## CHAPTER 1

## Flow of Fluid

Types of Manometers, Reynolds Number and its Significance, Bernoulli's Theorem and its Applications, Energy Losses, Orifice Meter, Venturi Meter, Pitot Tube and Rotameter.

## Types of Manometers

Manometer is one of the earliest devices used for measuring fluid pressure difference consisting of a hollow U-tube having one or more fluid (may be liquid or gas) of different specific gravities. It can provide a very accurate result. A U-tube manometer is recognized by NIST as a primary standard due to its inherent accuracy and simple operation. The manometer has no movable parts that can subject to wear. Manometers works on the Hydrostatic Balance Principle: a liquid column of known height will exert a known hydrostatic pressure when the weight per unit volume of the fluid is known. The fundamental relationship for pressure expressed by a fluid column is:

$$
p=P_{1}-P_{2}=\rho \times h \times g
$$

Where
p is the differential pressure,
$\mathrm{P}_{1}$ is the pressure at the low-pressure connection
$P_{2}$ is the pressure at the high-pressure connection
$\rho$ is the density of the liquid
g is acceleration due to gravity, and
$h$ is the height of the liquid column
All forms of manometers such as Piezometer, U-tube manometers, differential manometer, well-type, and inclined, etc. contain two liquid surfaces. Pressure determinations are made by how the fluid moves when pressures are applied to each surface. For gauge pressure, $\mathrm{P}_{2}$ is equal to zero (atmospheric reference), simplifying the above equation to

$$
\mathrm{p}=\mathrm{\rho hg}
$$

In manometer, a known pressure (which is generally atmospheric pressure) is applied to one end of the manometer tube and the unknown pressure (to be determined) is applied to the other end.

The manometers measure the differential pressure; that is, only the difference between the two pressures. There are primarily four types of manometers:

1. Simple u-tube manometer - Piezometer
2. Differential U-tube manometers
3. U-tube with one leg enlarged (Well type manometer), and
4. Inclined U-tube manometer

The U-tube manometer may be of three types:
(a) Two fluid U-tube manometer,
(b) Four-fluid U-tube manometer, and
(c) Differential U-tube manometer

1. Simple u-tube manometer - Piezometer: It is used to measure the pressure in a static fluid by using the height of a column of a liquid as shown in Figure 1.1.
Pressure at $\mathrm{A}=$ Pressure at B

$$
=\text { p.g.h }
$$

Where, $\rho$ is the density of the liquid g is acceleration due to gravity h is the height of the liquid column
2. Differential U-tube manometers: The principles of manometry can be easily established in the U tube manometer shown in Figure 1.2. It is simply a glass-tube


Fig. 1.1 Piezometer bent to form the letter $U$ and partially filled with a liquid. If both the legs of the manometer are kept open to atmosphere or exposed to the same pressure, the levels of the liquid remain exactly on the same line or with a zero difference. As illustrated in Figure 1.3, if the pressure is more to the left side of the apparatus, the fluid level comes down to a lower level in the left leg and raises in the right leg. The fluid moves until the weight of the fluid as indicated in figure exactly balance the pressure. This is known as hydrostatic balance. The height of fluid from one side to the other is the actual height of fluid opposing the pressure. The pressure is always indicated by the height of fluid regardless of the shape or size of the tubes, as illustrated in Figure 1.3. Manometers in Figure 1.3 are open to atmosphere on both legs indicating the fluid level in both legs. Because of the variations in the volume of the manometer legs, the distances moved by the fluid columns are different. However, the total distance between the fluid levels, H , remains identical in all the manometers.


Fig. 1.2 U-tube manometer


Fig. 1.3 U-tube manometer


Fig. 1.4 U-tube manometer

## 3. U-tube with one leg enlarged (Well type manometer):

 To provide convenience and to meet other requirements this type of manometer has been used. The well-type manometer is one of the various types of manometers. The schematic diagram of this type of manometer has been shown in Figure 1.5. The cross-sectional area of one leg of the manometer is many times larger than that of the other leg. The leg with a larger area is called the well. As pressure is applied to the larger leg, the fluid moves down compared to the increase in height of the smaller leg.The true pressure reading follows the principles, previously described and is measured by the difference between the fluid surfaces $h$. As the pressure is applied at


Fig. 1.5 Well type manometer 1 there must be some drop in the well level from O to X . This is readily compensated for by spacing the scale graduations in the exact amount required to correct for this well drop. To ensure the accuracy of this correction, the well area and internal diameter of the indicating tube must be carefully controlled.
Thus, the well type manometer offers itself to be used with direct reading scales graduated in units for the process or test variable involved. It does require certain operational restrictions not found on the U-tube. A pressure higher than atmospheric pressure is always connected to the well; a pressure lower than atmospheric is always connected to the top of the tube. For a differential pressure, the higher pressure is connected at the well. A raised well manometer, however, allows both gauge and vacuum measurements off of the well port.
4. Inclined U-tube manometer: In many cases, accurate measurement of low pressure such as drafts and very low differentials are required. To handle these applications better the manometer is arranged with the indicating tube inclined, as shown in Figure 1.6. It provides better resolution. This arrangement can allow 12 " of scale length to represent 1 " of vertical liquid height. With subdivisions of scale, even up to a pressure of 0.00036 psi (one-hundredth of an inch of water) can be read.

## Absolute Manometer

Absolute zero pressure refers to the pressure at a perfect vacuum. In an absolute pressure manometer, the pressure being measured is compared to absolute zero pressure in a sealed leg above a mercury column, as shown in Figure 1.7. The term absolute zero pressure is derived from the definition, that is, a perfect vacuum is the complete absence of any gas. The most common form of sealed tube manometer


Fig. 1.7 Absolute manometer
is the conventional mercury barometer used to measure atmospheric pressure. Mercury is the only fluid used in this application. In this type of manometer, there is only one connection from which both pressures above atmospheric pressure and pressure below atmospheric can be measured. Absolute manometers are available in well type or U tube configurations.

## Indicating Fluid

The sensitivity, range, and accuracy of the manometer can be improved by selecting an appropriate indicating fluid. Indicating fluids are available with densities varying from 0.827 $\mathrm{g} / \mathrm{cm}^{3}$ (Red Oil) to $13.54 \mathrm{~g} / \mathrm{cm}^{3}$ (Mercury). The pressure range would become three times greater and the resolution would be one third as great if an indicating fluid is three times heavier than water. If the density of an indicating fluid is less than water, the pressure range will decrease and the resolution (sensitivity) will increase. Thus, for a given size of the instrument, the pressure range can be increased by using a fluid with higher density and reduced by using a fluid with lower density.

## Correction of the Manometer

Manometry measurements depend on both density and acceleration due to gravity. The values of the density and acceleration due to gravity are not always constant. Density is a function of temperature, and acceleration due to gravity is a function of latitude and elevation. Due to this relationship, specific ambient conditions must be selected as standard, so that a fixed pressure can be maintained.

Standard conditions for mercury: Density of mercury at $0^{\circ} \mathrm{C}$ is $13.5951 \mathrm{~g} / \mathrm{cm}^{3}$;
Acceleration due to gravity at sea level and at $45.544^{\circ}$ latitude is $980.665 \mathrm{~cm} / \mathrm{sec}^{2}$
Standard conditions for water:
Density of water at $4^{\circ} \mathrm{C}$ is $1.000 \mathrm{~g} / \mathrm{cm}^{3}$;
Acceleration due to gravity at sea level and at $45.544^{\circ}$ latitude is $980.665 \mathrm{~cm} / \mathrm{sec}^{2}$
Generally, the manometers may be read outside standard temperature and acceleration due to gravity and corrections should be made to improve the accuracy of a manometer reading at any given condition.

## Correction of Fluid Density

Manometers indicate the correct pressure at only one temperature. This is because the indicating fluid density changes with temperature. If water is the indicating fluid, an inch scale indicates one inch of water at $4^{\circ} \mathrm{C}$ only. On the same scale, mercury indicates one inch of mercury at $0^{\circ} \mathrm{C}$ only. A reading using water or mercury taken at $20^{\circ} \mathrm{C}\left(68^{\circ} \mathrm{F}\right)$ is not an accurate reading. The error introduced is about $0.4 \%$ of reading for mercury and about $0.2 \%$ of reading for water. Since manometers are used at temperatures above and below the standard temperature, corrections are needed. A simple way of correcting for density changes is to ratio the densities. At a particular place where the value of $g$ is constant,

$$
\text { (Standard) } \rho_{\mathrm{c}} \mathrm{gh}_{\mathrm{c}}=(\text { Ambient }) \rho \mathrm{gh}
$$

$$
h_{c}=\frac{\rho}{\rho_{c}} \times h
$$

Where,
$\rho_{c}=$ Density of indicating fluid at standard temperature,
$\mathrm{h}_{\mathrm{c}}=$ Corrected height of the indicating fluid at standard temperature,
$h=$ Height of the indicating fluid at experimental temperature, and
$\rho=$ Density of indicating fluid at the experimental temperature
This method is very accurate when density-temperature relations are known. Data is readily available for water and mercury.

The density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ of mercury can be corrected as a function of temperature $\left(0^{\circ} \mathrm{C}\right)$ as

$$
13.556786[1-0.0001818(\mathrm{~T}-15.5556)]
$$

Density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ of water can be corrected as a function of temperature $\left(0^{\circ} \mathrm{C}\right)$ as 0.9998395639 $+6.798299989 \times 10^{-5}(\mathrm{~T})$

$$
\begin{aligned}
& -9.10602556 \times 10^{-6}\left(\mathrm{~T}^{2}\right)+1.005272999 \times 10^{-7}\left(\mathrm{~T}^{3}\right)- \\
& 1.126713526 \times 10^{-9}\left(\mathrm{~T}^{4}\right)+6.591795606 \times 10^{-12}\left(\mathrm{~T}^{5}\right)
\end{aligned}
$$

## Reynolds Number

The Reynolds number is defined as the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.

When liquids flow through a pipe or tube or channel, the character of the flow varies depending on certain conditions. This was first demonstrated by Osborne Reynolds in the year 1883. He carried out the experiment using a long glass tube. The tube was connected to water reservoir to maintain a constant supply pressure. A jet was inserted into the inlet of the tube as shown in figure 8 . The position of the jet was so adjusted that it can inject the dye solution at the center of the tube. The outlet of the tube was attached to a valve to control the flow of water. Thus, two liquids (water and dye solution) were flowing through the tube. Reynolds observed how the flow property of the dye solution varied with flow property (velocity) of water.

When the velocity of water was less, the dye solution flew steadily, having a thread-like appearance and remained undisturbed in the center of the water stream. This can be imagined as if water has been flowing in a series of concentric layers, like the draw-tube of a telescope. This type of flow is called streamline, laminar, or viscous flow.

When the velocity of water was increased moderately, a point was reached at which the thread started to become a wave-like appearance without mixing with water. The velocity of water at this point was called critical velocity and the phase of the flow was called as transitional flow.

When the velocity of water was further increased to a high degree, eddies (vortices) began to start in the flow and the dye solution started mixing with water just after leaving the orifice of the jet. This was the state of complete turbulence and flow is called turbulent flow.

Figure 1.8 shows the schematic diagram of Reynolds apparatus. Based on the observations made in his experiment, Reynolds concluded that the flow of liquid could be affected by the following four factors;

Diameter of the pipe,
Velocity of the fluid,
Density of the fluid, and
Viscosity of the fluid.


Fig. 1.8 Reynolds apparatus
Reynolds connected all these factors, grouped together and expressed in the form of the following equation:

$$
\operatorname{Re}=\frac{\rho u d}{\mu}
$$

Where Re is the Reynolds Number, $\rho$ is the density of the fluid, $\mathrm{kg} / \mathrm{m}^{3}$
$u$ is the velocity of the fluid, $\mathrm{m} / \mathrm{s}$
$d$ is the diameter of the pipe, $m$ and
$\mu$ is the viscosity of the fluid, $\mathrm{kg} / \mathrm{m} . \mathrm{s}$.
Hence, the unit of Reynolds Number would be $\frac{\mathrm{kg}}{\mathrm{m}^{3}} \times \frac{\mathrm{m}}{\mathrm{s}} \times \mathrm{m} \times \frac{\mathrm{m} . \mathrm{s}}{\mathrm{kg}}$. Thus, Reynolds Number has no unit; that is, dimensionless.

## Significance of Reynolds Number

It is very important to know whether the flow is streamlined or turbulent and the significance of Reynolds Number is that it can be used to predict the character of flow in a particular set of conditions. Generally, the significances of the Reynolds Number are that:

If the Reynolds Number is less than 2000, the flow will be streamlined or laminar.

If the Reynolds Number is more than 4000 , the flow will be turbulent.
If the Reynolds Number is more than 2000 but less than 4000 , the type of flow will depend on the form of the flow channel.

If there is no disturbance of any kind the flow pattern may be streamlined even at Reynolds Number is more than 2000.

If the internal surface of the pipe is rough, the flow may be turbulent.
If there are bends in the pipe, the flow may be turbulent.
If there are other pipe fittings attached to the pipe, the flow may be turbulent even the Reynolds Number is less than 2000.

The major difference between the streamline and turbulent flow is that in streamline flow there is no velocity component at right angles to the direction of flow. Hence, in case of streamline flow, there will no movement of fluid between the walls and center of the pipe or tube, and the dye solution will appear like a thread at the center of the tube in the Reynolds experiment. While in case of turbulent flow there will be greater movement of fluid across the direction of flow, eddies would be set up, and there will be a mixing of two liquids.

## Bernoulli's Theorem

According to Bernoulli's theorem, if there is an increase in the speed of the liquid, there would be a simultaneous decrease in the potential energy of the fluid or there is a decrease in the pressure of the fluid. Basically, it is the principle of conservation of energy in the case of ideal fluids. If the fluid flows horizontally such that there is no change in the gravitational potential energy of the fluid then increase in velocity of the fluid results in a decrease in pressure of the fluid and vice versa. In the year 1726 Daniel Bernoulli postulated three basic equations of hydrodynamics (properties of moving fluids - liquids and gases).Bernoulli's principle is applied to various types of fluid flow and there are different forms of Bernoulli's


Fig. 1.9 Schematic diagram of Bernoulli's equation equation for different types of flow. In fact, Bernoulli's equation can be derived from the principle of conservation of energy. It states that, in a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline. That is, the sum of kinetic energy, potential energy, and internal energy would remain constant.

## Derivation of Bernoulli's Equation

Let us assume that a fluid is flowing through a pipe; at a point $X$ the velocity, pressure, and area of the fluid column are $v_{1}, p_{1}$, and $A_{1}$ respectively. Similarly, let us assume that at another point $Y$ the velocity, pressure, and area of the fluid column are $\mathrm{v}_{2}, \mathrm{p}_{2}$, and $\mathrm{A}_{2}$ respectively.

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Let the volume of the fluid is bounded by XY and it is moving towards MN, and there the fluid is bounded by MN.

Let us further assume that $\mathrm{XM}=\mathrm{L}_{1}$ and $\mathrm{YN}=\mathrm{L}_{2}$.
Now, if we can compress the fluid; then,

$$
\mathrm{A}_{1} \times \mathrm{L}_{1}=\mathrm{A}_{2} \times \mathrm{L}_{2}
$$

We know that the work done by the pressure difference per volume of the unit is equal to the sum of the gain in kinetic energy and gain in potential energy per volume of the unit.
This indicates that

Or, $\quad$ Work done $=p \times$ volume
Therefore, network done per volume $=\mathrm{p}_{1}-\mathrm{p}_{2}$
The kinetic energy per unit volume $=\frac{1}{2} \mathrm{~m} \mathrm{v}^{2}$
Therefore, we have,
The kinetic energy gained per unit volume $=\frac{1}{2} \rho\left[\left(\mathrm{v}_{2}\right)^{2}-\left(\mathrm{v}_{1}\right)^{2}\right]$
And potential energy gained per unit volume $=\mathrm{pg}\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right)$
where $h_{1}$ and $h_{2}$ are heights of X and Y above the reference level taken in common.
Finally, we have

$$
\begin{aligned}
& p_{1}-p_{2}=\frac{1}{2} \rho\left[\left(v_{2}\right)^{2}-\left(v_{1}\right)^{2}\right]+p g\left(h_{2}-h_{1}\right) \\
& \text { or, } p_{1}+\frac{1}{2} \rho\left(v_{1}\right)^{2}+\rho g h_{1}=p_{2}+\frac{1}{2} \rho\left(v_{2}\right)^{2}+\rho g h_{2} \\
& \text { or, } p+\frac{1}{2} \rho v^{2}+\rho g h \text { is a constant }
\end{aligned}
$$

When $\mathrm{h}_{1}=\mathrm{h}_{2}$
Then, we have, $\mathrm{p}+\frac{1}{2} \rho \mathrm{v}^{2}$ is constant.

## This proves the Bernoulli's Theorem

Therefore, the Bernoulli's equation is,

$$
\mathrm{p}_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g \mathrm{~h}_{1}=\mathrm{p}_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho \mathrm{gh}_{2}
$$



The Bernoulli's equation no 7 is applicable in the following situations:
The velocity of fluid flow is high and the pressure is low
Flow rate is small but the pressure is high
The fluid is automatically flowing from high pressure to low pressure.
An ideal fluid flows as a stationary if the level of fluid flow or the influence of the height difference is not significant (such as gas flow).

Bernoulli's Theorem basically relates the pressure, velocity, and elevation in a moving fluid (liquid or gas), the compressibility and viscosity (internal friction) of which are negligible and the flow becomes steady, or laminar.

Flow velocity Flow velocity


Fig. 1.10 Increased fluid velocity, decreased internal pressure

## Applications of Bernoulli's Theorem

It can be used to calculate the pressure or velocity of the fluid:
Air flight: When an aircraft flies, the air flight can be explained by Bernoulli's principle with the architecture of the wings of the plane. The top of the wing is somewhat curved, while the bottom of the wing is completely flat. In the sky, air moves across both the top and the bottom simultaneously. Different design of both the top part and the bottom part of the plane allows the air to move on the bottom at a relatively slower rate, which creates more pressure on the bottom, and allows the air on the top to move faster, which creates less pressure. This ultimately creates lift, which allows the planes to fly. An airplane is also acted upon by a pull of gravity in which opposes the lift, drag, and thrust. Thrust is the force that enables the airplane to move forward while drag is air resistance that opposes the thrust force.
Baseball: Baseball is another example of an application of Bernoulli's principle which is very visible in everyday life. In the case of baseball, the curveball plays the role. Since the stitches of the ball actually form a curve, it helps the pitcher to grip the seams of the baseball. By gripping the baseball, the pitcher can make the ball to spin. This allows the friction to cause a thin layer of air to engulf the misunderstanding of the baseball as it is spinning; since the ball is spinning in a certain manner, this allows for more air pressure on the top of the ball and less air pressure on the bottom of the ball. Therefore, according to Bernoulli's principle, there should be less speed on the top of the ball than on the bottom of the ball. What emerges out of this is that the bottom part of the ball accelerates downwards faster than the top part, and this phenomenon allows for the ball to curve downward, causing the batsman to miscalculate the ball's position.


Fig. 1.11 Application of Bernoulli's theorem (Air fight)


Fig. 1.12 Application of Bernoulli's theorem.

Draft: Another example of Bernoulli's principle in our everyday lives is in the case of someone feeling a draft. We all at least one time or another, have experienced the feeling of a draft, and it is because of Bernoulli's principle. Let's say that a room is really hot, but it is nice and cool both outside of the window and outside the door. If the window is opened up and fresh air is allowed to enter into the room. Then, there would not be much change in temperature, unless the door of the room is opened up to let the hot air out of the room. The reason why it works in this way is that if the front door is closed, the room will become an area of high pressure built up from the hot air, and outside the door, there is little pressure, which means that the rate at which the air enters with high speed. When the door is opened up, the inside pressure is reduced and the hot air exits quickly. When the hot air exits the pressure of air in the room becomes more because within a while the cool air come in. Once the hot air flows out, the cool air comes in at a faster rate. Thus, a draft result.
Engines: In many reciprocating engines the carburetor used contains a venturi that creates a region of low pressure to draw fuel into the carburetor and mix it thoroughly with the incoming air. The low pressure in the throat of a venturi can be explained by Bernoulli's principle; in the narrow throat, the air is moving at its fastest speed and therefore it is at its lowest pressure.
Venturi meter: The flow speed of a fluid can be measured using a device called venturi meter. It works on Bernoulli's principle.

## Energy Losses

When a fluid flows through a pipe, some of the energy of the fluid is lost due to resistance to the flow. Two types of energy-losses are there - major and minor. The major energy loss takes place due to friction. While minor energy losses take place due to sudden expansion, sudden contraction, bent in the pipe, pipe fittings, and obstruction in flow in the pipe.


Horizontal pipe in which the fluid is in steady flow

## Major Energy Loss

Loss in energy due to friction in the flow through pipe is caused due to the viscosity. Viscosity causes frictional loss. The frictional loss can be expressed using Bernoulli's equation for real fluid at points 1 and 2 (as shown in the figure) as:

$$
\frac{\mathrm{p}_{1}}{\mathrm{p}_{\mathrm{g}}}+\frac{v_{1}^{2}}{2 \mathrm{~g}}+\mathrm{z}_{1}=\frac{\mathrm{p}_{2}}{\rho_{\mathrm{g}}}+\frac{v_{2}^{2}}{2 \mathrm{~g}}+\mathrm{z}_{2}+\mathrm{h}_{f}
$$

where, $L=$ length of the pipe between section 1 and 2
$d=$ diameter of the pipe
$\mathrm{f}=$ friction factor
$\mathrm{h}_{\mathrm{f}}=$ loss of head due to friction
$\mathrm{p}_{1}=$ pressure at section 1
$\mathrm{v}_{1}=$ velocity at section 1
$\mathrm{p}_{2}=$ pressure at section 2 , and
$\mathrm{v}_{2}=$ velocity at section 2.
But, $\mathrm{z}_{1}=\mathrm{z}_{2}$ and $\mathrm{v}_{1}=\mathrm{v}_{2}$, since the pipe is horizontal and at both the sections the diameter is same, the above equation is reduced to

$$
\mathrm{h}_{\mathrm{f}}=\frac{\mathrm{p}_{1}}{\rho_{\mathrm{g}}}-\frac{\mathrm{p}_{2}}{\rho_{\mathrm{g}}}
$$

According to Darcy-Weishbach, head loss due to friction can be expressed as:

$$
\mathrm{h}_{\mathrm{f}}=\mathrm{f} \frac{\mathrm{LV}^{2}}{\mathrm{~d}_{2} \mathrm{~g}}=4 \mathrm{C}_{\mathrm{f}} \frac{\mathrm{LV}^{2}}{\mathrm{~d}_{2} \mathrm{~g}}
$$

Where,
$\mathrm{L}=$ length of the pipe,
$\mathrm{V}=$ mean velocity of the flow,
$\mathrm{d}=$ diameter of the pipe, and
$\mathrm{C}_{\mathrm{f}}=$ the skin friction coefficient
For fully developed laminar flow when f is the friction factor:

$$
\mathrm{F}=\frac{64}{\mathrm{Re}}\left(\text { for } \operatorname{Re}<2000 \text { and } \operatorname{Re}=\frac{\rho \mathrm{u}_{\mathrm{avg}} \mathrm{~d}}{\mu}\right)
$$

Minor energy loss due to sudden expansion
Let $\quad \mathrm{A}_{1}=$ Area at section 1,
$\mathrm{p}_{1}=$ pressure at section 1 ,
$\mathrm{v}_{1}=$ velocity at section 1 ,
$\mathrm{A}_{2}=$ Area at section 2,
$\mathrm{p}_{2}=$ pressure at section 2 , and

$\mathrm{v}_{2}=$ velocity at section 2.
Applying Bernoulli's equations for real fluid at section 1 and 2, we get

$$
\begin{array}{ll} 
& \frac{p_{1}}{p_{g}}+\frac{v_{1}^{2}}{2 g}+z_{1}=\frac{p_{2}}{\rho_{g}}+\frac{v_{2}^{2}}{2 g}+z_{2}+h_{e} \\
\text { or, } & h_{e}=\frac{p_{1}-p_{2}}{\rho_{g}}+\frac{v_{1}^{2}-v_{2}^{2}}{2 g}
\end{array}
$$

Minor energy loss due to sudden contraction
Let, $\mathrm{A}_{1}$ is the area at section $1, \mathrm{p}_{1}$ is pressure at section $1, \mathrm{v}_{1}$ is the velocity of fluid at section $1, \mathrm{~A}_{2}$ is the area at section $2, \mathrm{p}_{2}$ is the pressure at section $2, \mathrm{v}_{2}$ is the velocity of fluid at section 2 , and $\mathrm{A}_{3}, \mathrm{p}_{3}, \mathrm{v}_{3}$ are the corresponding values at section 3 .


$$
\text { Now, } A_{2} \mathrm{~V}_{2}=\mathrm{A}_{3} \mathrm{~V}_{3}
$$

Or,

$$
\frac{v_{2}}{v_{3}}=\frac{A_{2}}{A_{3}}=\frac{1}{C_{a}}
$$

Hence, the head loss due to expansion from section 2 to 3 can be expressed as:

$$
\mathrm{h}_{3}=\frac{\left(v_{2}-v_{3}\right)^{2}}{2 \mathrm{~g}}=\frac{v_{2}^{2}}{2 \mathrm{~g}}\left(\frac{v_{3}}{v_{2}}-1\right)^{2}=\frac{v_{2}^{2}}{2 \mathrm{~g}}\left(\frac{1}{\mathrm{C}_{2}}-1\right)^{2}=\mathrm{k} \frac{v_{2}^{2}}{2 \mathrm{~g}}
$$

Where, $\mathrm{k}=\left(\frac{1}{\mathrm{C}_{2}}-1\right)^{2}$; the value of k varies from 0.5 to 0.7 .
Head loss at the entrance of the pipe

$$
\mathrm{h}_{\mathrm{i}}=0.5 \frac{v^{2}}{2 \mathrm{~g}}
$$

Where, $v$ is the velocity of the fluid in the pipe
Head loss at the exit of the pipe

$$
h_{\rho}=\frac{v^{2}}{2 g}
$$

Head loss due to bend in the pipe

$$
\mathrm{h}_{\mathrm{b}}=\frac{\mathrm{k} v^{2}}{2 \mathrm{~g}}
$$

Where $v$ is the velocity of the flow, k is the coefficient of the bend which depends on the angle of the bend, radius of curvature of the bend and diameter of pipe.

Head loss due to pipe fittings

$$
\mathrm{h}_{\mathrm{f}}=\frac{\mathrm{k} v^{2}}{2 \mathrm{~g}}
$$

Where $v$ is the velocity of the flow, k is the coefficient of pipe fitting.

## Orifice Meter

An orifice meter consists of a flat plate that has a sharp-edged hole accurately machined in it. It is placed concentrically in a pipe as shown in Figure 13. As the liquid flows through the pipe, the flow suddenly contracts as it approaches the orifice and then suddenly expands after the orifice back to the full pipe diameter. This forms a vena contracta or a throat immediately past the orifice. This reduction in flow pattern at the vena contracta causes increased velocity and hence lower pressure at the throat, similar to the venturi meter.

The pressure difference between section 1 , with the full flow and section 2 at the throat, can then be used to measure the liquid flow rate, using equations developed for the venturi meter and the flow nozzle. Because of the sudden contraction at the orifice and the subsequent sudden expansion after the orifice, the coefficient of discharge for the orifice meter is much lower. Moreover, depending on the pressure tap locations, section 1 and section 2, the value of coefficient of discharge becomes different for orifices. The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.


Fig. 1.13 Schematic diagram of Irifice meter

## Advantages of the Orifice Meter

Well documented in standards;
It has wide acceptance; people working in the industry should have the knowledge about use and maintenance;

Relatively low cost,
Easy to install;
No moving parts in the flow stream; and
When built to standards' requirements, does not require calibration beyond confirming mechanical tolerances.

## Disadvantages of the Orifice Meter

Low rangeability with a single device;
Relatively high-pressure loss for a given flow rate, particularly at lower beta ratios;
More sensitive to flow disturbances at higher beta ratios than some meters; and
Flow pattern in the meter does not make meter self-cleaning.

## Calculation

Let us assume that, $\mathrm{d}_{1}$ and $\mathrm{d}_{2}=$ Diameter at section 1 and section 2 respectively,
$\mathrm{p}_{1}$ and $\mathrm{p}_{2}=$ Pressure at section1 and section2 respectively,
$\mathrm{v}_{1}$ and $\mathrm{v}_{2}=$ Velocity of fluid at section 1 and section 2 respectively,
$\mathrm{A}_{1}$ and $\mathrm{A}_{2}=$ Area at section1 and section2 respectively,
On application of the Bernoulli's equation at section1 and section2,

$$
\frac{\mathrm{p}_{1}}{\rho_{\mathrm{g}}}+\frac{v_{1}^{2}}{2 \mathrm{~g}}+\mathrm{z}_{1}=\frac{\mathrm{p}_{2}}{\rho_{\mathrm{g}}}+\frac{v_{2}^{2}}{2 \mathrm{~g}}+\mathrm{z}_{2}
$$

Or, $\quad\left(\frac{p_{1}}{\rho_{g}}+z_{1}\right)-\left(\frac{p_{2}}{\rho_{g}}+z_{2}\right)=\frac{v_{2}^{2}-v_{1}^{2}}{2 g}$
Or, The differential head, $\mathrm{h}=\frac{\mathrm{v}_{2}^{2}-\mathrm{v}_{1}^{2}}{2 \mathrm{~g}}$
and

$$
\mathrm{v}_{2}=\sqrt{2 \mathrm{gh}+\mathrm{v}_{1}^{2}}
$$

Say, $A_{0}$ is the area of the orifice; then Coefficient of contraction, $C_{c}=\frac{A_{2}}{A_{0}}$
Now,

$$
\begin{aligned}
& \mathrm{A}_{1} \mathrm{v}_{1}=\mathrm{A}_{2} \mathrm{v}_{2} \\
& \mathrm{v}_{1}=\frac{\mathrm{A}_{0} \mathrm{C}_{\mathrm{C}}}{\mathrm{~A}_{1}} \mathrm{~V}_{2}
\end{aligned}
$$

Hence,

$$
\mathrm{V}_{2}=\sqrt{2 \mathrm{gh}+\frac{\mathrm{A}_{\mathrm{o}}^{2} \mathrm{C}_{\mathrm{C}}^{2} \mathrm{~V}_{2}^{2}}{\mathrm{~A}_{1}^{2}}}=\frac{\sqrt{2 \mathrm{gh}}}{\sqrt{1-\frac{\mathrm{A}_{\mathrm{o}}^{2} \mathrm{C}_{\mathrm{C}}^{2}}{\mathrm{~A}_{1}^{2}}}}
$$

If $Q$ is the discharge, then $Q=A_{2} v_{2}=v_{2} A_{0} C_{c}$

$$
=\frac{\mathrm{A}_{\mathrm{o}}^{2} \mathrm{C}_{\mathrm{C}}^{2} \sqrt{2 \mathrm{gh}}}{\sqrt{1-\mathrm{C}_{\mathrm{C}}^{2} \frac{\mathrm{~A}_{\mathrm{o}}^{2}}{\mathrm{~A}_{1}^{2}}}}
$$

If $C_{d}$ is the coefficient of discharge for orifice meter, then

Therefore,

$$
\mathrm{C}_{\mathrm{d}}=\mathrm{C}_{\mathrm{C}}=\frac{\sqrt{1-\frac{\mathrm{A}_{\mathrm{o}}^{2}}{\mathrm{~A}_{1}^{2}}}}{\sqrt{1-\mathrm{C}_{2}^{2} \frac{\mathrm{~A}_{\mathrm{o}}^{2}}{\mathrm{~A}_{1}^{2}}}} \quad \text { or, } \quad \mathrm{C}_{\mathrm{c}}=\mathrm{C}_{\mathrm{d}}=\frac{\sqrt{1-\mathrm{C}_{2}^{2} \frac{\mathrm{~A}_{\mathrm{o}}^{2}}{\mathrm{~A}_{1}^{2}}}}{\sqrt{1-\frac{\mathrm{A}_{\mathrm{o}}^{2}}{\mathrm{~A}_{1}^{2}}}}
$$

$$
\mathrm{C}_{\mathrm{d}}=\frac{\mathrm{A}_{0} \mathrm{~A}_{1} \sqrt{2 \mathrm{gh}}}{\sqrt{\mathrm{~A}_{1}^{2}-\mathrm{A}_{0}^{2}}}
$$

The coefficient of discharge of orifice meter is much smaller than that of a Venturimeter.

## Venturi Meter

Venturi meter and orifice meter are commonly used to measure the flow of liquid or to measure the flow rate of the fluid flowing. These flow meters are also known as variable head meters.

The venturi meter has a converging conical inlet, a cylindrical throat and a diverging recovery cone. It has no projections into the fluid, no sharp corners and no sudden changes in contour. The figure 1.14 shows the venturi meter with uniform cylindrical section before converging entrance, a throat and divergent outlet.


Fig. 1.14 Schematic diagram of Venturimeter

The converging inlet section decreases the area of the fluid stream, resulting the increase of velocity and decrease in the pressure. The low pressure is measured at the center of the cylindrical throat where the pressure is the lowest. Neither the pressure nor the velocity would change. As the fluid enters the diverging section the pressure increases with lowering the velocity of the fluid. The Venturi effect is the reduction in fluid pressure that occurs when a fluid flows through a constricted section of pipe. The velocity of fluid increases through the constriction to satisfy the equation of continuity; while its pressure must decrease due to conservation of energy: the gain in kinetic energy is balanced by a drop-in pressure or a pressure gradient force. An equation for the drop-in pressure due to Venturi effect may be derived from a combination of Bernoulli's principle and the equation of continuity. The equation for venturi meter is obtained by applying Bernoulli equation and equation of continuity assuming an incompressible flow of fluids through manometer tubes. If $V_{1}$ and $V_{2}$ are the average upstream and downstream velocities and $\rho$ is the density of the fluid, then using Bernoulli's equation we get,

$$
\begin{equation*}
\alpha_{2} V_{2}^{2}-\alpha_{1} V_{1}^{2}=\frac{2 g\left(p_{a}-p_{b}\right)}{\rho} \tag{1.2}
\end{equation*}
$$

Where $\alpha_{1}$ and $\alpha_{2}$ are kinetic energy correction factors at two pressure tap positions. Assuming density of fluid to be constant, the above equation can be written as;

$$
\begin{equation*}
\mathrm{V}_{1}=\left(\frac{\mathrm{D}_{2}}{\mathrm{D}_{1}}\right)^{2} \mathrm{~V}_{2} \tag{1.3}
\end{equation*}
$$

Where $D_{1}$ and $D_{2}$ are the diameter of pipe and throat in meters respectively. Eliminating $V_{1}$ from equation 1.2 and 1.3, we get

$$
\begin{equation*}
\mathrm{V}_{2}=\frac{1}{\sqrt{\alpha_{2}-\alpha_{1} \beta^{4}}} \sqrt{\frac{2\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)}{\rho}} \tag{1.4}
\end{equation*}
$$

Where $\beta$ is the ratio of the diameter of throat to that of diameter of the pipe.
If we assume small friction lose between two pressure taps, the above equation 4 can be corrected by introducing imperial factor $\mathrm{C}_{\mathrm{v}}$ and written as;

$$
\begin{equation*}
\mathrm{V}_{2}=\frac{\mathrm{C}_{\mathrm{v}}}{\sqrt{1-\beta^{4}}} \sqrt{\frac{2\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)}{\rho}} \tag{1.5}
\end{equation*}
$$

The small effect of the kinetic energy factors $\alpha_{1}$ and $\alpha_{2}$ are also taken into account in the definition of $\mathrm{C}_{\mathrm{v}}$. The volumetric flow rate $\mathrm{Q}_{a}$ can be calculated as:

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{a}}=\mathrm{V}_{2} \mathrm{~S}_{2}=\frac{\mathrm{C}_{\mathrm{v}} \mathrm{~S}_{2}}{\sqrt{1-\beta^{4}}} \sqrt{2 \mathrm{~g} \Delta \mathrm{H}} \tag{1.6}
\end{equation*}
$$

Where $\Delta \mathrm{H}$, the manometric height difference $=$ specific gravity of manometric fluid - specific gravity of manometric fluid of water.

## Pitot Tube

Pitot tube is composed of two tubes inserted into a pipe carrying a liquid as shown in Fig 1.15. Tubes inserted at right angle to the direction of flow can measure the pressure head inside the pipe. The tube pointing up stream shall measure the pressure head and velocity head. The reading $h$ of the manometer will therefore indicate the velocity head, and $\Delta H_{P}$ measures the pressure head. $\Delta \mathrm{H}_{\mathrm{P}}$ corresponding to h can be calculated as;

$$
\begin{equation*}
\Delta \mathrm{H}_{\mathrm{p}}=\frac{\mathrm{u}^{2}}{2 \mathrm{~g}_{\mathrm{c}}} \tag{1.7}
\end{equation*}
$$

The orifice meter and venturi meter measure the average velocity of the whole stream of fluid, whereas pitot tube measures the velocity at single point only. The velocity of the fluid depends on the cross section of the pipe. Hence, the true average velocity of the fluid flowing in the pipe can be measured by one of the two methods:

The pitot tube should be inserted at the centre of the pipe and the average velocity is to be calculated from the semi-logarithmic graph indicating the relation between $u / u$ ' and Reynolds number. When this method is followed, the pitot tube must be inserted carefully so that the velocity of the fluid is not disturbed in any way. There should be normal velocity distribution in the pipe.

The pitot tube should be adjustable and readings should be taken at different points in the cross section of the pipe. The mean velocity is to be calculated by graphic integration. This method is commonly used when gases flow in large ducts at low pressure.

In fact, the pitot tube as shown in Fig 1.15 is not practically suitable. The pitot tube itself causes too much of disturbance in the movement of fluid; and hence in the measurement. For measuring the flow of low-pressure gases, the apparatus commonly used is shown in Fig 1.16. The diameter of the static holes should not be more than 0.02 in . The leading edge of the pitot tube may be sharp or hemispherical.


Fig. 1.15 Schematic diagram of Pitot tube


Fig. 1.16 Pitot tube

A perfect pitot tube should obey exactly the eqn.7, but all the practical instruments should be calibrated and the corrections should be made.

## Advantages

Pitot tube is most convenient to use,
If the pressure of a liquid or gas is high, pitot tube can be suitable for measurement,

## Disadvantages

It fails to provide the average velocity directly,
In case of low-pressure gases, its readings are small.
For measurement of low pressure, either differential or inclined manometer should be used.

## Rotameter

The measurement of a variable pressure difference by orifice meter and venturi meter depends on the fixed constriction placed in the flow in a pipe. These are frequently called "variable-head meters"; because the velocity of the fluid varies with pressure-difference. In case of rotameter, the area of flow of fluid is varied to produce constant pressure difference; for which rotameters are alternatively called "variable-area meters".

It can only be used in a vertical pipeline. Its accuracy is also $98 \%$ of other types of flow meters. As shown in Fig 1.17, a rotameter primarily consists of the following measurement components - Float 1. meter tube 2 . flowrate scale 3 . fittings 5 . flanged or threaded, for mounting in the pipeline and O Rings 7. to provide the seals. The movement of float is restricted by the float stops 4 . and a protective housing 6 . surrounds the meter tube.

The flowrate scale for flowmeters with glass meter tubes is preferably applied directly on the


Fig. 1.17 Schematic diagram of a rotameter meter tube.

When the fluid flows upward through the gap between the tube and the float, the float moves up or down depending on the rate and results a change in the gap; as a result, the area of the orifice changes. In fact, the float settles down at a position, where the pressure drop across the orifice creates an upward thrust and that will balance the downward force due to the gravity. The position of the float is calibrated with the flow rate.

The float position, which is a measure of the instantaneous flow-rate, is indicated external to the metal meter tube by a transmission system.

The tapered tube's gradually increasing diameter provides a related increase in the annular area around the float, and is designed in accordance with the basic equation for volumetric flow rate:

$$
\mathrm{Q}=\mathrm{kA} \sqrt{\mathrm{gh}}
$$

Where,
Q is the volumetric flow rate,
k is a constant,
A is the annular area between float and the tube wall,
g is the gravitational force, and
$h$ is the pressure drop across the float.

The performance of a rotameter depends greatly on the two basic components that are the tapered metering tube and the float. Tube sizes vary from ${ }^{1 / 16}$ to 4 in., most commonly ${ }^{1 / 8}$ to 2 in. Of course, each model has limitations as to capacity, temperature, pressure, and, in the case of liquids, viscosity. In general, a float should be designed in such a way that: it must be held vertical, it should create uniform turbulence so as to make it insensitive to viscosity, it should make the rotameter least sensitive to the variation of the fluid density.

As regards tapered tubes, there are different types of tapered tubes made of different materials for different purposes.

Glass tube rotameters are commonly used for simple but reliable indication of flow rate with a high level of repeatability. Alarm contacts can be easily added to provide high-, and/or low-flow signals, in which the contact is activated as the flow rate either drops below or rises above the set point.

Plastic tube rotameters can be an entirely suitable, very cost-effective alternative to glass or metal meters for a wide variety of fluid measurements. One popular model is made of a single piece of clear acrylic that is practically unbreakable in most industrial process applications. Frequently used as a purgemeter, this type is a low-cost and reliable.

Metal Tube Rotameters, also known as armored meters, are designed for applications where the temperature or pressure exceeds the limits of glass tubes. Flow rate is indicated by a pointer on an indicating scale by means of a magnet inside the float, magnetically linked to the pointer. Designed for indication only, metal tube meters require no external source of electric power. They may also be specified in applications requiring remote transmission of the measured flow rate, a feature not generally available with glass tube meters.

## Advantages

It is simple in construction, ready to install and the flow rate can be directly seen on a calibrated scale, without the help of any other device, e.g. differential pressure sensor etc.

It is useful for a wide range of variation of flow rates (10:1).

## Disadvanges

Accuracy of rotameter is about $98 \%$ of that of other instruments used to measure the flow of fluid.

## Short Questions

1. 'The manometers measure the differential pressure.' Explain the statement.
2. What are different types of manometers? Mention their applications.
3. How does a suspended particle flow with a flowing liquid?
4. What is Reynold's Number? Name the factors on which flow of fluid depends.
5. What are the significances of Reynold's Number?
6. Define Bernoulli's theorem. Under which situations Bernoulli's theorem is applicable?
7. What is meant by the term 'energy loss'?
8. What is 'vena contracta or throat'? Write down the functions of the apparatus to which the terms 'vena contracta or throat' are related.
9. What for a Venturimeter is used? Write down the advantages and limitations of a venturi meter.
10. What are the advantages and disadvantages of a metal tube rotator?

## Long Questions

1. Is Bernoulli's theorem is applicable in all cases? Explain briefly the applications of Bernoulli's theorem.
2. Explain the terms, major energy loss and minor energy loss under different situations.
3. Draw a schematic diagram of a venturi meter and explain how does it work.
4. With the help a diagram, write down how does an orifice meter work. What are its advantages and disadvantages?
5. For what a pitot tube is used? How is it used? What are its merits and demerits?
6. Why is a rotameter used? With the help of a diagram explain how does it work.
7. What is armored meter? What are its advantages and limitations?
8. What are different types of manometer? Draw the diagram of a well type manometer and describe how does it work.
9. What is an 'indicating fluid'? How can a manometer be corrected?
10. With the help of a diagram explain the Reynold's Number.
