)$$

$$V_S = V_{C_R} + I_{C_R} R + I_{C_R}(jX_L)$$

3. **Vector Diagram:**

Let V_{R_0} be the Reference vector

The vector diagram is shown in Figure 1.17.

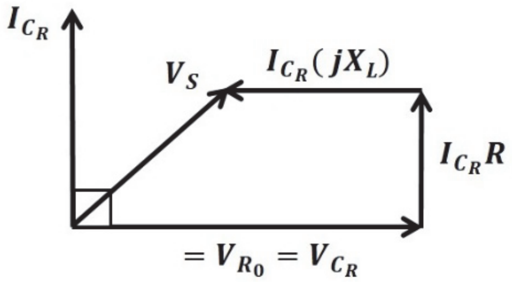


Figure 1.17 Vector diagram of Ferranti Effect for Nominal–π method

* $|V_S| < |V_{R_0}|$ i.e., Ferranti effect

4. **No–Load Receiving End Voltage:**

$$V_{R_0} + V_R = V_{C_R} = I_{C_R}(-jX_{C_R})$$

$$= \left\{ \frac{V_S}{R + jX_L (-jX_{C_R})} \right\}$$

$$= \left\{ \frac{V_S}{R + j\omega_L - \frac{j}{\omega C_R}} \right\} = \left\{ \frac{V_S}{R + j\omega L - \frac{j}{\omega \frac{C}{2}}} \right\} \left(-\frac{j}{\omega \frac{C}{2}} \right)$$

Note: Operator 'j' rotates a vector in anticlockwise direction by 90^0.

1.7 IMPEDANCE OF TRANSMISSION LINE FOR OPEN CIRCUIT CONDITION & SHORT CIRCUIT CONDITION

Case 1: With receiving end open–circuited

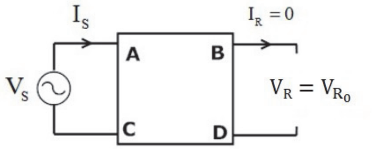


Figure 1.18 With receiving end open–circuited

As receiving end is open circuited, $I_R = 0$ and $V_R = V_{R0}$

From the basic two port equations,

$$V_S = AV_R + BI_R$$

$$V_S = A\,V_{R_o} \quad (\because I_R = 0) \qquad \text{…..(1.5)}$$

$$A = \frac{V_S}{V_{R_0}}$$

$$I_S = CV_R + DI_R$$

$$I_S = C\,V_{R_o} \qquad \text{…..(1.6)}$$

$$C = \frac{I_S}{V_{R_0}}$$

Divide (1.5) and (1.6):

$$\frac{V_S}{V_S} = \frac{A}{C} \qquad \text{…..(1.7)}$$

Case 2: With Receiving end short–circuited

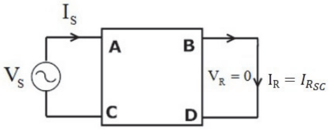


Figure 1.19 With receiving end short–circuited

As receiving end is short circuited, $V_R = 0$ and $I_R = I_{R_{SC}}$

$$V_S = AV_R + BI_R$$

$$V_S = B\,I_{R_{SC}} \qquad \text{…..(1.8)}$$

$$B = \frac{V_S}{I_{R_{SC}}}$$

$$I_S = CV_R + DI_R$$

$$I_S = D.I_{R_{SC}} \qquad \text{…..(1.9)}$$

$$D = \frac{I_S}{I_{R_{SC}}}$$

$$\frac{B}{D} = \frac{V_S}{I_S} \qquad \text{…..(1.10)}$$

Multiply (1.7) & (1.10)

$$\frac{V_S}{I_S} \times \frac{V_S}{I_S} = \frac{A}{C} \times \frac{B}{D}$$

$$\left(\frac{V_S}{I_S}\right)^2 = \frac{AB}{CD}$$

Assuming symmetrical line i.e., A = D

$$\left(\frac{V_S}{I_S}\right)^2 = \frac{B}{C}$$

$$\left(\frac{V_S}{I_S}\right) = \sqrt{\frac{B}{C}}$$

$$\left(\frac{V_S}{I_S}\right) = Z_C\sqrt{\frac{B}{C}} = \sqrt{\frac{Z_{SC}}{Y_{OC}}}$$

$$\frac{V_S}{I_S} = Z_C\sqrt{\frac{B}{C}} = \sqrt{\frac{Z_{SC}}{Y_{OC}}}$$

$$Z_C = \frac{V_S}{I_S} = \sqrt{Z_{SC}Y_{OC}}$$

Characteristic Impedance, Z_c: The Impedance of a transmission line with losses is known as Characteristic Impedance, Z_c

* $Z_C = 400\ \Omega$ for overhead transmission lines

* $Z_C = 40\ \Omega$ for underground cables

Surge impedance Z_S: The impedance of a transmission line without losses is known as Surge impedance Z_S.

i.e., the impedance of an ideal transmission line.

1.8 CONDITIONS FOR ZERO VOLTAGE REGULATION AND MAXIMUM VOLTAGE REGULATION OF A TRANSMISSION LINE

* Consider a short transmission line, by taking I_R as reference.

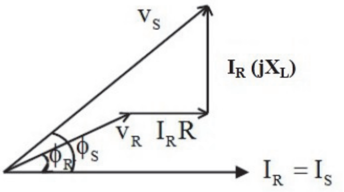


Figure 1.20 Vector diagram of short transmission line

From the above vector diagram,

$$V_S \cong V_R + I_R R \cos\phi_R + I_R X_L \sin\phi_R$$

$$V_S - V_R \cong I_R R \cos\phi_R + I_R X_L \sin\phi_R$$

$$\% \in = \frac{V_S - V_R}{V_R} \times 100$$

$$= \frac{I_R R}{V_R} \cos\phi_R \times 100 + \frac{I_R X_L}{V_R} \sin\phi_R \times 100$$

$$= \left(\frac{I_R R}{V_R}\right) \cos\phi_R \times 100 + \left(\frac{I_R X_L}{V_R}\right) \sin\phi_R \times 100$$

$$= v_{rpu} \cos\phi_R \times 100 + v_{xpu} \sin\phi_R \times 100$$

$$= \% V_R \cos\phi_R + \% V_X \sin\phi_R \qquad \text{.....(1.11)}$$

In general, % Regulation

$$\% \in = \% V_R \cos\phi_R + \% V_X \sin\phi_R$$

In general the sign is positive for lagging power factor load and negative for lead power factor load.

Case 1: Condition for maximum voltage regulation

Maximum %voltage regulation occurs at lagging power factor load

$$\% \in = \% V_R \cos\phi_R + \% V_X \sin\phi_R$$

$$\frac{d}{d\phi_R}(\% \in) = 0$$

$$\% V_R (-\sin\phi_R) + \% V_X (\cos\phi_R) = 0$$

$$\% V_R \sin\phi_R = \% V_X \cos\phi_R$$

$$\text{Tan } \phi_R = \frac{\% V_X}{\% V_R}$$

$$= \frac{\dfrac{I_R X_L \times 100}{V_R}}{\dfrac{I_R R \times 100}{V_R}}$$

$$\text{Tan}\ \phi_R = \frac{X_L}{R}$$

$$\phi_R = \text{Tan}^{-1}\left(\frac{X_L}{R}\right)$$

Impedance of the short line: $Z = R + j\,X_L$

Impedance Triangle:

Where θ = Impedance phase angle

From the Figure 1.21,

$$\text{Tan}\ \phi_R = \frac{X_L}{R}$$

$$\theta = \phi_R$$

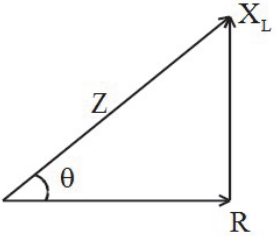


Figure 1.21 Impedance triangle of short line

Case 2: Condition for zero voltage regulation

* Zero voltage regulation occurs at leading p.f load st

$$\% \in = \%\ V_R \cos\phi_R - \%\ V_X\ \sin\phi_R$$

$$\% \in = 0 \Rightarrow \%\ V_R \cos\phi_R - \%\ V_X\ \sin\phi_R = 0$$

$$\%\ V_R \cos\phi_R - \%\ V_X\ \sin\phi_R$$

$$\text{Tan}\ \phi_R = \frac{\%\ V_R}{\%\ V_X} = \frac{\frac{I_R\ R}{V_R}\times 100}{\frac{I_R\ X_L}{R}\times 100} = \frac{R}{X_L}$$

$$\theta = \phi_R$$

$$\phi_R = \text{Tan}^{-1}\left(\frac{R}{X_L}\right) \qquad \text{.....(1.12)}$$

From Impedance triangle,

$$\text{Tan}\ \phi_R = \frac{R}{X_L} = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\phi_R = \frac{\pi}{2} - \theta$$

1.9 RELATION BETWEEN ACTUAL LOADING OF TRANSMISSION LINE AND SURGE IMPEDANCE LOADING

(A) Characteristic Impedance Loading (CIL): Consider a transmission line with losses connected to a load as in the Figure 1.22.

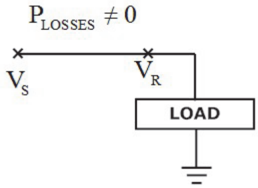


Figure 1.22 A practical transmission line

$$\text{CIL} = \left|\frac{|V_S|\,|V_R|}{Z_C}\right| \text{ W (or) kW (or) MW}$$

"The maximum active power transmitted through a line with losses and further through the load at unity power factor is known as Characteristic Impedance Loading" (CIL).

(B) Surge Impedance Loading (SIL): Consider a transmission line without losses connected to a load as in the figure 1.23.

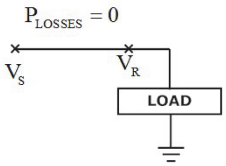


Figure 1.23 An ideal transmission line

$$\text{SIL} = \left|\frac{|V_S|\,|V_R|}{Z_S}\right| = \frac{|V|^2}{Z_S} \text{ W or kW or MW}$$

$\because \quad |V_S| = |V_R| = |V|$

"The maximum active power transmitted through a loss less line and further to the Load at unity p.f. is known as Surge Impedance Loading"

Condition 1 – Actual Loading of a transmission line is greater than surge impedance Loading of transmission line

1. As power carrying capacity increases, current flowing through the transmission line increases, electromagnetic energy stored by inductor in the magnetic field = $\frac{1}{2}LI^2$ increases

$\therefore$ Inductor is dominant, p.f is Lagging, $|V_R| < |V_S|$ and Ferranti effect do not occur.

Condition 2 – Actual Loading Actual Loading of a transmission line is lesser than surge impedance Loading of transmission line

1. As power carrying capacity decreases, current flowing through the transmission line decreases & Electromagnetic energy stored by inductor in the magnetic field = $\frac{1}{2}LI^2$ decreases

$\therefore$ Capacitor is dominant, p.f is Leading, $|V_R| > |V_S|$ and Ferranti effect occurs.

$\therefore$ Ferranti effect can be eliminating by loading the transmission line beyond their surge impedance loading capacity.

Q1. The sending end voltage & receiving end voltage of a transmission line are 220kV, 200kV respectively. Determine the Characteristic Impedance Loading of transmission line.

Solution:

$$CIL = \frac{|V_S||V_R|}{Z_C}$$

$$CIL = \frac{(220\times10^3)(200\times10^3)}{400}$$

$$CIL = 110MW$$

Q2. Determine the surge impedance loading of an underground cable operating at 400 kV.

Solution:

$$SIL = \frac{|V|^2}{Z_S} = \frac{(400\times10^3)}{40}$$

$$SIL = 4000MW$$

Q3. The open circuit& short circuit impedance of a transmission line are $16 \times 10^4\ \Omega$, & 1 Ω respectively. Determine the characteristic impedance of the transmission line.

Solution:

$$Z_C = \sqrt{Z_{OC}Z_{SC}} = \sqrt{16\times10^4\times1}$$

$$Z_C = 400\ \Omega$$

Q4. The ABCD constants of a 220kV transmission line are A = D = $0.94\angle 1^0\ \angle 1^0$, B = $130\angle 73^0$ (Ω) and c = $0.0001\angle 90^0$ (Ω). If the sending voltage of the Line for a given Load delivered at nominal voltage is 240kV then %voltage regulation of the line is

Solution:

For a No – Load condition $I_R = 0$

$$V_S = AV_R + BI_R = A\ V_{R_o}$$

$$V_{R_o} = \frac{V_S}{A}$$

$$\%\in=\frac{|V_{R_0}|-|V_R|}{|V_R|}\times 100$$

$$=\frac{\left|\frac{V_S}{A}\right|-|V_R|}{|V_R|}\times 100$$

$$=\frac{\left|\frac{240}{0.94}\right|-|220|}{|220|}\times 100$$

$$\%\in=16.05\%$$

Q5. The ABCD–constants of a three phase transmission line are A = D = $0.8\angle 1^0$, B = $170\angle 85^0$ Ω, C = $2\times 10^{-3}\angle 90.4^0$ S. The sending end voltage is $V_S = 400$kV. Determine the receiving end voltage under no–load condition.

Solution:

No – Load receiving end voltage,

$$V_{R_0}=\frac{V_S}{A} \quad (\because \text{No–Load condition } I_R = 0)$$

$$=\frac{400}{0.8} \quad V_S = AV_R + BI_R$$

$$V_{R_0} = 500 \text{ kV}$$

Q6. A 220kV transmission line is represented by nominal π–parameters A = $0.9\angle 5^0$, B = $80\angle 65^0$ Ω, the sending end voltage is maintained at 220kV. Calculate the rise in voltage

Solution:

The no–load receiving end voltage is

$$V_{R_0}=\frac{V_S}{A}$$

$$=\frac{200}{0.9}$$

$$V_{R_0} = 244.4 \text{ kV}$$

The no–load rise in the voltage is

$$V_{R_0} - V_S = 244.4 - 220 = 24.4\text{kV}$$

Q7. For a 500Hz frequency excitation, how can a 50km long power transmission line be modeled ?

Solution:

As frequency increases by 10 times, physical length decreases by 10 times,

Short line: 0 – 8 km

Medium line: 8 – 16 km

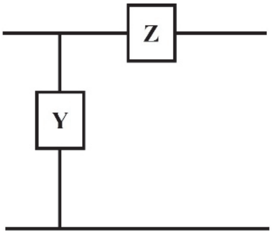
Long line: greater than 16 km

∴ 50 km line can be modeled as a long line.

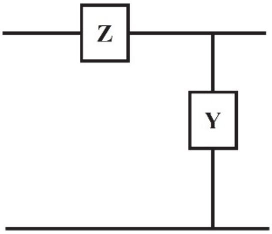
Q8. In the matrix form, equations of a 4– terminal network representing a transmission line is given by

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The two –networks considered are



and



Determine the matrices for the networks A & B

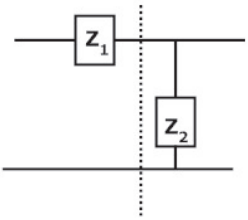
Solution:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}$$

Q9. Two networks are connected as shown in the figure, the equivalent ABCD–constants are further obtained, given that $Z_1 = 10\angle 30^0\ \Omega$, $C = 0.025\angle 45^0$, find the value of Z_2?

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{Z_2} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1+\frac{Z_1}{Z_2} & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$

$$C = \frac{1}{Z_2} \Rightarrow Z_2 = \frac{1}{C} = \frac{1}{0.025\angle 45^0}$$

$$Z_2 = 40\angle 30^0\ \Omega$$

Q10. A 220kV, 20km Long three phase transmission line has the following ABCD constants. A $= D = 0.96\angle 3^0$, $B = 55\angle 65^0\ \Omega$, $C = 0.510^{-4}\angle 80^0$ S. Determine the charging current per phase.

Solution:

Definition: The source current with receiving end open–circuited in source condenser method is called "Charging Current".

$$I_R = 0$$

$$I_C = I_S = CV_R + DI_R$$

$$= CV_R$$

$$= (0.5 \times 10^{-4})\left(\frac{220\times 10^3}{\sqrt{3}}\right)$$

$$= \frac{11}{\sqrt{3}}\ \text{A}$$

Q11. A 50HZ, 3–ϕ transmission line of length 100km has a capacitance of $\frac{0.03}{\pi}\frac{\mu F}{km}$. It is represented by π– model. Determine the shunt admittance at each end of the transmission line?

Solution:

$$C = \frac{0.03}{\pi}\frac{\mu F}{km}\times 100km$$

$$C = \frac{3}{\pi}\mu F$$

The shunt admittance at the end of each transmission line is

$$\frac{Y}{2} = \frac{1}{2}\left(jB_C\right) = \frac{1}{2}\left(j\omega_C\right) = \frac{1}{2}(j2\pi\times 50\times\frac{3}{\pi}\times 10^{-6} = j150\times 10^{-6}S$$

Q12. The Generalized circuit constants of a 3–ϕ, 220kv rated voltage, medium length line are A $= D = 0.936\angle 98^0$, $B = 142\angle 76.4^0\ \Omega$, load at the receiving end is 50MW at 220kV with a p.f. of 0.9(lagging). Calculate the magnitude of Line–to–Line sending end voltage?

Solution:

$$V_R = 220\text{kV}$$

$$V_S = AV_R + BI_R$$

$$= (0.936\angle 98^0)\left(\frac{220}{\sqrt{3}}\right) + (142\angle 76.4^o)I_R$$

$$P_R = \sqrt{3}\, V_R I_R \cos\phi_R$$

$$50 \times 10^6 = \sqrt{3} \times 220 \times 10^3 \times I_R \times 0.9$$

$$I_R = 145.79\text{A} = 145.79\angle -\cos^{-1}(0.9) = 145.79\angle -25.84^0 \ \text{A}$$

$$V_{S_{L-N}} = A\,V_R + B\,I_R = (0.936\angle 98^0\,)\left(\frac{220}{\sqrt{3}}\times 10^3\right) + (142\angle 76.4^0\ (145.79\angle -25.84^0$$

$$V_{S_{LL}} = \sqrt{3}V_{S_{L-N}} = \sqrt{3}\times 133\text{kV} = 233\text{kV}$$

Q13. Calculate the % rise in voltage at the receiving end of transmission line of length 200km, operating at 50Hz?

Solution:

$$\%\text{ rise in voltage } = \frac{V_S - V_R}{V_R}\times 100 = \frac{\omega^2 l^2}{18}\times 10^{-8}$$

$$= \frac{(2\pi\times 50)^2(200)^2}{18}\times 10^{-8}$$

$$= 2.1932\%$$

Q14. Calculate the time taken by voltage wave to travel 600km long overhead transmission line is.

Solution:

$$\text{Time, t} = \frac{\text{length, } l}{\text{velocity, } v} = \frac{600}{3\times 10^5} = 2\text{ msec}$$

Q15. A transmission line is having resistance 18 Ω & reactance 12 Ω and supplies a load of 5MW at a voltage ‘v’. The supply voltage is “V_S”. If $V = V_s$ then determine the p.f. of the load.

Solution:

$$V_R = V \text{ and } V_s = V$$

$$V_S = V_R + I_R R\cos\phi_R \pm IR\, X_L \sin\phi_R$$

for $V_S = V_R \Rightarrow I_R R \cos\phi_R - I_R X_L \sin\phi_R = 0$

$$\tan\phi_R = \frac{R}{X_L} = \frac{18}{12} = 1.5 \rightarrow \tan^{-1}(1.5) = \phi_R \rightarrow \phi_R = 56.3$$

$\cos\phi_R = 0.53$, Lead

1.10 INTERCONNECTION OF TRANSMISSION LINE

1.10.1 Transmission Lines Connected in Cascade

The cascade connection of transmission lines is shown in the figure 1.24.

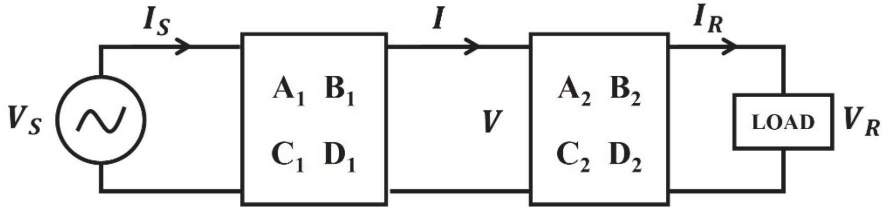


Figure 1.24 Cascade connection of transmission lines

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Q16. Two transmission lines are connected in cascade, whose ABCD parameters are

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 1 & 10\angle 30^\circ \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.025\angle -30^\circ & 1 \end{bmatrix}.$$

Determine the resultant ABCD Parameters.

Solution:

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 10\angle -30^\circ \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.025\angle -30^\circ & 1 \end{bmatrix}$$

1.10.2 Parallel Connection of Transmission Lines

The parallel connection of transmission lines is shown in the figure 1.25.

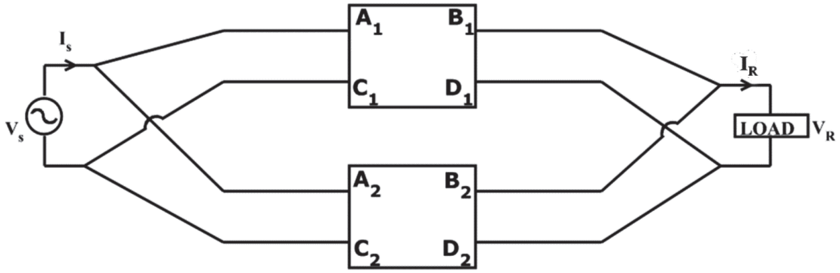


Figure 1.25 Parallel connection of transmission lines

$$A_0 = \frac{A_1B_2 + A_2B_1}{B_1 + B_2} \quad D_0 = \frac{D_1B_2 + D_2B_1}{B_1 + B_2}$$

$$B_0 = \frac{B_1B_2}{B_1 + B_2} \quad C_0 = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2}$$

Case – With identical transmission lines connected in parallel

$A_1 = A_2 = A$; $B_1 = B_2 = B$; $C_1 = C_2 = C$; $D_1 = D_2 = D$

$A_O = A$

$D_O = D$

$B_O = \frac{B}{2}$

$C_O = 2C$

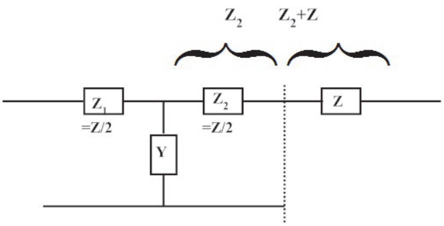
Q17. A medium line with parameters ABCD is extended by connecting a short line of impedance Z in series. Determine the overall ABCD–parameters of the series combination.

Solution:

Assume nominal–T line.

$$A_{new} = 1+\frac{YZ}{2} = 1+YZ_1 = A_{old}$$

$$C_{new} = Y = C_{old}$$



$$B_{old} = Z\left(1+\frac{YZ}{4}\right) = Z+\frac{YZ^2}{4}$$

$$= \left(\frac{Z}{2}+\frac{Z}{2}\right)+Y\left(\frac{Z}{2}.\frac{Z}{2}\right)$$

$$(Z_1 + Z_2) + Y(Z_1.Z_2)$$

$$B_{new} = \{Z_1 + (Z_2 + Z)\} + +YZ_1(Z_2 + Z)$$

$$= \{(Z_1 + Z_2 + YZ_1Z_2)\} + \{Z + YZ_1Z\}$$

$$B_{new} = B_{old} + ZA_{old}$$

$$D_{old} = 1 + YZ_2 = 1 + \frac{YZ}{2}$$

$$= 1 + Y(Z_2 + Z)$$

$$= 1 + YZ_2 + YZ$$

$$= D_{old} + YZ$$

$$D_{new} = D_{old} + Z\,C_{old}$$

OBJECTIVE QUESTIONS

1. A transmission line consists of R,L in ______________ and G,C in ___________.
2. Transmission lines are primarily classified based on ______________ of the travelling wave.
3. Transmission lines are secondarily classified based on ______________________________.
4. Ferranti effect do not occur in ___________________ transmission line.
5. Ferranti effect occurs with receiving end ________________________.
6. Ferranti effect results in no–load receiving end voltage _____________than sending end voltage.
7. The impedance of a practical transmission line is known as ____________________.
8. The impedance of an ideal transmission line is known as __________________________.
9. The characteristic impedance of an overhead transmission line is ____________ohms.
10. The characteristic impedance of an underground cable is _____________ ohms.
11. The maximum active power transmitted through a practical transmission line is known as ____________________________________.
12. The maximum active power transmitted through an ideal transmission line is known as _________________________________.
13. _________________________________varies inversely to the length of the line.
14. _______________________________ is independent of the length of the line.
15. The constants A,C can be determined with receiving _______________________.
16. The constants B,D can be determined with receiving end ____________________________.
17. Characteristic impedance is the geometric mean of _____________________ respectively.