Multifractal Detrended Fluctuation Analysis for Onset of Combustion Instability in Liquid Rocket Engines

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Abstract

Combustion instability is a potentially destructive phenomenon that leads to time and cost overruns in a rocket engine development project. Active control in open loop or closed loop configurations is being investigated to suppress combustion instability. The active control strategies can be made more effective if there is a forewarning of impending instability. In this work, multifractal detrended fluctuation analysis (MFDFA) is examined as a technique to provide forewarning of instability in liquid engines. The method is based on the decomposition of the chamber pressure data of the engine into singularity exponents and dimensions at different fractal orders. These parameters can be parametrically combined to generate the fractal spectrum. The fractal spectrum has a definite shape characterized by its height, width, differential height and apex location. The evolution of the multifractal spectrum characteristics is known to yield information on the onset of instability based on lab level experiments. The applicability of MFDFA to investigate the onset of instability in full-scale engines assessed in this work. Typical earth storable and cryogenic engines are used as case studies.

Keywords: *Multifractal detrended fluctuation analysis, multifractal spectrum, spectral characteristics*

1. Introduction

Combustion instability is a potentially destructive phenomenon that leads to time and cost overruns in a rocket engine development project. It is characterized by high frequency pressure fluctuations that can increase vibration and thermal loading on the combustion chamber. In extreme cases, the increased loading can result in failure of the engine. Conventionally, it is controlled by passive means wherein slots, resonators and liners are used to damp out the instability. Active control is also being investigated wherein the instability is controlled in open loop or closed loop configurations. Active control strategies are most effective if coupled with some sort of forewarning of the impending instability. This is because instability sets in very rapidly, within a few milliseconds to a few seconds. By the time the control system reacts, it may be too late to mitigate the deleterious effects of the instability. It would be desirable if the onset of instability can be anticipated before it happens. This gives sufficient time for corrective action to be taken. In this work, a technique based on the analysis of the fractal spectrum of chamber pressure oscillations is investigated to anticipate the combustion instability.

2. Literature Review

Combustion instability is highly nonlinear in nature. A stable combustion system exhibits aperiodic low amplitude pressure fluctuations known as combustion noise. On the other hand, combustion instability is characterized by high amplitude periodic oscillations in chamber pressure. As the system transitions from stability to instability, intermittency is observed

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wherein the system behaves in a quasi-periodic manner. The transition from combustion noise to combustion instability can be inferred from nonlinear time series analysis of the pressure perturbations [1].

The concept of fractals can be used to carry out nonlinear time series analysis for the onset of combustion instability. A fractal is an object whose geometry cannot be expressed by an integer dimension but it is rather a fractional quantity [2]. The fractional dimension of a fractal is due to its self-similar nature i.e., its properties repeat at smaller and smaller scales. Most objects in nature such as contours of coastlines, branching of the circulatory system, combustion oscillations and turbulence can be described by fractals [3].

A common measure of a fractal is the Hurst exponent [4]. An analysis of the time series chamber pressure data of an unstable combustion system reveals that it is multifractal. The time series has multiple fractal components each of which has a different Hurst exponent depending on the amplitude of the perturbations [5-7].

To adequately capture the behavior of multifractal time series it is more appropriate to generalize the definition of the Hurst exponent. Using the concept of structure functions [8] the generalized Hurst exponents can be obtained using multi-fractal detrended fluctuation analysis (MFDFA) [9-11]. The onset of combustion instability has been correlated to the loss of multifractality. The Hurst exponent is found to have a decreasing trend prior to the onset of instability [12-14].

3. Problem Definition

The generalized Hurst exponents can be Legendre transformed and expressed as a two-dimensional curve which typically has the shape of an inverted U. This multifractal spectrum is broad in the case of combustion noise, becomes narrower during intermittency and finally contracts to almost a spot during combustion instability [12]. In the process, the location of the apex of the multifractal spectrum shifts to the right. The variation in the shape and location of the multifractal spectrum forms the basis the present work.

MFDFA has been applied to a laboratory combustor [12] and a model liquid rocket combustor [14]. It is found that the variation of Hurst exponents of different orders are good indicators of the onset of instability. However, there is very little literature on the application of MFDFA to full-scale engines, which is the purpose of the present work.

4. Research methodology

To estimate the generalized Hurst exponents, the time series of chamber pressure, *P(t)*, is first mean adjusted and then cumulated over i intervals to get a deviated series x_i:

$$x_i = \sum_{t=1}^{i} \left(P(t) - \overline{P} \right) \tag{1}$$

The deviated series is then separated into m_w non-overlapping segments of equal width w where m_w is an integer. To measure the multifractal nature of the fractal spectrum, structure functions, S_w^q, of order q and span w, can then be obtained from the deviated series as:

$$S_w^q = \left(\frac{1}{m_w} \sum_{i=1}^{m_w} \left(\sqrt{\frac{1}{w} \sum_{j=1}^{w} \left(y_j(t) - \overline{y}_i \right)^2} \right)^q \right)^{\frac{1}{q}} \tag{2}$$

Here, $\overline{y_i}$ is a local polynomial fit to the deviate series y_j. The Hurst exponent of order q, H^q, is obtained from the log-log plot of S_w^q, by taking its slope in the linear regime, for a range of span sizes w. Applying a Legendre transformation, the spectrum of singularities, $f(\alpha)$, is obtained:

$$\tau_q = qH_q - 1 \tag{3}$$

$$\alpha = \frac{\partial \tau_q}{\partial q} \tag{4}$$

$$f(\alpha) = q\alpha - \tau \tag{5}$$

α is also called the singularity exponent while $f(\alpha)$ is called the singularity dimension. The multifractal spectrum is a plot of $f(\alpha)$ vs. α as seen in Fig. 1.

A typical multifractal spectrum is shown in Fig. 1. It has an inverted-U shape with a right limb and a left limb. The spectrum is not necessarily symmetric. The parameters used to describe the spectrum are named as the spectral characteristics and are indicated in the figure. Here the mean spectrum height is the average of the singularity dimensions of the ends of the limbs. In this work, attention is focused on the apex location and spectrum width.

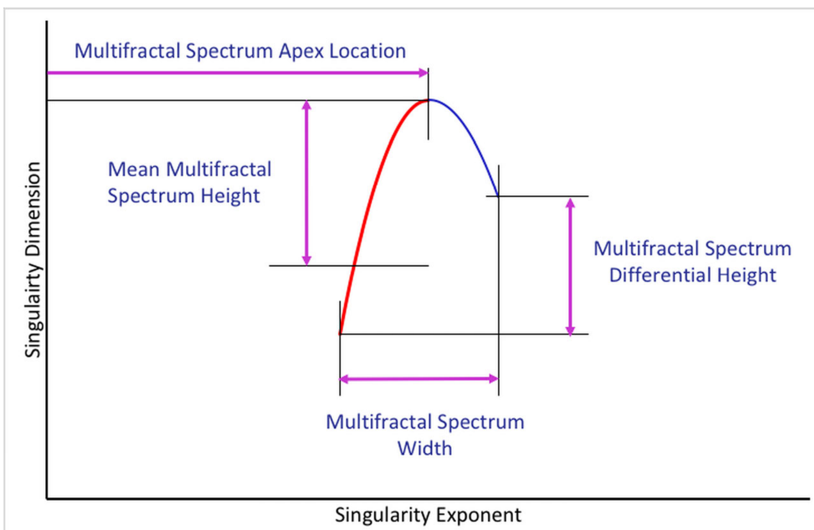

Figure 1: Multifractal spectrum with spectral characteristics

A MATLAB code is written to calculate the structure functions and thereby the multifractal spectrum. Reference [11] presents the results for three test cases viz. monofractal, multifractal and white noise signals. The present code was used to simulate these cases and the algorithm was validated.

5. Results and Fiscussions

MFDFA is applied to three liquid rocket engines of ISRO. The details of the engines are given in Table 1.

Table 1: Liquid engines analysed by MFDFA

SN	Designation	Propellant combination	Type of injector	Nominal chamber pressure	Nominal mixture ratio
1	High thrust earth storable engine (HTEE)	$UDMH/N_2O_4$	Doublet impinging	60 bar	1.7
2	Low thrust earth storable engine (LTEE)	MMH/MON	Triplet impinging	8 bar	1.7
3	Cryogenic engine (CE)	LH_2/LOX	Swirl coaxial	60 bar	5

5.1. Onset of Combustion Instability in a High Thrust Earth Storable Engine

Six tests of the high thrust earth storable engine were chosen for the study comprising of both short duration and long duration tests. No instability was observed in any of these tests based on FFT analysis, but MFDFA is used to examine the potential for development of instability. In each case, the multifractal spectral characteristics are determined in the start transient and a period thereafter. The results for all the tests are similar and are illustrated with reference to the hot test designated as HTEET. Salient spectral characteristics of HTEET are given in Fig. 2.

The apex location remains above 0.7 and does not show any tendency to approach zero. This clearly confirms that there is no instability as it does not approach zero. It can be inferred that the fluctuations observed are combustion noise. This is corroborated by the fact that the spectrum does not contract as time progresses, since the width also does not approach zero. Using MFDFA, it is thus conclusively established that there is no tendency to instability in the high thrust earth storable engine tests considered.

5.2. Onset of Combustion Instability in a Low Thrust Earth Storable Engine

The onset of combustion instability in a low thrust earth storable engine is now investigated. Two tests are considered, which are designated as LTEET1 and LTEET2. In these tests, signatures of instability were observed based on FFT analysis. This can be probed further by examining the spectral characteristics at higher ranges of scale (Fig. 3). Lower scale ranges are found not to yield information pertaining to instability but rather to combustion noise.

From Fig. 3a, it is seen that the apex location shows a downward trend at 0.5s. The apex location dips from 1.4 at 0.5s to almost 0.1 when the instability manifests at 3.2s. There is a forewarning of instability of about 2.7s if the apex location is appropriately considered. Prior to the instability, the width has a value of 0.5 on average, which decreases to about 0.25 when it sets in. The greater change in apex location during the onset of instability indicates that it is a more parameter definitive for establishing tendency for instability as compared to the width.

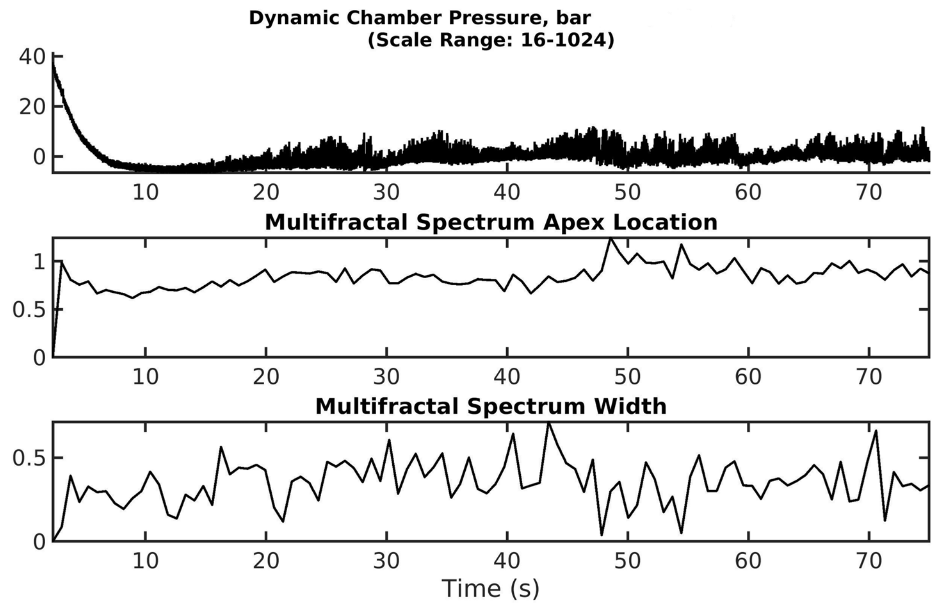

Figure 2: Apex location and width of multifractal spectrum of high thrust earth storable engine

In the case of LTEET2 (Fig. 3b), the FFT analysis shows a distinct dominant frequency. The apex location remains in the vicinity of 1 after 0.7s, which would indicate that the phenomenon is pronounced combustion noise and not instability. The behaviour of the width is more or less similar in both LTEET1 and LTEET2, being in the vicinity of 0.25 on average. The lateral contraction of the spectrum is thus not seen in the contrasting cases. This again indicates that the apex location is a key parameter from the point of view of the multifractal analysis of the onset of instability.

5.3 ONSET OF COMBUSTION INSTABILITY IN A CRYOGENIC ENGINE

A cryogenic engine has experienced instability issues in multiple instances. Two unstable tests were analysed by the present methodology. They are designated as CET1 and CET2. The former is a standalone test of the thrust chamber while the latter is a full engine test.

Both experienced catastrophic instability leading to failure of the hardware. The salient spectral characteristics for the two cases are given in Fig.4. Fig. 4a reveals a classic instability between 2.7s and 8.3s. The plotted spectral characteristics attain values very close to zero. Similarly, the initial 7s in the CET2 test are subjected to MFDFA. Again, it shows a classic instability as seen by examining the spectral characteristics (Fig. 4b). The spectral characteristics dip to zero at 5.6s. There is hardly any forewarning of the impending instability, with respect to the apex location and width, for both tests of the cryogenic engine considered. The spectral characteristics respond at or after the initiation of the exponentially growing pressure perturbations. This contrasts with the significant forewarning obtained in the LTEET1 test. It suggests that the more reactive the propellant combination, the less effective MFDFA becomes in anticipating instability.

Interestingly, after 6.8s, when the dynamic pressure trace in CET2 (Fig. 4b) shows increasing amplitude of oscillations, there seems to be a change in the nature of the instability. The spectral characteristics start to deviate from zero. This may be due to some unknown nonlinear effect that causes non-periodicity in the oscillations.

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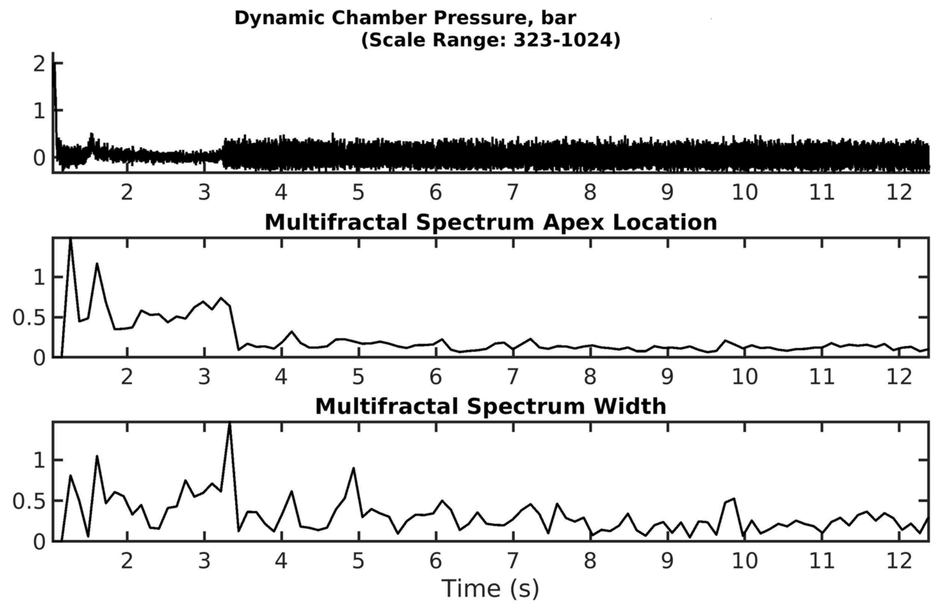
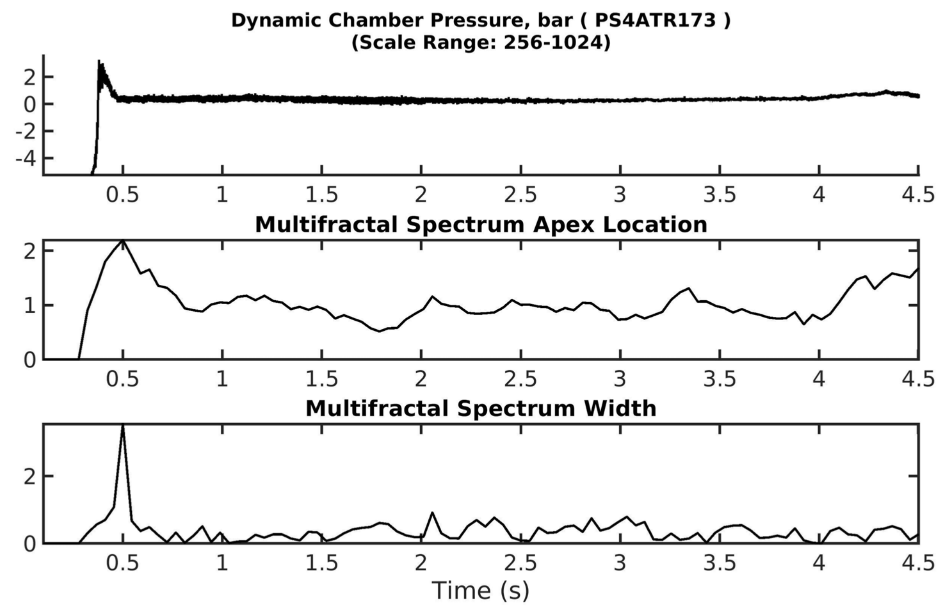


(a)

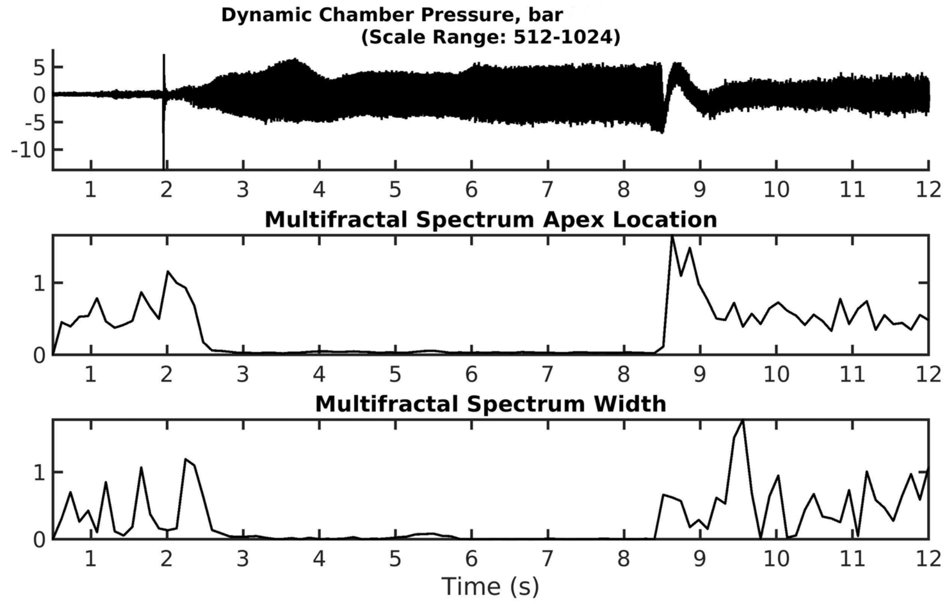


(b)

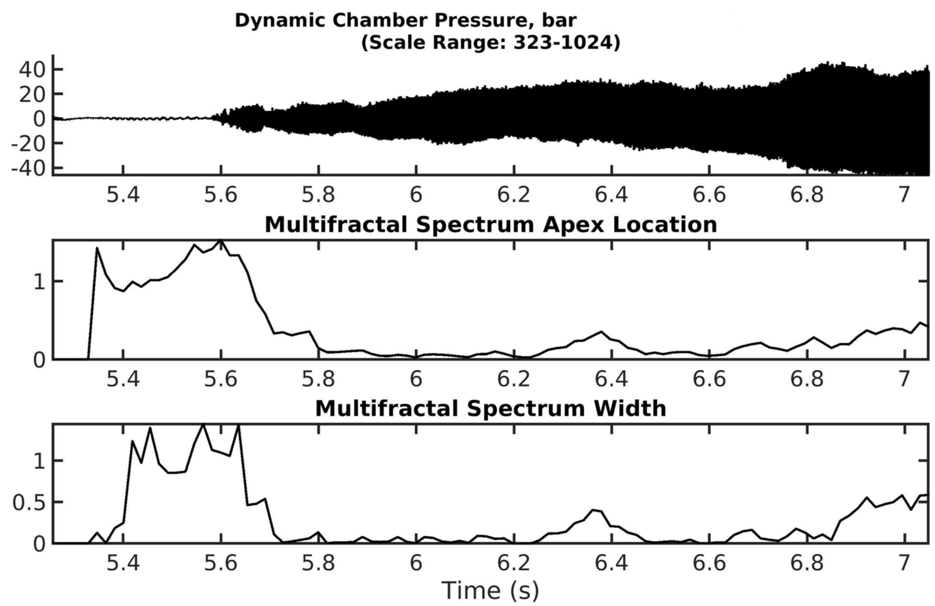
Figure 3: Apex location and width of multifractal spectrum (a) LTEET1 (b) LTEET2

6. CONCLUSIONS

The onset of combustion instability in typical liquid rocket engines was investigated using the technique of multifractal detrended fluctuation analysis. The multifractal spectrum of the chamber pressure oscillations was determined. The variation of two spectral characteristics viz. apex location and width is investigated is this work. Across all the engine tests, the apex location is the primary spectral characteristic with regard to the anticipation of the onset of instability and the width plays a secondary role.



(a)



(b)

Figure 4: Apex location and differential height of multifractal spectrum (a) CET1 (b) CET2

The high thrust earth storable engine is found to be stable with no tendency to instability. The low thrust earth storable engine instability was captured in one case but in the other case, the signature is of combustion noise. Classic instability is observed in the unstable tests of the cryogenic engine prior to catastrophic failure. There are indications of a different mechanism for combustion oscillations in CET2 using MFDFA that is not immediately apparent from the pressure trace.

The forewarning time for the onset of instability tends to be almost nil for the hydrogen-oxygen propellant combination. This may be attributed to the high reactivity of the hydrogen-oxygen system.

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The variation in multifractal spectral characteristics can be used to configure an expert system to trigger a combustion controller before, at, or shortly after the moment of inception of instability. This can prevent expensive loss of hardware. However, further optimization of the MFDFA algorithm is needed to ensure robustness of the active control system.

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