CHAPTER 1

INTRODUCTION

Mechanics is a subject dealing with our many experiences in daily life. It is introduced in high school and intermediate education as a part of physics. Engineering Mechanics is the introductory course for engineers associated with design of any mechanical, civil or aerospace component/structure against failure when subjected to operational loads. It deals with forces acting on bodies and their effects. It is taught in many universities in two parts as –

- Engineering Mechanics or **Mechanics of rigid bodies**, limited to the effects of forces in keeping the body in equilibrium or in producing desired motion and
- Strength of Materials or Solid Mechanics or **Mechanics of deformable solids** considering the deformations of the body such as
  - axial compression or buckling of columns
  - bending deformation of beams
  - angular twist of shafts etc..

The subject is broadly classified as

- Statics – involving calculation of forces & reactions in bodies at rest, even after forces are applied
- Dynamics – involving study of motion of bodies due to forces, covering
  Kinematics – dealing with motion of bodies
  Kinetics – dealing with forces which result in motion of bodies and
  Vibrations – dealing with repetitive or cyclic motion of bodies over an equilibrium position

The subject doesn't use empirical relations and the equations can be checked for matching dimensions on either side. A large variety of problems can be created in this subject. Solution to any problem in Engineering Mechanics involves proper understanding of the physical problem and identifying mathematical equations associated with the specific conditions/situations. Most problems can be solved from a set of 'n' simultaneous equations in 'n' unknowns. They can be solved by many different approaches, using different mathematical (trigonometric) theorems. Hence, if the number of unknown variables and number of equations from the specified conditions match, the problem is half solved.

It is very common in question papers, to find different quantities in different units. The student is expected to use consistent units for all quantities. A brief coverage of units of measurement, arithmetics, trigonometry, differential and integral calculus are added, for a quick reference while solving problems in this subject.

The subject appears to be very well understood, when a student goes through solved problems. **Practice with unsolved problems is essential** for good performance in the examinations. For a better exposure to a large variety of problems, every student is advised to refer to many books and old question papers.

1.1 FORCES

A Force is defined by **Newton's 1st law of motion** as one which changes state of rest or of uniform motion of any physical body. A force differs from other physical properties of a body, by the identifying parameters associated with it.

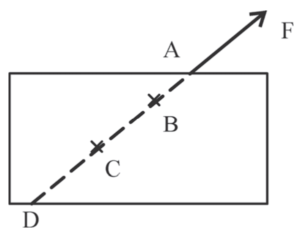
Every body has properties such as mass, temperature etc., distributed at various points. These properties have magnitudes but are not associated with any specific direction. These are called ***scalars***. They can be added together algebraically.

i.e., $\sum m = m_1 + m_2 + m_3 + m_4 + \ldots..$

A Force such as weight, reaction, centrifugal force etc.. has magnitude as well as direction associated with it. Such a directional quantity is called a ***vector***. Magnitude of a vector **P** is denoted by |P| or p.

A single scalar property such as mass (m) at a point may produce many vectors in different directions such as weight 'mg' (oriented towards center of earth), momentum 'mv' of a moving mass (in the direction of motion), centrifugal force 'mv^2/r' of a rotating mass, acting radially outwards etc.

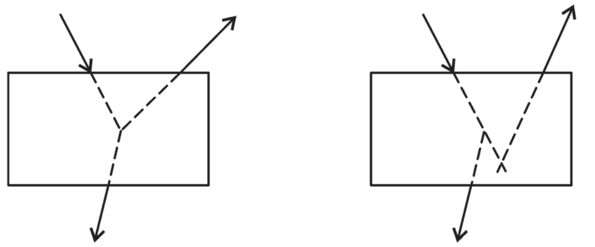
Effect of a force on a body is not dependent on the point of application, but only on the line of action or direction. A force (F) can be considered as applied from any point on the body (A, B, C or D in the figure) along its line of action. This is called **Principle of transmissibility of forces.**



1.1.1 CLASSIFICATION OF FORCES

Forces are broadly categorised, based on their orientation, as ***co-planar (2-D) forces*** (if all the forces lie in a single plane – horizontal, vertical or inclined) and ***spatial (3-D) forces***

They are also categorised, based on their lines of action, as ***concurrent forces*** with lines of action starting from a point or meeting at a point (Ref Fig 1.1 a) and ***non-concurrent forces*** (Ref Fig 1.1 b). Concurrency does not mean forces are acting at one point, but their lines of action, even when they act at different points of a body, meet at one point as shown in the fig 1.1 (a).



(a) Concurrent forces (b) Non-concurrent forces

FIGURE 1.1 Concurrent and non-concurrent forces

1.1.2 COMPONENTS OF FORCES

A force oriented in an arbitrary direction is identified by its magnitude and the direction of the force represented by the angles it makes with the coordinate axes. It is more conveniently described by its components along chosen coordinate axes (along X, Y for planar forces and along X, Y and Z for spatial forces). They are explained in more detail with examples in the subsequent chapters.

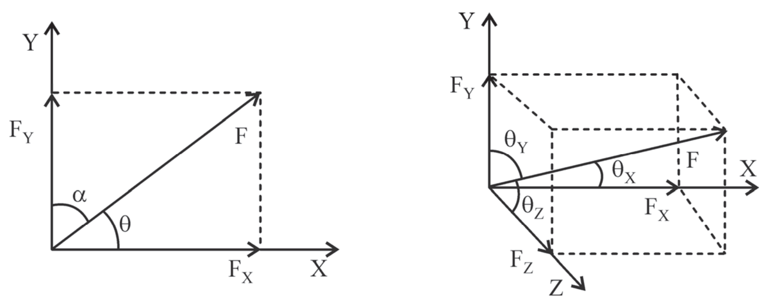


FIGURE 1.2 Components of a force in a 2-D plane & 3-D space

1.2 UNITS

The magnitude of any physical quantity loses its significance, unless associated with the appropriate units. Different countries may use different units. Engineers and scientists in most countries now use SI units – with basic units such as ‘meter (m)’ for length, ‘kilogram (kg)’ for mass, ‘second (sec)’ for time and ‘degree Kelvin (K)’ for temperature and derived units of ‘Newton (N)’ for force, ‘Pascal (Pa)’ for pressure or stress and ‘Watt (W)’ for power etc. The basic units of m, kg, sec for length, mass and time respectively are together called MKS system.

1.2.1 SOME DERIVED UNITS

(a) According to Newton’s 2nd law of motion, magnitude of force is proportional to rate of change of momentum

$$F \;\alpha\; d(m \times v) / dt$$

where

v is the velocity of the body due to the applied forcé

$F = k \times m \times (dv/dt)$ where, k is the proportionality constant

$= k \times m \times a$ if mass of the body (m) does not change with time

and

$a = dv/dt$ is the acceleration of the body

In SI units, F (1 N) = m (1 kg) × a (1 m/sec^2) is a derived unit from kg, m and sec with k =1 for the given set of units

We refer to weight (forcé with which mass, m is attracted to the ground) in kg_f

It is defined as F (1 kg_f) = m (1 kg) × a (9.81 m/sec^2)

Thus, 1 kg_f = 9.81 N ≈ 10 N

(b) Applied pressure (p) or internal stress (σ) is defined as force per unit area.

Thus, p or $\sigma = F / A$ or 1 Pa = 1 N / m^2

Magnitude of 1 Pa is very low and in many problems,

Pressure or stress is expressed as M Pa (Mega Pascal or 10^6 Pa)

Modulus of elasticity or modulus of rigidity (in the same units as stress) is expressed as G Pa (Giga Pascal or 10^9 Pa).

In our day-to-day life, we use MKS system of units. Accordingly, we are familiar with ***weight in 'kg_f', pressure in 'kg_f/sq cm'*** etc. This may pose some difficulty or confusion to the students. However, enginners and scientists would prefer to use unambiguous units like 'kg' for mass and 'N' for force (weight,..) as primary or basic units and 'Pa' or 'N/sq m' for pressure etc as derived units.

(c) Power (P), extensively used in this subject, is defined as

Power = Work done per unit time

where, work done, W = Force × distance

Thus, P (1 W) = Work done (1N × 1m) / time (1 sec)

or 1 W = 1 Nm/sec

1.2.2 METRIC SYSTEM OF UNITS

A brief understanding of metric system (with smaller and larger units as negative and positive powers of 10) will be useful in this subject. Table 1.1 gives standard symbols used in metric units system

Table 1.1 Metric system of units

10^{-9}	**10^{-6}**	**10^{-3}**	**10^{-2}**	**10^{-1}**	**Basic unit**	**10^{1}**	**10^{2}**	**10^{3}**	**10^{6}**	**10^{9}**
Nano (n)	**Micro (μ)**	**Milli (m)**	**Centi (c)**	**Deci (d)**		**Deca (da)**	**Hecta (h)**	**Kilo (k)**	**Mega (M)**	**Giga (G)**
nm	μm	mm	cm	dm	**meter**	da m	hm	km	Mm	Gn
ng	μg	mg	cg	dg	**gram**	da g	hg	kg	Mg	Gg
nW	μW	mW	cW	dW	**Watt**	da W	hW	kW	MW	GW
nPa	μPa	mPa	cPa	dPa	**Pascal**	da Pa	hPa	kPa	MPa	GPa

Conversion factor: A commonly used unit of velocity in Kinematics chapter is 1 m/sec = 1 × 10^{-3} km / (1/3600)hr = 3.6 Km/hr or kmph

1.3 ENGINEERING MATERIALS

A design engineer should have some knowledge of materials to be used in a particular component and their properties as well as limitations. Detailed study of the materials is outside the scope of this book.

1.3.1 MATERIAL TYPES

Many structures may consist of members made up of different materials with distinctly varying material properties, such as Modulus of elasticity (E), Modulus of rigidity (G), coefficient of linear thermal expansion. These properties may be

(a) constant (linear stress-strain relationship) or variable (non-linear stress-strain relationship) over the range of load
(b) same in all directions (isotropic) or vary in different directions (anisotropic or orthotropic)
(c) constant over the temperature range or vary with temperature, particularly when the temperature range over the component is large

These variations may be inherent in the material or induced by the manufacturing processes like rolling, casting, welding etc. Treatment of non-homogeneous material, with varying properties at different locations of a component, is difficult and is also very unusual. In most cases, variation of material properties with the direction and with temperature may not be significant and hence neglected. So, an isotropic, homogeneous material is most often used in the analysis of a component.

Engineering materials used in examples of this book are assumed to be

- **homogeneous** – having same properties at all points of the component (in reality most alloys are non-homogeneous at micro level, but their gross or average properties are considered homogeneous and used in calculations)
- **isotropic** – having same properties in all directions (in reality materials like wood, fiber reinforced composite materials etc are non-isotropic)
- **stress is within the linear elastic range** – when loads are applied, the resulting stress is proportional to strain, satisfying Hooke's law (even though at times, stress may be in the non-linear elastic or plastic range) and
- **properties are independent of type of load** – some materials may behave differently when subjected to tensile, compressive, shear and torsional loads (some materials like bricks, concrete, wood are strong in compression and weak in tension)

1.4 LINEAR ANALYSIS

It is based on linear stress-strain relationship (Hooke's law) and is usually applicable to stress below the elastic limit (or yield stress), at any point in the component. In this analysis, ***linear superposition of results*** obtained for individual loads on a component is valid in order to obtain stresses due to any combination of these loads acting simultaneously. In some designs, it is necessary to check for many combinations such as pressure and thermal loads at different times of a start-up transient of a steam turbine. In such a case, analysing for unit pressure; multiplying the stress results with the pressure corresponding to that particular time of the transient and adding to the stresses due to temperature distribution will be economical.

1.5 A BRIEF REVIEW OF MATHEMATICS

- **Solution of a quadratic polynomial** of the form $a \times x^2 + b \times x + c = 0$ is given by

$$x = -b \pm \sqrt{(b^2 - 4 \times a \times c) / (2 \times a)}$$

- **Sum of an arithmetic progression**

$$n + (n - 1) + (n - 2) + \ldots\ldots + 1 = n \times (n + 1)/2$$

1.5.1 OVERVIEW OF TRIGONOMETRY

Elementary concepts of trigonometry are used while solving many problems in this subject. A review of the basics is considered desirable, without proof for the same. Books on trigonometry may be referred for additional concepts as well as for proof of the theorems mentioned here.

- If two straight lines are inclined at an angle θ, then **opposite angle** is also θ (Ref Fig 1.3 a)
- If two parallel lines are cut by a straight line, the **corresponding angles** are equal (Ref Fig 1.3 b)
- If two parallel lines are cut by a straight line, **internal opposite angles** are equal, by a combination of the first two rules (Ref Fig 1.3 c)
- If two straight lines AB and AC are inclined at an angle θ, then their **perpendiculars DE and FG are also inclined at the same angle θ** (Ref Fig 1.3 d)

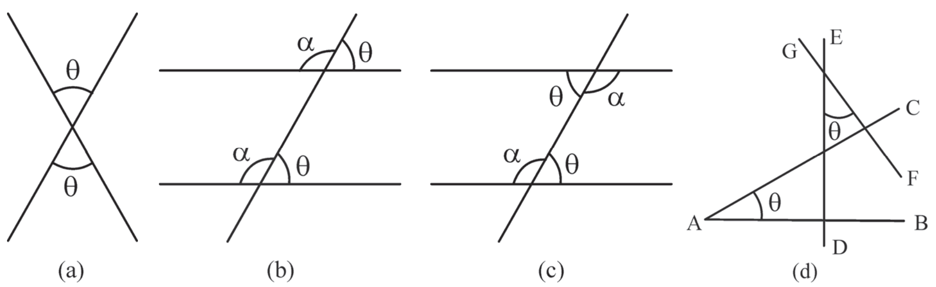


FIGURE 1.3 Some useful theorems of Trigonometry

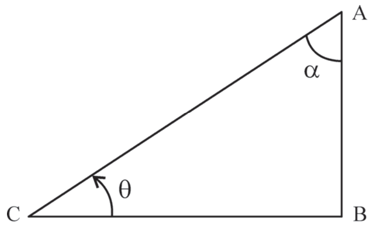
In a right angled triangle (one angle is 90^0),

Sine θ or Sin θ = Opposite side / Hypotenuse = AB/AC

Cosine θ or Cos θ = Adjacent side / Hypotenuse = BC/AC

Tangent θ or Tan θ = Opposite side / Adjacent side

= Sin θ/Cos θ = AB/BC



Cosecant θ or Csc θ = 1 / Sin θ = Hypotenuse / Opposite side = AC/AB

Secant θ or Sec θ = 1 / Cos θ = Hypotenuse / Adjacent side = AC/BC

Cotangent θ or Cot θ = 1 / Tan θ = Adjacent side / Opposite side = BC/AB

It is obvious that, Sin θ = Cos α and Cos θ = Sin α

Minimum of (Sin θ or Cos θ) = 0

and Maximum of (Sin θ or Cos θ) = +1 or -1

Angle is usually measured in the counter-clockwise direction from +ve X-axis. For different values of angle θ, nature of values of Sin θ, Cos θ and Tan θ are given in Table 1.2.

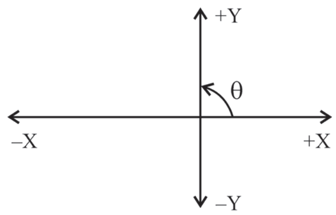
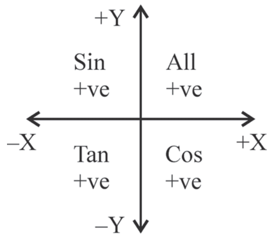


Table 1.2 Values of trigonometric functions

Angle θ (degrees)	Angle θ (radians)	Sin θ	Cos θ	Tan θ
0^0 or 360^0	0 or 2π	0	1	0
> 0^0 – < 90^0	> 0 – < π/2	+ ve	+ ve	+ ve
90^0	π/2	1	0	∞
> 90^0 – < 180^0	> π/2 – < π	+ ve	– ve	– ve
180^0	π	0	–1	0
> 180^0 – < 270^0	> π – < 3π/2	– ve	– ve	+ ve
270^0	3π/2	–1	0	∞
> 270^0 – < 360^0	> 3π/2 – < 2π	– ve	+ ve	– ve



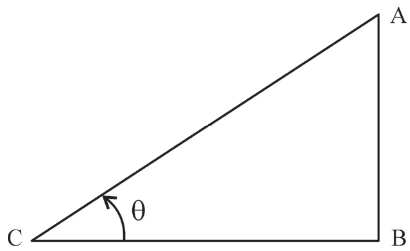
Signs of these trigonometric functions are remembered by the notation – All Silver Tea Cups, 1st letter of each word indicating +ve functions (All, Sin θ, Tan θ and Cos θ) in the four quadrants in sequence i.e., ALL in 1st quadrant (+X,+Y), Sin in 2nd quadrant (–X,+Y), Tan in 3rd quadrant (–X,–Y), Cos in 4th quadrant (+X,–Y)

Necessary changes must be made, when angles are measured from –ve X-axis, +ve Y-axis or –ve Y-axis

From Pythagoras theorem, $AB^2 + BC^2 = AC^2$

or $AB^2/AC^2 + BC^2/AC^2 = 1$

$\text{Sin}^2\,\theta + \text{Cos}^2\,\theta = 1$



If α and β are two adjacent angles, trigonometric properties (Sine and Cosine) for the sum of the angles are given by

$\text{Sin}\,(\alpha + \beta) = \text{Sin}\,\alpha \times \text{Cos}\,\beta + \text{Cos}\,\alpha \times \text{Sin}\,\beta$

$\text{Sin}\,2\,\alpha = 2\,\text{Sin}\,\alpha \times \text{Cos}\,\alpha$

$\text{Cos}\,(\alpha + \beta) = \text{Cos}\,\alpha \times \text{Cos}\,\beta - \text{Sin}\,\alpha \times \text{Sin}\,\beta$

$\text{Cos}\,2\,\alpha = \text{Cos}^2\,\alpha - \text{Sin}^2\,\alpha$

$= 2\text{Cos}^2\,\alpha - 1 = 1 - 2\text{Sin}^2\,\alpha$

$\text{Sin}^2\,\alpha = (1 - \text{Cos}\,2\alpha)\,/\,2$

$\text{Cos}^2\,\alpha = (1 + \text{Cos}\,2\alpha)\,/\,2$

$\text{Tan}^2\,\alpha = \text{Sin}^2\,\alpha\,/\,\text{Cos}^2\,\alpha = (1 - \text{Cos}^2\,\alpha)\,/\,\text{Cos}^2\,\alpha = \text{Sec}^2\,\alpha - 1$

$\text{Cot}^2\,\alpha = \text{Cos}^2\,\alpha\,/\,\text{Sin}^2\,\alpha = (1 - \text{Sin}^2\,\alpha)\,/\,\text{Sin}^2\,\alpha = \text{Csc}^2\,\alpha - 1$

1.5.2 DIFFERENTIAL & INTEGRAL CALCULUS

Some formulae of differential and integral calculus are given below, useful for solving problems in centroid, Moment of inertia, kinematics etc. Students interested in a detailed explanation and proof may refer to appropriate books in mathematics.

Differentiation	**Integration**
$d(x^n)/dx = n \times x^{n-1}$	$\int x^n\, dx = x^{n+1} / (n+1)$
$d[(ax+b)^n]/dx = a\times[n\times(ax+b)^{n-1}]$	$\int (ax+b)^n\, dx = (ax+b)^{n-1} / [(n-1)\times a]$
$d(\sin\theta)/d\theta = \cos\theta$	$\int (\cos\theta)\, d\theta = \sin\theta$
$d(\cos\theta)/d\theta = -\sin\theta$	$\int (\sin\theta)\, d\theta = -\cos\theta$
	$\int (\tan\theta)\, d\theta = -\log_e (\mathbf{cos}\ \theta)$
	$\int (\cot\theta)\, d\theta = \log_e (\sin\theta)$
$d(\tan\theta)/d\theta = \sec^2\theta = 1+\tan^2\theta$	$\int (\sec^2\theta)\, d\theta = \tan\theta$
$d(\cot\theta)/d\theta = -\mathrm{cosec}^2\theta = -(1+\cot^2\theta)$	$\int (\csc^2\theta)\, d\theta = -\cot\theta$
$d(\sec\theta)/d\theta = \sin\theta / \cos^2\theta$ $= \sec\theta \times \tan\theta$	$\int(\sin^2\theta)\, d\theta = \int [(1-\cos 2\theta)/2]\, d\theta$ $= \theta/2 - (\sin 2\theta)/4$
$d(\csc\theta)/d\theta = -\sin\theta / \cos^2\theta$ $= -\csc\theta \times \cot\theta$	$\int(\cos^2\theta)\, d\theta = \int [(1+\cos 2\theta)/2]\, d\theta$ $= \theta/2 + (\sin 2\theta)/4$
$d(\log_e x)/dx = 1/x$	$\int dx/x = \log_e x$
$d^2(x^m y^n)/(dx\, dy) = d(x^m)/dx \times d(y^n)/dy$ $= (mx^{m-1}) \times (ny^{n-1})$ if x and y are mutually independent	$\int\int x^m y^n\, dx\, dy = \int x^m\, dx \times \int y^n\, dy$ $= [x^{m+1}/(m+1)] \times [y^{n+1}/(n+1)]$ if x and y are mutually independent

1.6 PRECAUTIONS / USEFUL TIPS TO STUDENTS

- Questions may contain different quantities in different units. Care should be taken to ***use all physical quantities in one consistent system of units*** (N or kN for force; km or m or cm for distance; Pa or MPa or GPa for pressure; hr, min or sec for time; …). Units of quantities in all equations on either side should match. Otherwise results can be erroneous, even after using correct method and correct formulae.
- Mere numbers in answers will not fetch full marks. ***Students should practice using units along with all physical quantities*** like distance, forcé, stress, speed, velocity, acceleration, .. in answers. It will also help them get a physical feel of quantities like distance, force, stress etc in solutions and correct absurd answers.
- ***Angles are in degrees in some chapters / problems, while they are in radians in some other chapters. Students should use calculator in the appropriate mode***. Otherwise, solutions will be incorrect.

- Different students may use different coordinate systems (X, Y, Z axes) and differnt notations (Force components P_1, P_2, P_3,.. or F_1, F_2, F_3,..or P_X, P_Y, P_Z,.. ; angles α, β, γ, θ, φ etc) while solving problems in examinations.
- ***It is a good practice to draw Free body diagram (FBD) in all relevant problems*** which will also indicate notation of forces, angles and coordinate system used by a student thereby avoiding a posible misunderstanding with the examiner while evaluating answer paper