Radar Cross Section Modelling using Swerling Probability Density Function for Target Signal Strength Estimation

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ABSTRACT

SDSC SHAR, ISRO equipped with ground-based Precision Radars for tracking the launch vehicles for range safety operations and trajectory computations. The Design of Radar parameters, waveform generation and Receiver signal processing [4] always appears to be challenging owing to the wide effects it can cause on the performance of radars [3]. The radars under consideration are Tracking Radars, intended for track Launch Vehicle. It is designed for skin mode and Transponder mode operation which calls for accurate estimation of Signal strength to achieve sufficient link margin for a target complex Radar Cross Section (RCS) [6] at slant range.

In radar technology, scattering phenomenology is quantified by the target parameter RCS, σ can be modelled as random process. Probability density functions techniques are used to describe the target scattering and RCS estimation. Receiver that employs an envelope detector followed by a threshold decision technique [5]. The input to the receiver is composed of radar echo s(t) and additive zero mean white Gaussian noise n(t), with variance Ψ^2*. The input noise is assumed to be spatially incoherent and uncorrelated with signal. The IF filter output is a complex random variable that is composed of either noise alone or noise plus target return signal. The noise quadrature components* $n_I(t)$ *and* $n_Q(t)$ *are uncorrelated zero mean low pass Gaussian noise with equal variance* Ψ^2*. The Joint Probability Density Functions of random variables is Rician PDF if target is available or Rayleigh PDF if noise alone present. Variations in radar-target geometry [8], target vibration, radar frequency, and polarization changes can lead to variations in target RCS, resulting in fluctuating targets having Swerling models [9] of target RCS fluctuation, The effect of these models on radar detection, signal strength estimation for a given radar parameters are considered.*

Keywords: *Radar, RCS estimation, Probability Density Functions, Predicted Signal strength, SNR, Radar Range Equation, Swerling model, polarization etc.*

1. INTRODUCTION

The fundamental functions of radar systems are to search and track of detected targets. In all of these functions the radar performance is influenced by the strength of the signal coming into the radar receiver from the target of interest and by the strength of the signals that interfere with the target signal. In the special case of receiver thermal noise being the interfering signal, the ratio of target signal to noise power is called the signal-to-noise ratio (SNR). The ratio of the signal to the total interfering signal is the signal-to-interference ratio (SIR). Most of these interfering signals are noise-like and are therefore modelled as random processes [3]. To evaluate SIR the unknown, random, stochastic parameter is target RCS which need to be statistically estimated. The RCS is a function of target viewing angle relative to the transmitter and receiver antenna, the frequency of operation and polarization of the incident EM wave. RCS is a measure of not only how much of the incident EM wave is reflected from the target but also how much of the wave is intercepted by the target and how much is directed back toward the radar's receiver. Thus, these mechanisms interception, reflection, and directivity all interact to determine the RCS of a target. Radar Cross Section Modelling using Probability Density Functions [5] for Target Signal Strength Estimation discussed in detail in this paper. The paper describes a model which simulates and predicts the dynamic radar cross section (RCS) [6] for a moving Launch vehicle. The model simulates the Radar-Launch vehicle engagement dynamics which will affect the RCS. Consider a target (Rocket) consisting of a large number of individual scatterers as shown in Fig. 1 randomly distributed in space and each with approximately the same individual RCS.

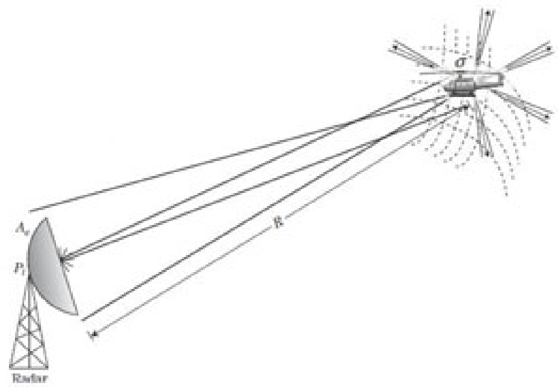

Figure 1: Radar and Rocket having multiple scatterers

The phase of the echoes from the various scatterers can then be assumed to be a random variable distributed uniformly on (0,2π).

The radar under consideration is a Tracking Radar having specifications mentioned in the table.1, intended for track Launch Vehicle having trajectory in three-dimensional spherical coordinate system as shown in Fig.2. It is designed for skin mode and Transponder mode operation which calls for accurate estimation of Signal strength to achieve sufficient link margin for a target Complex RCS at 1000Km slant range.

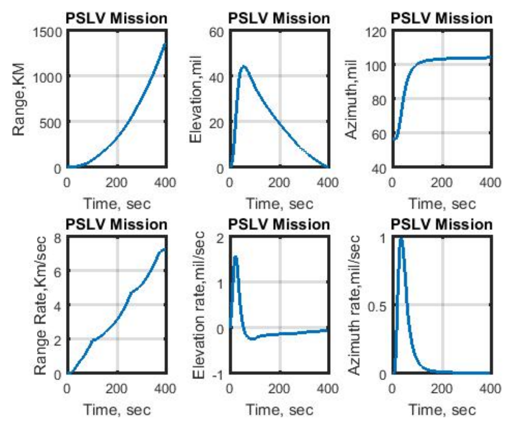

Figure 2: PSLV Mission Trajectory and its Rates

By taking into account the loses (L) involved during beam scanning, transmission, propagation and reception of radio waves. To these systems, clutter is considered to be an interfering return from a natural object such as precipitation, vegetation, soil and rocks, or the sea. The primary differences between clutter returns and receiver noise. Random noise, although varying with time. Clutter statistics can be similar to those of noise when the natural targets are composed of small, nearly equal-sized scatterers but can be quite different when the nature of the scatterers change or scatterers of differing types are present in the radar field of view. For this case, amplitude distributions having much longer “tails” than the Rayleigh distribution have been observed. Finally, although noise is independent of transmitted frequency, spatial position, and environmental parameters, clutter varies with all of these parameters, making clutter characterization very complex.

Table 1: Radar Parameters to estimate signal strength

Radar parameter	Symbol	Value
Peak Power (KW)	P_t	1000
Frequency of Operation (MHz)	f	5660
Pulse width (nano sec)	τ	1000
Gain of Antenna (dB)	G_t, G_γ	45
Target Range (Km)	R	Based on Target Trajectory
Target Cross section (RCS)(sqm)	σ	To be estimated
Boltzmann constant(watt-sec/K)	K	1.38×10^{-23}
System Noise Temperature(K)	T	570
Receiver Bandwidth (MHz)	B	1.4
Receiver noise figure	F	3.5
Radar Losses(dB)	L_s	3

The radar beam illuminates a number of scatterers, each having a different reflectivity, σ_i, and distance, d_i, from the radar. The electric field amplitude (horizontal or vertical polarization component) measured at the radar due to the echo from the i-th scatterer will be proportional to the square root of the received power given by the radar range equation [2].

$$\mathrm{SNR} = \frac{P_t G_t G_\gamma \lambda^2 \sigma}{(4\pi)^3 R^4 KTFBFL_s} \tag{1}$$

2. Radar Cross Section

Radar cross section [6] is a measure of power scattered in a given spatial direction when a target is illuminated by an incident wave. Another term for RCS is echo area. RCS is intended to characterize the target and not the effects of transmitter power, receiver sensitivity, and the distance or location between a target and the transmitter or receiver. Therefore, RCS is normalized to the power density of the incident wave at the target so that it does not depend on the distance from the illumination source to the target. This removes the effects of the transmitter power level and target distance. Mathematically, it is defined as 4π times the ratio of the power per unit solid angle scattered in a specified direction to the power per unit area in a plane wave incident on the scatterer from a specified direction. More precisely, it is the limit of that ratio as the distance from the scatterer to the point where the scattered power is measured approaches infinity.

$$\sigma = \underset{R \to \infty}{\mathrm{Lt}}\; 4\pi R^2 \frac{|E^{scat}|^2}{|E^{inc}|^2} \tag{2}$$

Where E^{scat} is the scattered electric field, and E^{inc} is the field incident at the target. Three cases are distinguished: monostatic or backscatter; forward scattering; and bistatic scattering. A formal cross section may also be defined for the energy that is scattered, absorbed, or removed from the incident wave. A total cross section can also be defined that includes all of these effects. The scattered energy is of greatest practical interest because it represents the energy available for radar detection.

The radar cross section of a target is a function of several attributes of the target like Target geometry and material composition, the radar observing the target called as aspect angle as shown in Fig.3 Transmitter and Receiver polarization, and frequency of operation.

Scattering Matrix for Linear Polarization

Radar cross section, as a scalar number, is a function of the polarization [1] of the incident and received wave. A more complete description of the interaction of the incident wave and the target is given by the polarization scattering matrix (PSM), which relates the scattered electric field vector E^{scat} to the incident field vector E^{inc}, component by component. In matrix notation, this is

$$\begin{bmatrix} E_v^{scat} \\ E_H^{scat} \end{bmatrix} = \begin{bmatrix} S_{VV} & S_{VH} \\ S_{HV} & S_{HH} \end{bmatrix} E^{inc} = \begin{bmatrix} S_{VV} & S_{VH} \\ S_{HV} & S_{HH} \end{bmatrix} \begin{bmatrix} E_v^{inc} \\ E_H^{inc} \end{bmatrix} \quad (3)$$

Scattering Matrix for Circular Polarization

In circular polarization, the electric field vector rotates in the plane perpendicular to the direction of propagation. The two orthogonal components of the electric field are called the right-handed circular [RC] and left-handed circular [LC].

Linear polarization can be transformed into circular polarization by shifting the phase of one of the two linear components by ±90 degrees. The choice between right- and left-handed circular is determined by the sign of the phase shift. Specifically, the RC and LC components of an electric field vector can be derived from the horizontal and vertical components

$$\begin{bmatrix} E_{RC}^t \\ E_{LC}^t \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} E_{HH}^t \\ E_{VV}^t \end{bmatrix} \quad (4)$$

The inverse transform to obtain linear polarization components from circular components is

$$\begin{bmatrix} E_{HH}^t \\ E_{VV}^t \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} E_{RC}^t \\ E_{LC}^t \end{bmatrix} \quad (5)$$

Received polarization can be defined in a similar manner. The LC and RC definitions change because the observer is now looking in the direction of propagation, which is from the target toward the receiver, while the radar system has defined LC and RC as viewed from the radar to the target. Therefore, with the superscript r denoting received field components

$$\begin{bmatrix} E_{RC}^{\gamma} \\ E_{LC}^{\gamma} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \begin{bmatrix} E_{HH}^t \\ E_{VV}^t \end{bmatrix} \quad (6)$$

which is seen to be the complex conjugate of the transmitted case. The circular polarization PSM contains no more information than the linear polarization PSM. If a linear PSM has been computed or measured, the corresponding circular PSM can be obtained by using equations.

$$\begin{bmatrix} S_{LL} & S_{LR} \\ S_{RL} & S_{RR} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \begin{bmatrix} S_{VV} & S_{VH} \\ S_{HV} & S_{HH} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \quad (7)$$

Radar receiver [5] considered an envelope detector followed by threshold detection. The Input signal to the receiver is composed of radar echo signal s(t) and additive zero mean white Gaussian Noise n(t) with variance ψ^2. The input noise is assumed to be spatially

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incoherent and uncorrelated with the signal. The output of the band pass IF filter is the signal v(t) and n(t).

$$v(t) = v_1(t) \cos \omega_0 t + v_Q(t) \sin \omega_0 t \tag{8}$$

In amplitude and phase form

$$v(t) = \gamma(t) \cos (\omega_o t) - \phi(t) \tag{9}$$

$\gamma(t)$ is the envelope of v(t), and phase $\phi(t)$, and $v_1(t)$, $v_0(t)$ are in phase and quadrature components as shown in Fig. 4

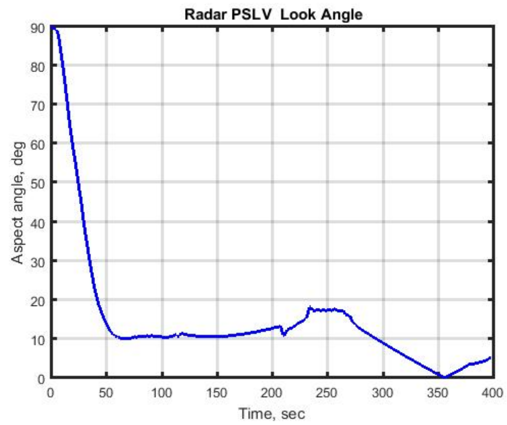

Figure 3: PSLV aspect Angle with respect to Radar

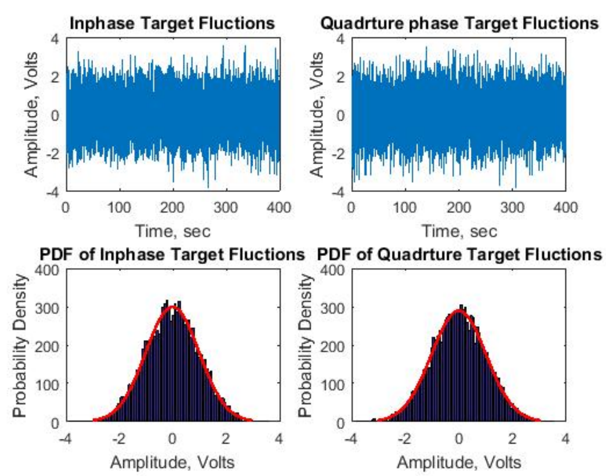

Figure 4: Receiver In & Quadrature Target Returns and Probability Density Functions

The Noise quadrature noise components $n_1(t)$, $n_0(t)$.

The Joint Probability Density Function of random variables.

$$f(n_1, n_Q) = \frac{1}{2\pi \psi^2} \exp\left(\frac{n_1^2 + n_Q^2}{2\psi^2} \right) \tag{10}$$

Signal amplitude and phase joint density PDF

$$f(\gamma, j) = \frac{1}{2\pi \psi^2} \exp\left(\frac{(r\cos j)^2 + (\gamma \sin j)^2}{2\psi^2} \right) \tag{11}$$

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They are related by

$$f(\gamma,\phi) = |J|\, f(n_1, n_Q) \tag{12}$$

Where J is Jacobian [7] target Signal amplitude is given by

$$J = \begin{bmatrix} \dfrac{\partial n_1}{\partial \gamma} & \dfrac{\partial n_1}{\partial j} \\ \dfrac{\partial n_Q}{\partial \gamma} & \dfrac{\partial n_Q}{\partial j} \end{bmatrix} \begin{bmatrix} \cos\phi & \gamma \sin\phi \\ \sin\phi & \gamma \cos\phi \end{bmatrix} \tag{13}$$

The PDF of r(t) is Rician PDF if signal is present obtained by

$$f(\gamma) = \frac{\gamma}{\Psi^2} I_0\left(\frac{\gamma}{\Psi^2}\right) \exp\left(-\frac{\gamma^2}{2\Psi^2}\right) \tag{14}$$

The PDF of r(t) is Rayleigh PDF if noise is present

$$f(\gamma) = \frac{\gamma}{\Psi^2} \exp\left(-\frac{\gamma^2}{2\Psi^2}\right) \tag{15}$$

For large value of r PDF will tends to Gaussian PDF. The target locations are resembled in received signal envelope as Rayleigh and signal strength as exponential PDF as shown in Fig. 5. The use of a statistical description for radar cross section [1–3] in which the composite RCS σ of the scatterers within a single resolution cell is considered to be a random variable with a specified probability density function (PDF). The radar range equation, or RCS modelling as described in above PDF equations is used to estimate the mean RCS from received echo power measurements, and one of a variety of standard PDFs is used to describe the statistical variations of the RCS. Combined with the uncertainty of aspect angle, particularly before a target is detected, it is much more practical to use a statistical model as a simple way to capture the complexity of the target RCS. Rayleigh/exponential model is strictly accurate only in the limit of a very large number of scatterers, in practice it can be a good model for a target having as few as 10 or 20 significant scatterers. Fig.5 compares a histogram of the RCS.

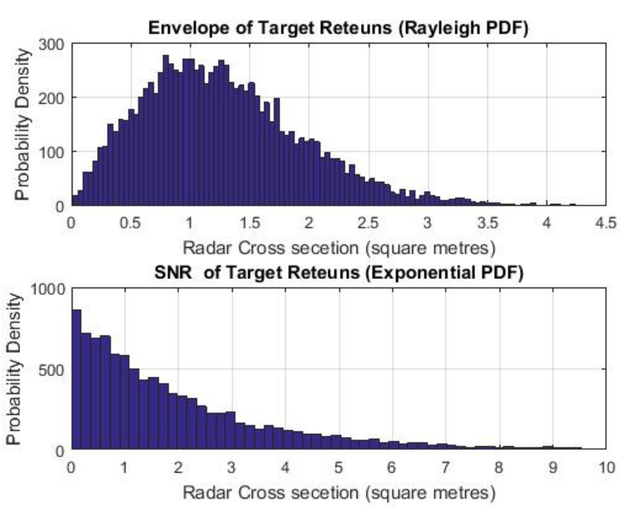

Figure 5: Envelope of Radar Detector and SNR PDFs

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RCS is modelled with considering polarisation, frequency operation, aspect angle, Target dimensions, scan to scan fluctuation of Launch vehicle. Furthermore, received power at the radar (target echo only, not including noise or other interference) is proportional to target RCS as seen in the radar range equation. Thus, RCS fluctuations shown in Fig.6 result in received target power fluctuations.

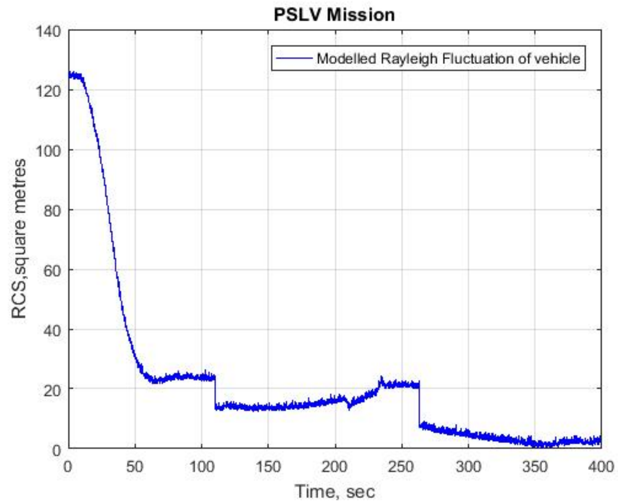

Figure 6: Modelled Radar Cross Section

After sizing of the Radar and Estimation of RCS (as modelled as random process) the signal strength as shown Fig. 7 is estimated for Radar for a given mission.

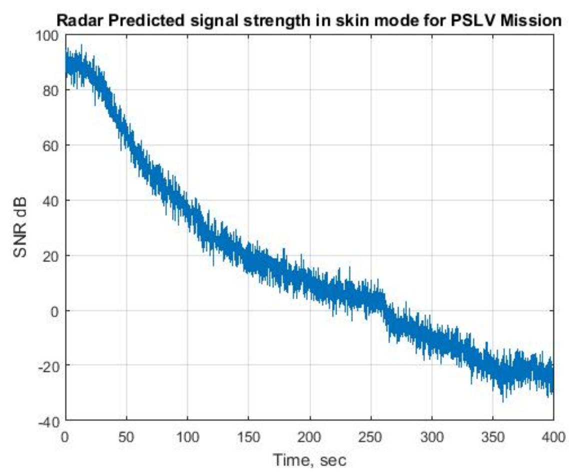

Figure 7: Estimated signal strength for Radar

3. CONCLUSIONS

The goal of this paper prior prediction of signal strength for Radar sizing. In the Radar Range Equation by modelling unknown Target RCS using swerling models with Rayleigh and Rician PDFs. The RCS evaluation of Launch vehicle by considering shape of launch vehicle in each stage, by considering input trajectory, Aspect angle of target. The Simulated Signal strength model is verified with real time trajectory the predicted signal strength by estimating RCS is matched with received signal strength in Real time.

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