Monte Carlo Based Tolerance Analysis of Sub-systems in Cryogenic Umbilicals

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Abstract

The space reforms in India have opened up the space market and in turn opened a window of opportunity for entry of private players into the market. The vision of Indian Space Research Organisation (ISRO) is to change the supply-based model of space sector to demand-based model. To capture the foreign and domestic launch vehicle market, the building costs of the launch vehicle needs to be optimised. The concepts of lean manufacturing, design optimisation, topology optimisation etc., need to be brought in at the design phase to achieve this while mass manufacturing is planned for the launch vehicles. Dimensional Tolerance analysis have been extensively used in mass manufacturing industries to model how the deviations in part level are propagating to assembly. Though computationally expensive, Monte Carlo method is one of the most common tools used for this analysis. Based on a Probability Distribution Function (PDF), number of part dimensions are generated within the tolerance band and the distribution of assembly parameter is evaluated based on geometrical dependency. The sensitivity of dimensional/ geometrical tolerances on assembly critical parameters can be effectively studied by this method. Fluid Control Components in launch vehicles employ a large number of fabricated parts and the assembly of these elements to form a sub-assembly will bring out huge number of critical assembly parameters. Not achieving these specifications can cause hardware rejection, thus increasing the cost of production & delay in project schedule. This study presents the tolerance analysis of certain sub-assemblies used in cryogenic umbilical systems employed in ISRO. The sensitivity analysis of geometrical parameters also presented. These results can of paramount importance to fine tune the design & fabrication drawings at the time of mass manufacturing of parts.

Keywords: *Tolerance Analysis, Monte Carlo Simulation, Probability Distribution Function, Skewness*

1. Introduction

Geometrical dimensioning and tolerancing has always been an important aspect in mechanical engineering design. The tolerances on each dimension is necessary since absolute dimensions cannot be achieved through any manufacturing process. In an assembly, the tolerances on part level combine to form critical dimensions termed as Functional Critical Dimensions (FCDs).

Tolerance analysis has been one of the major tools by which manufacturing industries improved their production & minimised the cost [1]. Various methods of tolerance analysis are employed by different researchers [2-5]. The dimensions in an assembly is represented as vectors in vector loop method [6,7]. The dimensional, kinematic & geometric variations are taken into considerations in vector loop method. The variation method employs a parametric model in which the geometry is modelled using mathematical equations and the shape & size of various features are studied using this [8,9]. The kinematic relations between two surfaces are modelled using six small displacement vectors and used for tolerance analysis in Small Displacement Torsor (SDT) method [10-12]. In the matrix method, displacement vectors are employed at each tolerance region to derive the mathematical model and analyse the tolerancing [13-15]. The Monte Carlo (MC) analysis gives a probabilistic approach of the tolerances and is the simplest tolerance analysis tool compared to other methods [16-19]. In

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MC analysis, functional relationships between the component dimensions is formed. The part dimensions are randomly generated conforming to a particular probability distribution function. The distributions of the FCD/ Assembly Critical Dimensions (ACD) are obtained from the functional relationship [18]. The sensitivity of different geometric tolerance features can also be studied using this tool.

Indian Space Research Organisation (ISRO) employs cryogenic umbilical for the servicing the cryogenic stages of its launch vehicles. Umbilicals are used in launch vehicles for electrical & propellant servicing, environmental conditioning etc., [20]. Essentially any of these systems consist of a flight segment (which will be fastened to the launch vehicle), ground segment (which is the interface to launch pad and separates at a specific time) and a lock with separation mechanism [21]. The cryogenic umbilicals contains two major sub-assemblies namely flight segment & ground segment, which are mated together through collet lock at the centre. The ground plate sub-assembly contains two closely toleranced guide pins and the other sub-assembly contains recesses for these pins. The clearance between the pin & the recess is an important dimension, since it can directly affect the assembly of ground plate sub-assembly to flight plate sub-assembly as well as the separation dynamics of the umbilical. This study finds out the distribution of guide pin clearances using the Monte Carlo tolerance analysis. The sensitivity of geometrical parameter is also studied. The ideal band of clearance is proven during the qualification studies of the umbilical and if the assembly does not meet this the parts will be rejected. The results of the MC analysis can be used to tune the dimensions & geometrical parameters such that the clearance distribution is well within the ideal band.

The paper is organised in four sections. The following section explains the methodology and problem formulation. Section 3 presents the results & discussion followed by conclusions in section 4.

2. Methodology

2.1 Dimensional relationships

The cryogenic umbilical sub-assembly houses two plate sub-assemblies which are mated against each other through centre. Flight plate sub-assembly consists of flight plate & allied sub-assemblies whereas ground plate sub-assembly consists of ground plate and allied sub-assemblies.

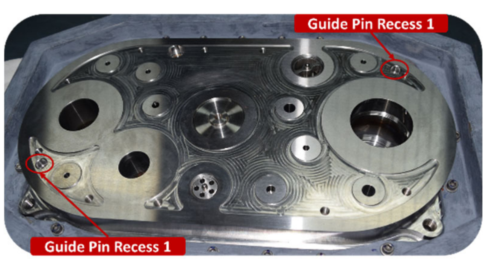

Figure 1: Flight Plate sub-assembly

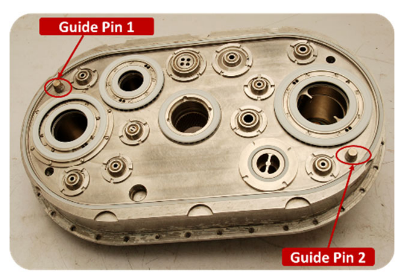

Figure 2: Ground Plate Sub-assembly

There are two guide pins which are assembled in to the ground plate and the flight plate houses the recesses for these pins. Figure 1 & Figure 2 shows the photographs of the plate sub-assemblies employed in Cryogenic Umbilicals. Figure 3 shows the dimensions of the parts in details. In both flight & ground plate sub-assemblies first guide pin location on the left side is dimensioned with respect to the centre (Refer Figure 3 (i), (ii)). The second guide pin location on the right side is dimensioned with respect to the first guide pin location. The ground plate sub-assembly has a female thread into which the guide pin is fastened (Refer Figure 3 (iii)). The assumptions taken for the analysis are as follows;

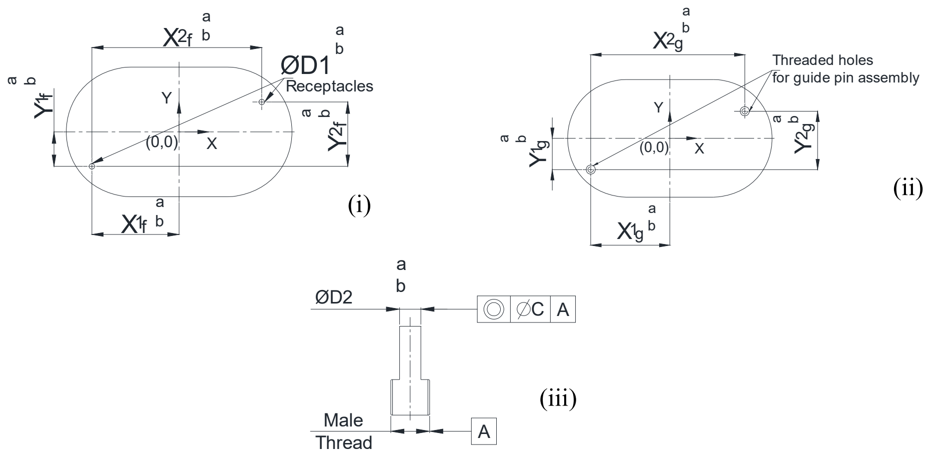

Figure 3: Part Dimensions (i) Flight Plate (ii) Ground Plate (iii) Guide Pin

- The deviation between the centre of the plate sub-assemblies during mating is neglected for the current study. Position of the holes are mentioned with respect to the centre in both flight & ground plate sub- assembly. Hence it is assumed that the centres are perfectly aligning.
- The tolerances on the threads are neglected.

Table 1 shows the details of the parameters from Figure 3. The tolerance band is described as (a, b) for representation. The line diagram to estimate the clearance between guide pin & recess is detailed in Figure 4.

Table 1: Dimensional details of parts

Plate	Dimension	Nominal Value
Flight Plate	Hole 1 X co-ordinate	$-X^1{}_f$
	Hole 1 Y co-ordinate	$-Y^1{}_f$
	Diameter of Hole 1 & 2	D_1
	Hole 2 X co-ordinate	$-X^1{}_f + X^2{}_f$
	Hole 2 Y co-ordinate	$-Y^1{}_f + Y^2{}_f$
Ground Plate	Threaded hole 1 X co-ordinate	$-X^1{}_g$
	Threaded hole 1 Y co-ordinate	$-Y^1{}_g$
	Threaded hole 2 X co-ordinate	$-X^1{}_g + X^2{}_g$
	Threaded hole 2 Y co-ordinate	$-Y^1{}_g + Y^2{}_g$
Guide pin	Concentricity for guide pin 1&2	C
	Angular location of guide pin center	θ
	Diameter of guide pin 1 & 2	D_2

From Figure 4, assuming that the guide pin centre falls at (0.5C, θ), the location of the first guide pin centre with respect to ground plate centre co-ordinate can be represented as;

$$\left(X^1{}_{gp}, Y^1{}_{gp}\right) = (\left(-X^1{}_g + (0.5C\cos(\theta))\right), \left(-Y^1{}_g + (0.5C\sin(\theta))\right)) \tag{1}$$

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The clearance for first guide pin can be obtained as;

$$Cgp1 = \left(0.5 * D1 - 0.5 * D2 - \sqrt{(X^1gp - X^1{}_f)^2 + (Y^1gp - Y^1{}_f)^2}\right) \quad (2)$$

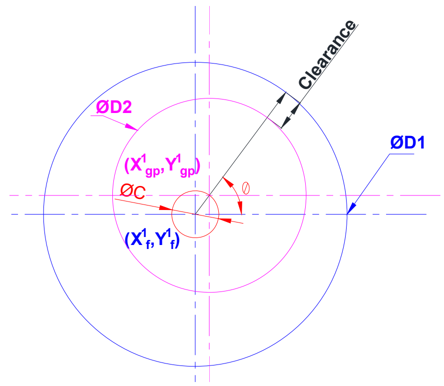

Figure 4: Guide Pin Clearance Details

Similarly, for the second guide pin, the center with respect to ground plate center is;

$$(X^2{}_{gp}, Y^2{}_{gp}) = (\left((-X^1{}_g + X^2{}_g) + (0.5C\cos(\theta))\right), \left((-Y^1{}_g + Y^2{}_g) + (0.5C\sin(\theta))\right)) \quad (3)$$

The clearance of second guide pin can be estimated as;

$$Cgp2 = \left(0.5 * D1 - 0.5 * D2 - \sqrt{(X^2{}_{gp} - (-X^1{}_f + X^2{}_f))^2 + (Y^2{}_{gp} - (-Y^1{}_f + Y^2{}_f)^2}\right) \quad (4)$$

Equations (2) & (4) represents the functional relationship of the guide pin clearances, *Cgp1* & *Cgp2* with part level dimensions & geometrical parameters.

2.2 MONTE CARLO ANALYSIS

The objective of the study is to carry out the tolerance analysis of guide pin clearances for the plates & conduct the sensitivity study. Equations (2) & (4) gives the functional relationships of the part level dimensions. The parameters taken for the case study is listed in Table 2.

Table 2: Case study parameters (All dimensions in mm)

Sl. No.	Dimension	Nominal Value	Upper Limit	Lower Limit
1	$-X^1{}_f$	190	190.01	189.99
2	$-Y^1{}_f$	75	75.01	74.99
3	D_1	12.025	12.05	12
4	$X^2{}_f$	370	370.01	369.99
5	$Y^2{}_f$	140	140.01	139.99
6	$-X^1{}_g$	190	190.01	189.99
7	$-Y^1{}_g$	75	75.01	74.99
8	$X^2{}_g$	370	370.01	369.99
9	$Y^2{}_g$	140	140.01	139.99
10	C	0	+0.03	0
11	D_2	11.965	11.98	11.95

The steps for the analysis are as follows;

- Random numbers are generated for all the 11 parameters listed in Table 2 assuming normal distribution. The mean of the distribution is the nominal value and the standard deviation is 1/6th of the tolerance band.
- The clearance is estimated for each of these set of dimensions. A computer program was written to get this using MATLAB 2022 software.
- The mean, standard deviation of the clearance data was obtained and the tolerance band is predicted using ± 3σ where σ is the standard deviation.
- The concentricity is varied and the normal plots for each clearance were plotted.

3. Results & Discussions

3.1 Guide pin clearance

The Monte Carlo analysis was done with a sample size of 2000. The random dimensions were generated conforming to normal distribution. The distributions of the generated data for flight plate recess co-ordinates are shown in Figure 5. Similar data was produced for guide pin co-ordinates, hole diameter, concentricity & guide pin diameter.

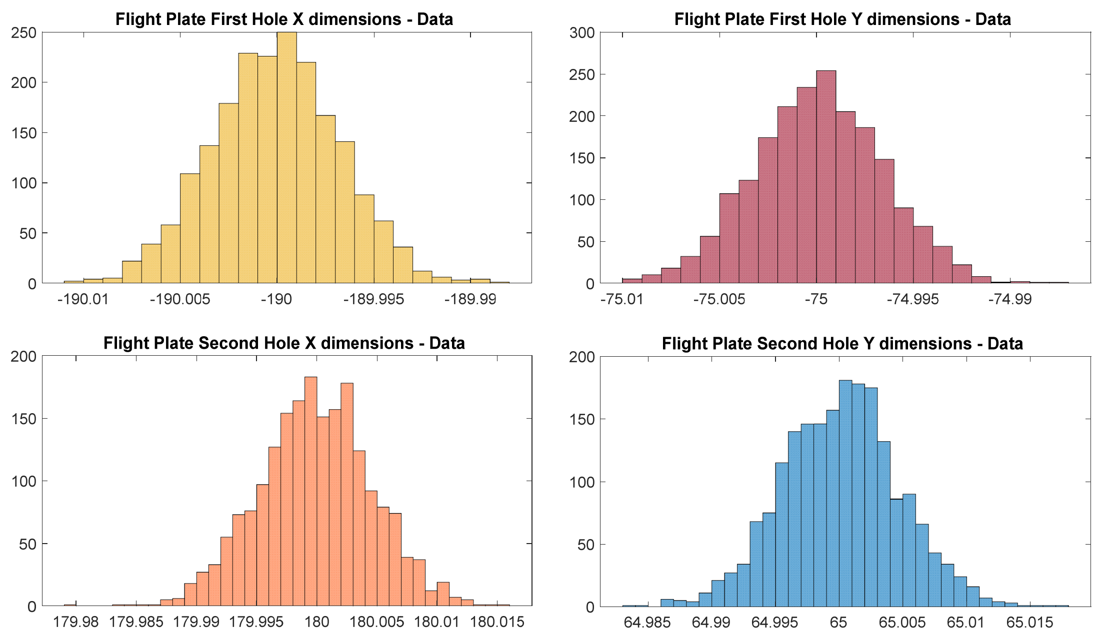

Figure 5: Flight plate hole co-ordinates distribution

Using the MATLAB program, the distribution of guide pin clearances was found out. The Figure 6 shows the histogram plots of the clearance distribution. From the histogram plot, it is evident that the guide pin clearance data follows a normal distribution. The co-efficient of skewness for both the distributions are between -0.5 & 0 and the co-efficient of kurtosis is approximately 3 which ensures the normal distribution.

Table 3: Guide Pin Clearance Distribution

Parameter	Mean (mm)	Standard Deviation	Lower Limit (mm)	Upper Limit (mm)	Co-efficient of Skewness	Co-efficient of Kurtosis
Guide Pin 1 Clearance	*0.026*	*0.006*	*0.009*	*0.043*	*-0.22*	*3.1*
Guide Pin 2 Clearance	*0.024*	*0.006*	*0.006*	*0.045*	*-0.3*	*3.2*

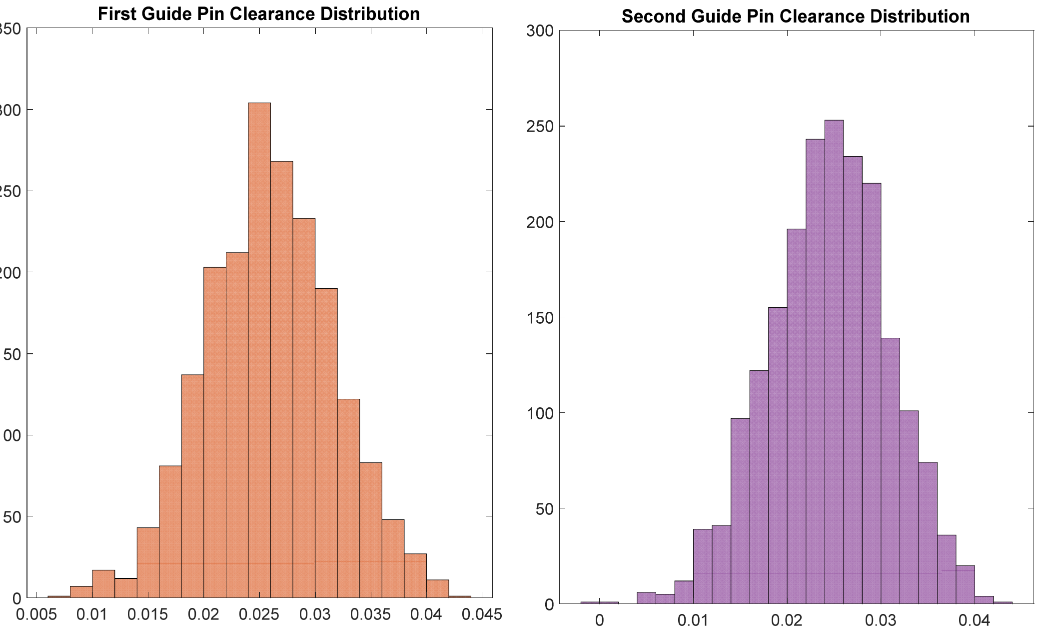

Figure 6: Guide Pin Clearance Data Histograms for Guide Pin 1 & 2

3.2 Sensitivity Study

The sensitivity analysis was carried out for the concentricity value. The analysis was done for different values. Table 4 shows the results. Figure 7 shows the normal plot of the guide pin 1 & 2 clearances for different concentricity values.

Table 4: Senistivity Study Results

Concentricity (mm)	Guide Pin 1 Mean (mm)	Guide Pin 1 Standard Deviation	Guide Pin 2 Mean (mm)	Guide Pin 2 Standard Deviation
0.03	*0.026*	*0.006*	*0.024*	*0.006*
0.06	*0.025*	*0.006*	*0.024*	*0.007*
0.09	*0.024*	*0.006*	*0.023*	*0.007*
0.12	*0.023*	*0.007*	*0.022*	*0.008*

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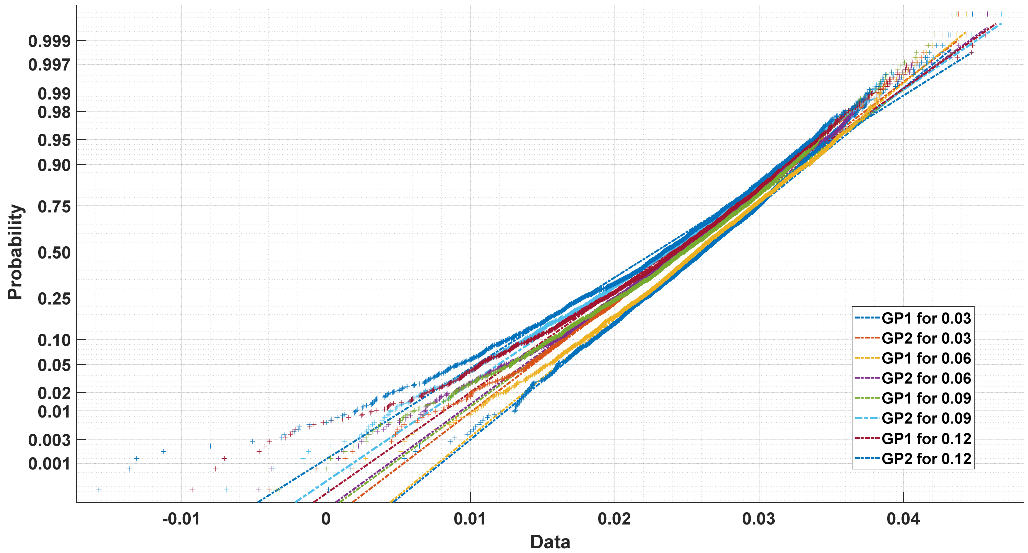

Figure 7: Normal plot of guide pin clearances for different concentricity values

The results indicate that during mass manufacturing, the guide pin clearance will have a mean of 0.026 & 0.024 for guide pin 1 & 2 respectively. The 99% confidence intervals are also estimated from the analysis which is shown in Table 3 upper & lower limit columns. The concentricity variation for four different values did not affect the guide pin clearances significantly.

4. CONCLUSIONS

The conclusions of the study can be listed as follows;

- The Monte Carlo based tolerance analysis was used to find out the distribution of guide pin clearances of plates used in cryogenic umbilicals. The analysis shows that the resulting data follows a normal distribution. The mean and standard deviation of the distribution were found out. The clearance values are well within the clearances qualified for the umbilicals. Hence the dimensions need not be revised.
- The upper & lower limit for the 99% confidence interval of guide pin clearances are found out (refer Table 3). During mass manufacturing, keeping the FCD clearances within this band can maintain the rejection during assembly to <1%.
- Sensitivity study was carried out for the concentricity of the guide pin. Though the variation of concentricity did not affect the mean and standard deviation, as the concentricity error increases, the distribution was found to become long tailed.
- The current study is restricted to a particular case study in a sub-system. This can be extended to various sub-systems to optimize the assembly tolerances & optimize FCD dimensions.

REFERENCES

1. Chase, K.W., Parkinson, A.R. "A survey of research in the application of tolerance analysis to the design of mechanical assemblies", Research in Engineering Design 3, 23–37 (1991), https://doi.org/10.1007/BF01580066
2. Marziale, M., Polini, W. "A review of two models for tolerance analysis of an assembly: Jacobian and Torsor", Int J Comput Integr Manuf, 2011;24(1):74-86

3. Laperrière L., Desrochers, A. “Modeling assembly quality requirements using Jacobian or screw transforms: a comparison Assembly and Task Planning”, 2001, Proceedings of the IEEE International Symposium on IEEE, 2001: 330-336
4. Ameta, G., Serge, S., Giordano, M. “Comparison of spatial math models for tolerance analysis: tolerance-maps, deviation domain, and TTRS[J]”, Journal of Computing and Information Science in Engineering, 2011, 11(2): 021004.
5. Chen, H., Jin, S., Li, Z. “A comprehensive study of three-dimensional tolerance analysis methods[J]”, Computer-Aided Design, 2014, 53: 1-13.
6. Chase, K.W., Gao, J. and Magleby, S.P., 1995. “General 2-D tolerance analysis of mechanical assemblies with small kinematic adjustments”, Journal of Design and Manufacturing, 5(4), 263-274.
7. Chase, K.W., Gao, J., Magleby, S.P. and Sorensen, C.D., “Including geometric feature variations in tolerance analysis of mechanical assemblies”, IIE Transactions, 1996, 28(10), 795- 807.
8. Gupta, S., Turner, J U. “Variational solid modeling for tolerance analysis[J]”, Computer Graphics and Applications, IEEE, 1993, 13(3): 64-74.
9. Li, B., Roy, U. “Relative positioning of toleranced polyhedral parts in an assembly[J]”, IIE Transactions, 2001, 33(4): 323-336.
10. Bourdet, P., Mathieu, L., Lartigue, C. “The concept of the small displacement torsor in metrology[J]”, Series on Advances in Mathematics for Applied Sciences, 1996, 40: 110-122.
11. Bourdet, P., Ballot, E. “Geometrical behavior laws for computer-aided tolerancing [M]”, Computer-Aided Tolerancing, Springer Netherlands, 1996: 119-131.
12. Legoff, O., Villeneuve, F., Bourdet, P. “Geometrical tolerancing in process planning: a tridimensional approach[J]. Proceedings of the Institution of Mechanical Engineers, Part B”, Journal of Engineering Manufacture, 1999, 213(6): 635-640.
13. Desrochers, A., Rivière, A. “A matrix approach to the representation of tolerance zones and clearances[J]”, The International Journal of Advanced Manufacturing Technology, 1997, 13(9): 630-636.
14. Whitney, D E., Gilbert, O L., Jastrzebski, M. “Representation of geometric variations using matrix transforms for statistical tolerance analysis in assemblies [J]”, Research in Engineering Design, 1994, 6(4): 191-210.
15. Serrano-Mira, J., Rosado-Castellano, P., Romero-Subirón, F., Bruscas-Bellido, G M., Abellán-Nebot, J.V. “Incorporation of form deviations into the matrix transformation method for tolerance analysis in assemblies”, Procedia Manufacturing 41, 2019, 547-554.
16. DeDoncker, D., Spencer, A. “Assembly tolerance analysis with simulation and optimization techniques[R]”, SAE Technical Paper, 1987.
17. Grossman, D D. “Monte Carlo simulation of tolerancing in discrete parts manufacturing and assembly[J]”, 1976.
18. Huiwen, Yan., Xiangji, Wu., Jiangxin, Yang, “Application of Monte Carlo Method in Tolerance Analysis”, 13th CIRP conference on Computer Aided Tolerancing, Procedia CIRP 27 (2015) 281 – 285
19. Samuel, H. Huang., Qiu, Liu., Rami Musa, “Tolerance-based process plan evaluation using Monte Carlo simulation”, International Journal of Production Research, 42:23, 4871-489, 2004, DOI: 10.1080/0020754042000264608
20. SP-8061, NASA, "Interaction with Umbilicals & Launch Stand," NASA Space Vehicle Design Criteria (Structures), August 1970.
21. Hemendra Kumar Dhurandher, Vaibhav Ahire, Dilip V, B Sathis Kumar and Krishnanandan V, "Fall Trajectory Finalisation for Cryogenic Umbilical," in LAMSYS, Trivandrum, 2017.