Aerodynamic Parameter Estimation of RLV from Telemeter Sensor Output

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ABSTRACT

An important applications of System Identification in aerospace science is aerodynamic parameter estimation from flight data. Though plenty of literature exist for aircraft on this topic, scanty for flight vehicles. An unified methodology of aerodynamic parameter estimation of flight vehicles is developed using Estimation Before Modeling technique under practical constraints of limited external measurements and mild or no excitations during flight. This technique has been extensively validated using flight data of different types of flight vehicles. In this paper, aerodynamic parameter estimation of Reusable Launch Vehicle during descend phase of flight is presented. Available onboard measurements are very noisy and also telemeter accelerometer and gyro signals are with intermittently missing data. Here no external measurements are available and it underwent maneuver only along pitch plane, considered for aerodynamic parameter estimation. Under these practical constraints, aerodynamic moment coefficients have been estimated and compared with preflight values in present research.

Keywords: *Aerodynamic Parameter Estimation, Estimation Before Modeling, Extended Kalman Filter, Filter Tuning, Multiple Linear Regression*

NOMENCLATURE

(a_x, a_y, a_z)	Accelerameter Sensed acceleration for SDINS
J	NIS Cost function
M, N	Mach Number, Total measurements
(I_{xx}, I_{yy}, I_{zz})	Moment of Inertia about (X,Y,Z) axes
R	Measurement Noise Covariance
S	Reference Area
V	Resultant Velocity
h	Altitude
m	FV mass
v_k	Innovation of kth measurement sample
(p,q,r)	Gyro sensed body rates from SDINS
(α_b,β_b)	Angle of attack, slip angle (Body Frame)
$(\alpha_{db}, \beta_{db})$	Angle of attack, slip angle (DB, Body Frame)
(α_f, β_f)	Angle of attack, slip angle (Fin Frame)
$(\alpha_{df}, \beta_{df})$	Angle of attack, slip angle (DB, Fin frame)
(α_r, ϕ_r)	Resultant α, Aero-phi
(α_{dr}, ϕ_{dr})	Resultant α, Aero-phi (DB)
P	Density

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ABBREVIATION

ADC	AeroDynamic Control
APE	Aerodynamic Parameter Estimation
CFD	Computational Fluid Dynamics
DB	Data Base
DOF	Degrees of Freedom
EBM	Estimation Before Modeling
EKF	Extended Kalman Filter
FV	Flight Vehicle
GM	Gauss Markov
GPS	Global Positioning Systems
MBFS	Modified Bryson Frazier Smoother
SDINS	Strap Down Inertial Navigation System
MLR	Multiple Linear Regression
NIS	Normalized Innovation Sequence
PFA	Post Flight Analysis
RLV	Reusable Launch Vehicle
SFV	Strategic Flight Vehicle
TFV	Tactical Flight Vehicle

1. INTRODUCTION

Techniques for Aerodynamic Parameter Estimation (APE) have led to three different, but complementary techniques 1) Analytical / Computational Fluid Dynamics (CFD) 2) Wind Tunnel Testing 3) Flight Data Extraction. Analytical and Wind Tunnel measurements though offer a good estimate of the aerodynamic parameters, extraction of same from flight data will be considered as more authentic as there are no scaling down or numerical approximations. Therefore it is required to use it for estimation of these parameters. But flight data has the inherent problems like sensor noise, Intermittent data loss due to high dynamics during descend phase, bias and scale factor in data and so on. However, the estimates from flight are more authentic. So it need to be processed and the relevant parameters need to be extracted. Based on flight measurements, APE is very important and necessary to reduce the limitations and uncertainties of aforementioned two methods. This study results in typically airframe / autopilot modification through post flight analysis (PFA). APE of aircraft from many onboard measurements like linear accelerometers, angular accelerometers, rate gyros, free gyros, air speed sensor, angle of attack sensors in both pitch and yaw plane and altimeter under manueuver has been attempted by lot of researchers. Plenty of literatures exist in this topic (1, 2). But same from flight test data of flight vehicle (FV) are very scanty. The practical difficulties encountered in different types of FVs for APE in authors' organization are 1) Tactical FVs (TFVs) are equipped with only telemeter on board accelerometer and gyro without any available external measurement. They undergo some maneuvers not continuously but only while any event switch over based on guidance requirement. But in aircraft pilot executes 3-2-1-1 doublet maneuver in required plane to excite it for parameter estimation which is of much larger magnitude than in FV [3] (pp. 29-46). 2) Strategic FVs (SFVs) are equipped with on board accelerometer and gyro outputs from strap down inertial navigation system (SDINS). Sometimes radar tracked position data is only available as external measurement. But during entire flight regime angle of attack and sideslip angle very low and it undergoes a less amount of maneuver. So input accelerometer and gyro data for parameter estimation consist of constant or DC biases and scale factors along with random noise. Under such practical constraints on available flight data, an unified methodology is developed to estimate crucial aerodynamic coefficients with some engineering assumptions by exploiting available data judiciously (Fig. 1). So limited sensor and mild maneuver throws challenge on APE of FV. The aerodynamic parameters estimated for different TFVs and SFVs were aerodynamic coefficients (a_r) as a part of PFA [4, 5]. Present research is extension of above activities for an Experimental Reusable Launch Vehicle (RLV). The new problems associated with present flight data are 1) Sensor outputs are very noisy along with intermittently missing data in all channels simultaneously. 2) Aerodynamic model is nonlinear and function of resultant angle of attack and aero-phi ($\sigma\rho$, $\phi\alpha$) in It underwent maneuver in Pitch plane, which is only considered for the analysis. So moment coefficients $\left(C_{DO}C_M^{\alpha}, C_M^{\delta}\right)$ along with Roll Moment Coefficients have been estimated. These are novel elements in present research over previously ones as cited above. There is a strong assumption in present activity that preflight force coefficients are invariant during flight. At first overall problem formulation is described. Next APE along pitch and roll channel both from simulated as well as flight data have been discussed. The paper concludes with immediate future activities being carried out for further improvement on present results.

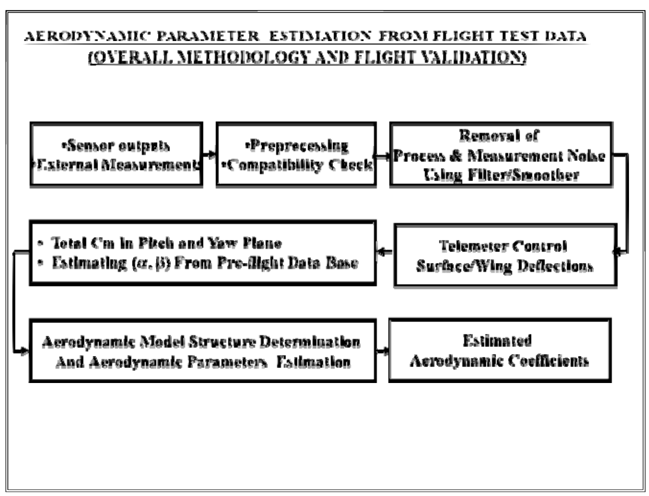

Figure 1: Steps in FV Aero. Parameter Estimation

PROBLEM FORMULATION

Present estimation problem is described in a nutshell consists of three major steps such as a) To obtain $\left(\hat{a}_x, \hat{a}_y, \hat{a}_z, \hat{p}, \hat{q}, \hat{r}\right)$ using EKF/MBFS as Estimation Before Modeling (EBM) from accelerometer and gyro outputs (a_{xm}, a_{ym}, a_{zm}, p_m, q_m, r_m) of SDINS b) Estimation of angle of attack and sideslip angle (α_{db}, β_{db}) by equating aerodynamic data base with flight sensed lateral acceleration (Latax) components $\left(\hat{a}_x, \hat{a}_y\right)$ c) Estimation of moment coefficients from $(\hat{p}, \hat{q}, \hat{r})$ using Multiple Linear Regression (MLR) based model structure determination.

PLANT AND MEASUREMENT MODEL FOR EKF/MBFS

Plant is 3rd order Gauss Markov (GM) (1, 4), SDINS sensed accelerations and body rates are the present measurements.

STATE EQUATIONS

Equivalent state equations of form $\dot{X}(t) = f(X(t), u(t), t) + G\,w(t)$ and size (21 × 1) are

$\dot{p} = C_1pq + C_2qr + N_1C_4 + L_1$

$\dot{q} = C_5pr + C_6(r^2 - p^2) + M_1$

$\dot{r} = C_8pq + C_9qr + L_1C_{11} + N_1$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} wx_1 \\ wx_2 \\ wx_3 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ \dot{Y}_2 \\ \dot{Y}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} + \begin{bmatrix} wy_1 \\ wy_2 \\ wy_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} + \begin{bmatrix} wz_1 \\ wz_2 \\ wz_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{L}_1 \\ \dot{L}_2 \\ \dot{L}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} + \begin{bmatrix} wl_1 \\ wl_2 \\ wl_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{M}_1 \\ \dot{M}_2 \\ \dot{M}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} + \begin{bmatrix} wm_1 \\ wm_2 \\ wm_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{N}_1 \\ \dot{N}_2 \\ \dot{N}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} + \begin{bmatrix} wn_1 \\ wn_2 \\ wn_3 \end{bmatrix}$$

MEASUREMENT EQUATIONS:

It can be written in the form $Z(t_k) = h(X, t_k) + \eta(t_k)$, $k = 1,2, \ldots.. N$ as

$$\begin{bmatrix} a_{xm} \\ a_{ym} \\ a_{zm} \end{bmatrix} = \begin{bmatrix} X_1 - (r^2 + q^2)x_{cg} + (pq - r)y_{cg} + (pr + q)z_{cg} \\ Y_1 - (p^2 + r^2)y_{cg} + (pq + r)x_{cg} + (qr - p)z_{cg} \\ Z_1 - (q^2 + p^2)z_{cg} + (pr - q)x_{cg} + (qr + p)y_{cg} \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} \qquad (2)$$

$$\begin{bmatrix} p_m \\ q_m \\ r_m \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} n_4 \\ n_5 \\ n_6 \end{bmatrix}$$

The available measurements are contaminated by Gaussian white noise of covariance $\left(\sigma_{a_x}^2,\ \sigma_{a_y}^2,\ \sigma_{a_z}^2,\ \sigma_p^2,\ \sigma_q^2,\ \sigma_r^2\right)$. These are diagonal elements of Measurement Covariance Matrix R. The notations used in above Eqns. (1-2) are

$$C_1 = \frac{\left[I_{zz}\left(I_{zz} + I_{zz} - I_{yy}\right)\right]}{I^2} \qquad C_2 = \frac{\left[I_{22}\left(I_{yy} - I_{zz}\right)I_{xx}^2\right]}{I^2} \qquad C_4 = \frac{I_{xx}}{I_{zz}}$$

$$C_5 = \frac{\left(I_{zz} - I_{zz}\right)}{I_{yy}} \qquad C_6 = \frac{I_{22}}{I_{yy}} \qquad C_8 = \frac{\left[I_{xx}\left(I_{xx} - I_{yy}\right) + I_{zz}^2\right]}{I^2}$$

$$C_9 = \frac{\left[I_{xx}\left(I_{yy} - I_{zz}\right) - I_{zz}^2\right]}{I^2} \qquad C_{11} = \frac{I_{22}}{I_{xx}};\ I^2 = \left(I_{xx}I_{zz} - I_{xx}^2\right)$$

EKF/MBFS has been mechanized with 21 state variables (Eqn. 1) and 6 measurements (Eqn. 2).

FILTER INITIALIZATION AND FILTER TUNING

In filtering /estimation problem evaluation of **(X_0, P_0)** is filter initialization and proper choice of **(X_0, P_0, Q, R)** is Filter Tuning [6]. The methodology is discussed now.

Evaluation of (X_0, P_0): First seven points of a measured signal from t_0 is approximated to a pararabola.

$$y_s(t_3) = c_0 + c_1(t_3 - t_0) + c_2(t_3 - t_0)^2 \qquad j = 1, ...,7 \tag{3}$$

Coefficients (C_0, C_1, C_2) and their variances $\left(\sigma_0^2, \sigma_1^2, \sigma_2^2\right)$ are determined by least square and is shown as

$$\sigma_y^2,(t) = \sigma_0^2 + \sigma_1^2(t - t_0)^2 + \sigma_2^2(t - t_0)^4;\ \sigma_{yx}^2(t) = \sigma_1^2 + 4\sigma_2^2(t - t_0)^2;\ \sigma_{yx}^2(t) = 4\sigma_2^2 \tag{4}$$

For all measurements above method is used to compute (X_0, P_0) [4]. Based on Eqn. 3 (position, velocity, acceleration) components $\left(y_a, \dot{y}_a, \ddot{y}_a\right)$ of a measured signal are evolved. Eqn. 4 is used to compute diagonal $\left(\sigma_y^2, \sigma_{\dot{y}}^2, \sigma_{\ddot{y}}^2\right)$ of above measurement channel. This is carried out for all measurement channels and (X_0, P_0) are evolved at $t = t_0$. Initially only the diagonal elements of P_0 at keeping off-diagonal elements to zero.

Evaluation of Q: During time interval (t_1, t_f) measured signal is analyzed in frequency domain to find out f_{max} upto 4 Hz sufficient enough to regenerate peak variations of the valid signal in measurements. So all measured signals are analyzed in frequency domain to calculate amplitudes within [0-f_{max}]. Then Q-matrix diagnonal elements of Eqn. 1 are evaluated [4].

Evaluation of R: The measurement variance can be calculated as $r_{aa} = \left(\frac{\Delta w}{3}\right)^2$ where Δy_i is the maximum dispersion of noise over mean. The diagonal elements of R-matrix are depicted in Table 2.

Proper filter tuning is based on Normalized Innovation Sequence (NIS) cost function which should be close to the number of measurement channels with near optimal filter tunning parameter [7].

Computation of Angle of Attack and Sideslip Angle from Aerodynamic Data Base

For APE, it is always customary to estimate angle of attack and sideslip angle (α_{bf}, β_{bf}) in fin frame from aerodynamic Data Base (DB). This experimental RLV undergoes no specific maneuver during initial phase and then followed by guidance demanded planned maneuver during the later part of the flight path regime. Moreover, maneuver is along pitch plane along body frame. The yaw plane along body frame is only for disturbance rejection. This makes fins along yaw plane are of lesser size (Fig. 2). Telemeter (α_b, β_b) from SDINS will be having effect of Bias and can not be used for aerodynamic coefficient estimation. So (α, β) is estimated from DB assuming invariance of preflight force coefficients. This is carried out by computing (α_{db}, β_{db}) in body frame from DB values of (C_N, C_S) along with telemeter flight control deflections (δ_1, δ_2, δ_3, δ_4). These angles are transformed to fin frame and (α_{df}, β_{df}) is used for aerodynamic moment coefficient estimation using Multiple Linear Regression (MLR) through model structure determination. It is to be noted that here fins are in (+) position. So body and fin frame are identical. The sensors are in body frame. Also complete aerodynamic data is in body frame with respect to total angle of attack, roll orientation and mach number (α_r, ϕ_r, M).

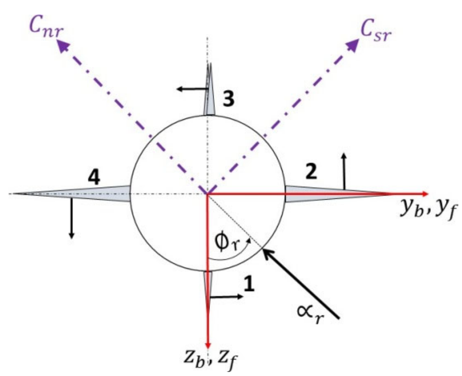

Figure 2: (C_N,C_S), (ϕ_r, α_r) ADC Sign Convention

Aerodynamic control (ADC) along (roll, pitch, yaw) are carried out by these fin surfaces (δ_1, δ_2, δ_3, δ_4). The sign convention of (δ_r, δ_p, δ_y) are given in Table 1. Corresponding sign convention is

$$\delta_{pb}^{(+)} \rightarrow q_b^{(+)} \rightarrow a_b^{(+)} \rightarrow C_{Nb}^{(+)} \rightarrow a_{zb}^{(-)} \quad \text{(pitch plane)} \tag{5}$$

$$\delta_{yb}^{(+)} \rightarrow r_b^{(-)} \rightarrow \beta_b^{(+)} \rightarrow C_{Yb}^{(+)} \rightarrow a_{yb}^{(-)} \quad \text{(yaw plane)}$$

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Table 1: ADC Sign Convention (Fig.2)

Descrip.	Expression	Direction
(δ_r) roll (+) ve Counterclockwise	$\delta_r = (\delta_1 + \delta_2 + \delta_3 + \delta_4)$	p (–)ve
(δ_{pf}) pitch (+ ve) along zf	$\delta_{pf} = (-\delta_2 + \delta_4)$	qf (+) ve
(δ_{yf}) yaw (– ve) along yf	$\delta_{yf} = (-\delta_1 + \delta_3)$	rf (+) ve

Methodology of (α_{db}, β_{db}) computation is explained now. Measured sensor outputs are smoothened using EKF/MBFS filter/smoother algorithm has been discussed before. The smoothened $\left(\hat{a}_x, \hat{a}_{yb}, \hat{a}_{zb}, \hat{p}, \hat{q}b, \hat{r}b, \hat{p}, \hat{q}_b, \hat{r}_b\right)$ will be used now for present study. Total aerodynamic force coefficients along (pitch, yaw) in body and fin plane are

$$C_{zb}^{Total}(\text{flight}) = \frac{m\dot{a}_{zb}}{\frac{1}{2}\rho V^2 S} \qquad C_{zf}^{Total}(\text{flight}) = \frac{m\dot{a}_{zf}}{\frac{1}{2}\rho V^2 S}$$

$$C_{yb}^{Total}(\text{flight}) = \frac{m\dot{a}_{yb}}{\frac{1}{2}\rho V^2 S} \qquad C_{yf}^{Total}(\text{flight}) = \frac{m\dot{a}_{yf}}{\frac{1}{2}\rho V^2 S} \tag{6}$$

$$C_N^R(\text{flight}) = \sqrt{C_{yb}^{Total}(\beta_b)^2 + C_{zb}^{Total}(a_b)^2}$$

for given $(\alpha_{df}, \beta_{df})$ from DB along fin frame using Eqn. 6

$$C_{yf}(\beta_{df}) = C_{yf}^{Total}(\text{flight}) - C_{yf}^{\delta yf \frac{(\delta_1 - \delta_2)}{2}}$$

$$C_{zf}(\alpha_{df}) = C_{zf}^{Total}(\text{flight}) - C_{z}^{\delta yf \frac{(-\delta_2 + \delta_4)}{2}}$$

$$C_N^R(\text{DB}) = \sqrt{C_{yf}(\beta_{df})^2 + C_z(\alpha_{df})} \tag{7}$$

Here $C_N^R(\alpha_r, M)$ is resultant normal force coefficient along α_r plane for given Mach number. Our aim is to evolve $(\alpha_{dr}, \phi_{dr}, \alpha_{df}, \beta_{df})$ by equating $\left(C_{yf}^{Total}, C_{zf}^{Total}, C_N^R\right)$ from flight and nominal zero DB using Eqns. (6-7). By defining cost function, this will be discussed now.

$$C_{yb}^{Total}(\beta_{db}) = -C_N^R(\alpha_{dr})\sin\phi_{dr}$$

$$\tan\beta_{db} = \tan\alpha_{dr}\sin\phi_{dr}$$

$$C_{zb}^{Total}(\alpha_{db}) = -C_N^R(\alpha_{dr})\cos\phi_{dr}$$

$$C_{yf} = C_{yb}; C_{zf} = C_{zb}$$

$$\tan\beta_b = \tan\alpha_r \sin\phi_r \tag{8}$$

$$\tan\alpha_b = \tan\alpha_r \cos\phi_r$$

$$\beta_f = \beta_b; \alpha_f = \alpha_b$$

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$$\tan\alpha_{db} = \tan\alpha_r \cos\phi_r$$

$$e_1 = C_{yf}\left(\text{flight}\right) - C_{yf}^{Total}\left(DB, \beta_{df}\right)$$

$$e_2 = C_{zf}\left(\text{flight}\right) - C_{zf}^{Total}\left(DB, \alpha_{df}\right)$$

$$e_3 = C_N^R\left(\text{flight}\right) - C_N^R\left(DB\right)$$

Cost function to be optimized for equating total normal force from flight and DB and to evolve the parameters $\Theta = \left(\alpha_{dr}, \phi_{dr}, \alpha_{df}, \beta_{df}\right)$ is,

$$J\left(\Theta\right) = \left(J_1 + J_2 + J_3\right) = \mathbf{e}^{\mathbf{T}}\mathbf{Re} \tag{9}$$

where $e = \left(e_1\ e_2\ e_3\right)^T$ and **R** = Diagonal [1 1 1]. Eqns (6.9) have been solved together using **fmincon** of **MATLAB Optimization Toolbox** to evolve Θ (Fig. 3).

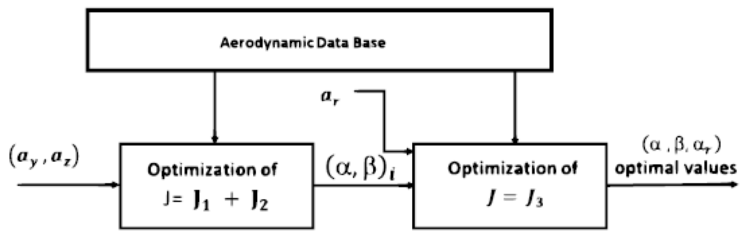

Figure 3: Optimization for $\left(\alpha_{dr}, \phi_{dr}, \alpha_{df}, \beta_{\partial f}\right)$ (DB)

AERODYNAMIC MODEL STRUCTURE FOR APE USING MLR

Total moment coefficients in fin frame along (roll, pitch and yaw) channels are computed as

$$C_{mx}^{Total} = \frac{I_{zz}\dot{p}}{\frac{1}{2}\rho V^2 Sd};\ C_{myf}^{Total} = \frac{I_{yy}\dot{q}_F}{\frac{1}{2}\rho V^2 Sd};\ C_{mzf}^{Total} = \frac{I_{zz}f_f}{\frac{1}{2}\rho V^2 Sd} \tag{10}$$

The aerodynamic model structure in fin frame along pitch plane with inputs as $\left(\alpha_{df}, \delta_{pf}\right)$ is

$$C_{zf}^{Total} = C_N^O + C_N^\alpha\ \alpha_{df} + \frac{C_N^\delta}{2}\delta_{pf}$$

$$C_{myf}^{Total} = C_{my}^O + C_{my}^\alpha\ \alpha_{df} + \frac{C_{my}^{\delta p}}{2}\delta_{pf} \tag{11}$$

Yaw plane model with inputs as $\left(\beta_{df},\ \delta_{yf}\right)$ is

$$C_{yf}^{Total} = C_Y^O + C_Y^\delta\ \beta_{df} + \frac{C_Y^\delta}{2}\delta_{yf}$$

$$C_{mzf}^{Total} = C_{mz}^O + C_{mz}^\beta\ \beta_{df} + \frac{C_m^\delta y_z}{2}\delta_{yf} \tag{12}$$

Roll model structure with inputs as (α_{dr}, δ_r) is

$$C_i^{Total} = C_i^O + C_i^{ar}\ \alpha_{dr} + C_i^{\phi r}\cos(4\phi_{dr}) + C_i^{\delta} r\delta r \tag{13}$$

$(\delta_r, \delta_{pf}, \delta_{yf})$ are function of $(\delta_1, \delta_2, \delta_3, \delta_4)$ (Table 1).

RESULTS AND DISCUSSION

Three major steps of present APE have been discussed before. Now they have been validated first using simulated flight data and later used to process flight data.

CASE STUDY WITH SIMULATED DATA

Noisy sensor outputs based on Table 2 obtained from flight data have been processed by EKF/MBFS filter smoother, NIS cost is high while filtering by EKF. But through smoothing the cost becomes very close to total number of measurement channels (Table 2). So smoothened estimates are very close to truth model. Due to brevity discussion on filtering results are skipped. Based on smoothened $(\hat{a}_z f, \hat{a}_z f), C_N^R$ along with (α_{dr}, ϕ_{dr}, α_{df}, β_{df}) have been estimated using cost function optimization (Eqn. 6). Time history of (C_N^R, α_{dr}) along with very small estimation error are depicted in Figs. (4-5). Using smoothened qf total pitching moment coefficients in fin frame C_{myf}^{Total} are also evolved (Eqn. 10). Now (α_{df}, δ_{pf}) are used to estimate aerodynamic moment coefficients along pitch (Eqn. 11). Comparison of estimated moment coefficients $(C_m^{\alpha}, C_m^{\delta})$ along with preflight values is shown in Table 3. C_{myf}^{Total} from flight and model output are compared in Fig. 6. Being simulated data the goodness of fit is very good because of high $F_{statistics}$ value. Study of roll channel being similar, not discussed herein. No attempt has been made to estimate yaw channel coefficient due to no excitation.

Table 2: Sensor Noise and NIS Cost

(1 – σ) Sensor Measurement Noise					
a_{xm} (m/s²)	**a_{ym} (m/s²)**	**a_{zm} (m/s²)**	**p_m (d/s)**	**q_m (d/s)**	**r_m (d/s)**
0.80	1.00	2.00	4.00	2.00	1.00
NIS Cost Function					
Description		Sim. Data		Flt. Data	
EKF Filter		12.16		14.25	
MBFS Smoother		6.35		6.86	
True		6		6	

Table 3: Estim. Aero. Coeff. (Simulation, MLR) ($M = 12.4$, $\alpha = 1.6^o$, $\frac{x_{cg}}{d} = 1.675$)

Pitch Channel (a_f Effect)				
Case	C_N^α (/deg)	C_m^α nose (/deg)	$C_m^\alpha\big\|_{cG}$ (/deg)	F_{stat}
Preflight	0.0305	–0.0578	–0.0067	-
Flight	0.0318 (4.5×10^{-5})($^{+}$)	-0.0579	-0.0047 (6.6×10^{-6}) (+)	5306089 C_N^{Total}
Pitch Channel (δp Effect)				
Case	C_N^δ (/deg pair)	$C_m^\delta\big\|_{nose}$ (/deg pair)	$C_m^\delta\big\|_{CG}$ (/deg pair)	F_{stat}
Preflight	0.0012	0.00277	0.00075	
Flight	0.0013 (6.0×10^{-6})$^{(t)}$	0.00278	0.00061 (8.8×10^{-6})	18929 C_M^{Total}

(†) is (Standard Deviation (±a))

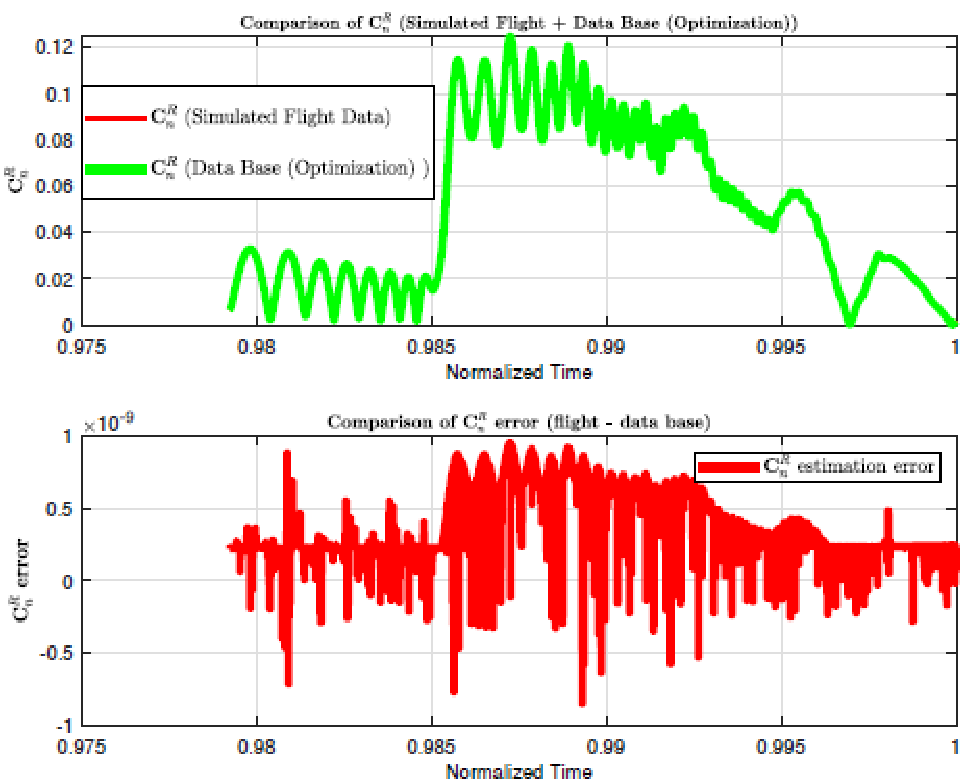

Figure 4: Time history comparison of C_N^R and estimation error (Simulated Flight + DB)

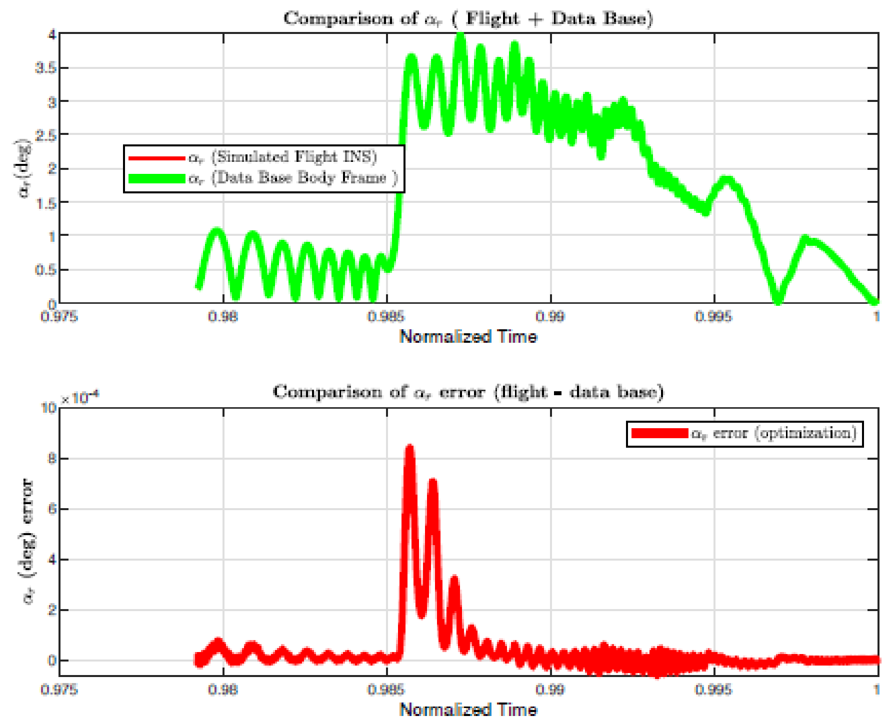

Figure 5: Time history comparison of $\alpha_{dr}\,(\text{deg})$ and estimation error (Simulated Flight + DB)

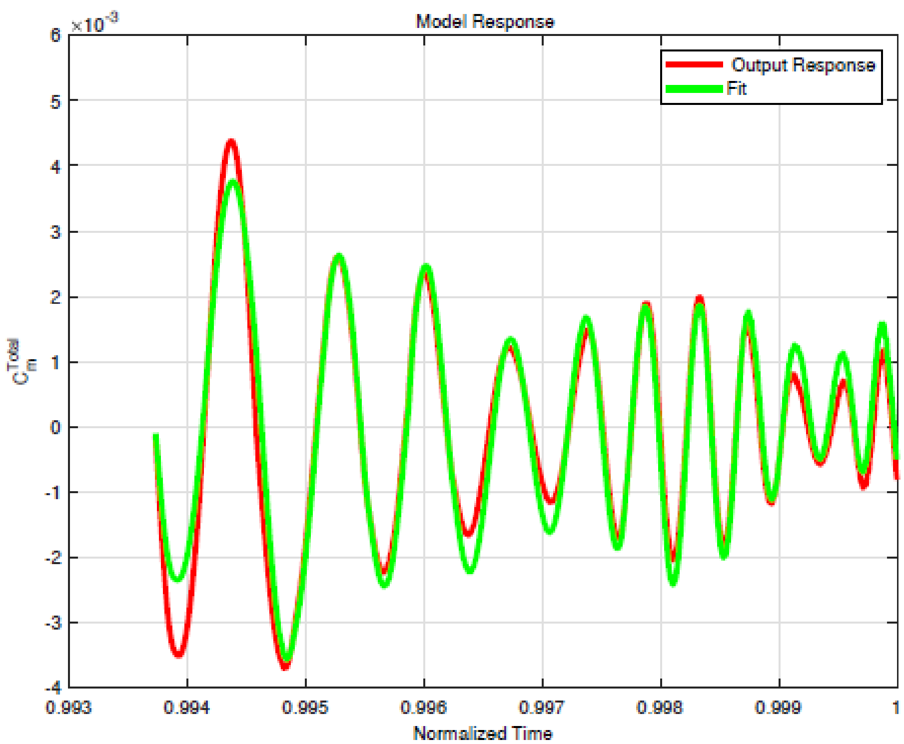

Figure 6: Time history comparison of C_m^{Total} (Simulated Flight + Pitch Plane model response)

Case study with Flight Data

Carefully noisy flight data with lot of missing data points and outliers have been preprocessed. Then EKF/MBFS filter / smoother has been used to estimate the states and their derivatives as discussed before. Smoothened pitch rate used for C_{myf}^{Total} estimation is shown in Fig. 7. NIS cost during smoothening is close to total number of measurement channels (Table 2) which ensures estimates are near optimal. Based on smoothened latax components, C_N^R along with (α_{dr}, ϕ_{dr}, α_{df}, β_{df}) have been estimated. Time history of C_N^R along with very small estimation error evolved during optimization are depicted in Fig. 8. Now using smoothened $(\hat{p}, \hat{q}f), (C_l^{Total}, C_{myf}^{Total})$ have been evolved (α_{df}, α_{dr}, ϕ_{dr}) along with (δ_{rf}, δ_{pf}) are used to estimate aerodynamic moment coefficients both along pitch and roll channel (Eqns. (11-13)). Comparison of estimated aerodynamic moment coefficients along with preflight values is shown in Table. 4. $(C_l^{Total}, C_{myf}^{Total})$ from flight and model output are compared in Figs. (9-10) respectively. Inferences based on present flight data analysis are 1) Due to lack of strong maneuver unlike in aircraft, the $F_{statistics}$ for moment coefficient estimation along pitch and roll channel are (750, 666) which is of good acceptance level. Based on authors flight data analysis experience it is of order of (500-800) for SFVs. Generally for TFVs it is of order of (1500-5000). Had there been strong maneuver, $F_{statistics}$ which dictates goodness of fit would have been more. 2) Control effectiveness for both channels are less than preflight and configuration experienced considerable C_i^{ar} as disturbance during flight. 3) All estimated parameters can be reasoned out as authentic due to small standard derivation.

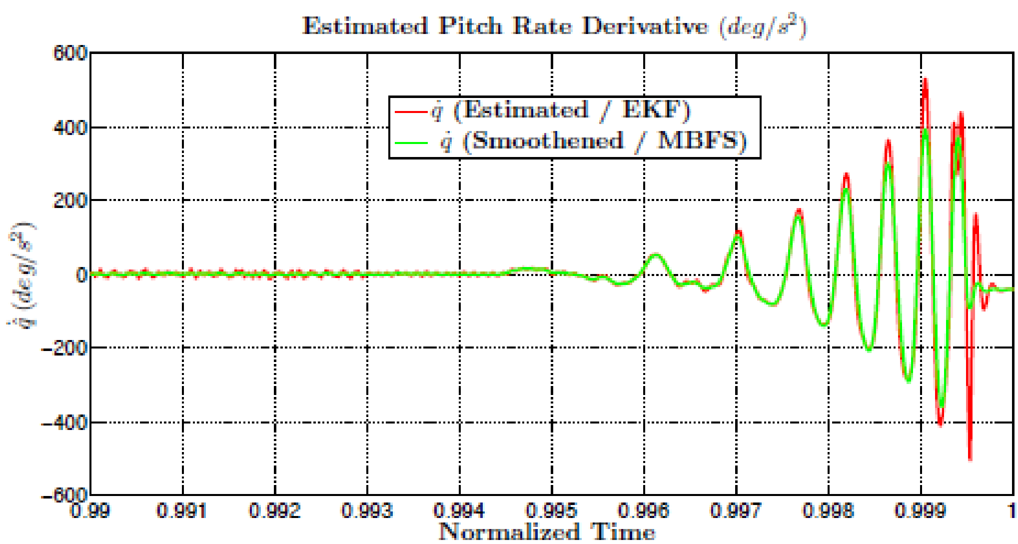

Figure 7: Time history comparison of $\dot{q}$ (Flight, EKF + MBFS)

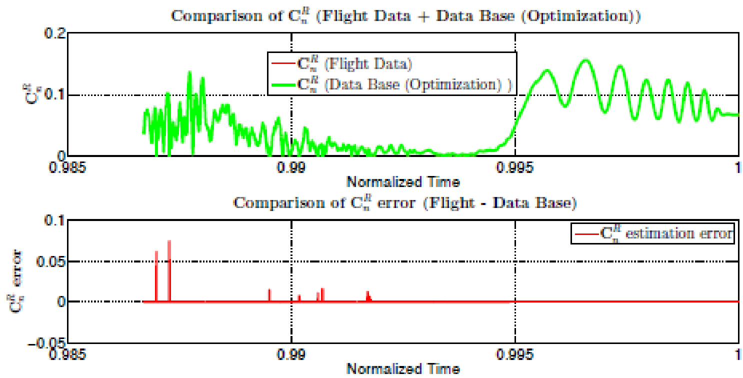

Figure 8: Time history comparison of C_{RN} and estimation error (Flight + DB)

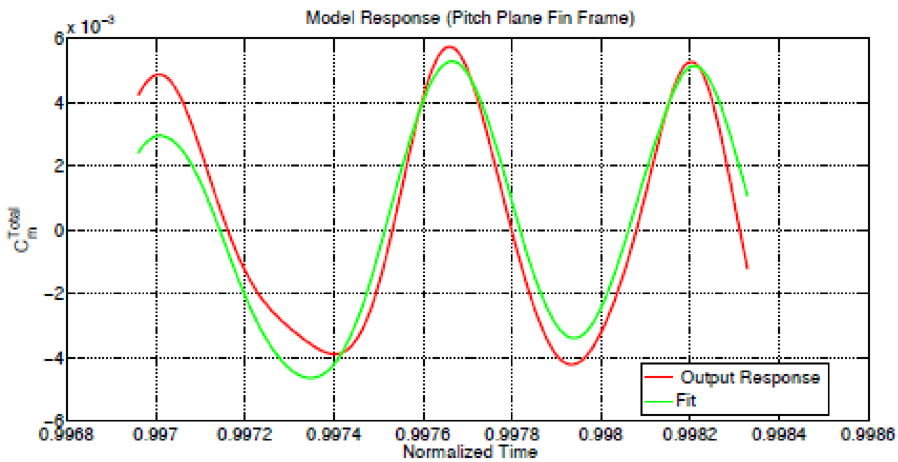

Figure 9: Time history comparison of C_M^{Total} (Flight + Pitch Plane model response)

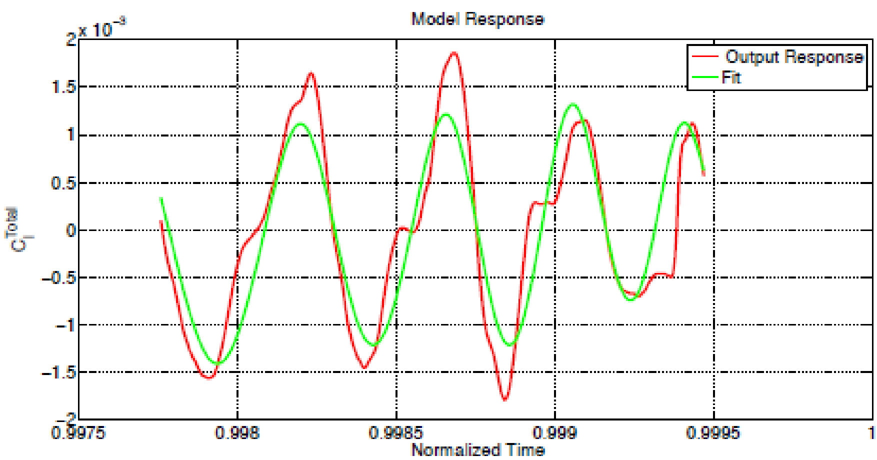

Figure 10: Time history comparison of C_l^{Total} (Flight + Roll Channel model response)

Table 4: Estim. Aero. Coeff. (Flight, MLR) (M=12.4,$\alpha = 1.6^0$, $\frac{x_{cg}}{d} = 1.675$)

Pitch Channel α_f Effect				
Case	C_N^{α} (/ deg)	$C_m^{\alpha}\big\|_{nose}$ (/deg)	$C_m^{\alpha}\big\|_{CG}$ (/deg)	F_{stat}
Preflight	0.0305	-0.0578	-0.00057	-
Flight	0.0310 (5.0×10^{-5}) (+)	-0.0550	-0.0020 (1.0×10^{-4})(+)	1769555 C_N^{Total}
Pitch Channel δ_p Effect				
Case	C_N^{δ} (/ deg pair)	$C_m^{\delta}\big\|_{nose}$ (/deg pair)	$C_m^{\delta}\big\|_{CG}$ (/deg pair)	F_{stat}
Preflight	0.0012	0.00278	0.00071	-
Flight	0.0013 (5.5 × 10−6)(†)	0.00252	0.00030 (1.4×10^{-5}) (†)	750 C_m^{Total}
Roll Channel (α_r, ϕ_r, δ_r) Effect				
Preflight	-	-	0.000332	-
Flight	0.00011 (3.0×10^{-5})(+)	-	0.0000 (1.3×10^{-5})(+)	666 C_t^{Total}

(†) is (Standard Deviation ($\pm\sigma$))

CONCLUSION AND FUTURE WORK

APE problem is carried out using MLR where pitch and roll channel have been processed separately through EBM. The same activity will be carried out by mechanizing complete 6 DOF plant state equations with available measurements using EKF/MBFS filter smoother ([8,9]). There the aerodynamic parameters will be augmented in plant equations as time-invariant during this phase of flight. Also in present experimental RLV, GPS aided SDINS is used for Navigation. So sensor outputs have negligible bias and this technique should give meaningful estimates. Estimated parameters using present method as well as proposed method should be consistent before using them for subsystem modifications as part of PFA.

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