Evolution of Radar and Seeker Based Reentry Target Tracking For Online State Estimation*

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***Abstract*—In this review paper complete state of the art literature survey on non-maneuvering and maneuvering Reentry Vehicle target tracking using Radar and Seeker for State Estimation has been carried out since its inception from early seventies. It is well known that accurate state estimation of target is mandatory to design guidance and control law of interceptor for near hit to kill mission. So the estimation problem focuses on accurate estimation of target position and velocity components along with accurate acceleration estimation by processing noisy radar and seeker measurements. Hit to kill interception can only be achieved by advanced guidance laws which needs additional estimated target acceleration along with estimated position and velocity components as input. This study consists of evolving Extended Kalman Filter and Interacting Multiple Models based adaptive estimator for tracking this class of objects.**

***Index Terms*—Filter Tuning, Input Estimation, Interacting Multiple Models, Maneuvering Reentry Target, Radar Estimator, Seeker Estimator.**

I. Introduction

Estimation of target's kinematic state variables such as (position, velocity, acceleration) components along Inertial Frame from available radar or seeker measurements is mandatory for successful missile target interception [1], [2]. Based on the estimated states of pursuer and evader, guidance law is evolved. It generates demanded lateral acceleration (latax) along yaw and pitch plane. Demanded latax is tracked by missile autopilot to steer it to the target for successful interception. The guidance demanded latax is evolved based on pursuer-evader estimated states by processing radar or seeker measurements in real time. Radar measurements are target range, azimuth and elevation angles (R_m, A_m, E_m) assumed to be contaminated by Gaussian noise. Onboard seeker measurements are a) missile-target relative range (r_m) and range rate $(\dot{r}_m)$ contaminated by Gaussian noise b) Gimbal angles along yaw and pitch plane (ϕ_{ym}, ϕ_{zm}) and Line of Sight (LOS) rates in the gimbal frame $(\omega_{ym}, \omega_{zm})$ contaminated by time-correlated non-Gaussian noise.

Zarchan (2000) highlighted that advanced guidance laws are required to have knowledge of various target states plus the target acceleration with weave frequency [3]. This can be only achieved by near optimal state estimation using Extended Kalman Filtering (EKF) and its variants ([4], [5]). In context of radar estimator the estimated states are target (position, velocity, acceleration) components $\left(\hat{x}, \hat{y}, \hat{z}, \hat{\dot{x}}, \hat{\dot{y}}, \hat{\dot{z}}, \hat{a}_{tx}, \hat{a}_{ty}, \hat{a}_{tz}\right)$ from available measurements. From seeker estimator the estimated states are pursuer-evader relative (position, velocity) components along with target acceleration components $\left(\Delta\hat{x}, \Delta\hat{y}, \Delta\hat{z}, \Delta\hat{\dot{x}}, \Delta\hat{\dot{y}}, \Delta\hat{\dot{z}}, \hat{a}_{tx}, \hat{a}_{ty}, \hat{a}_{tz}\right)$ from available measurements. These estimated states will be input to guidance law for successful interception.

At first we will describe brief history of EKF as recursive least square form of Estimation Theory (ET) and its evolution over the years in Section II . Then during the nineties Unscented Kalman Filter (UKF) and Cubature Kalman Filter (CKF) have been invented for state estimation from noisy measurements which will also be discussed. During nineties Interacting Multiple Model (IMM) based adaptive estimator with different models also has been invented for tracking randomly maneuvering target with minimum tracking error and will be discussed in brief. Then in Section III literature survey of both EKF based Radar and Seeker estimators will be discussed. At last in Section IV research carried out on both radar and seeker estimators in missile complex will be included. The paper concludes in Section V with possible futuristic research. From henceforth tracker, estimator and filter are synonyms for state estimation from available noisy measurements.

II. Evolution of EKF, Radar and Seeker Estimators

In this section we will briefly describe a) Evolution of EKF and its variants UKF and DDF and b) Detailed literature survey of Radar and Seeker Estimator till date. While discussing evolution of EKF, we will discuss evolution of its variants also as more popular while tracking maneuvering targets.

A. Evolution of EKF from ET

It was astronomy which gave rise to ET. Ceres, the largest of the minor planets was tracked from 1st January 1801 for 41 days by the Italian astronomer Giuseppe Piazzi. Later he could not find Ceres again. Luckily the problem caught the attention of Gauss who provided least square estimate of the orbit of Ceres and sent the results to Piazzi who could locate it on the last day of the same year! It is interesting to note that almost every aspect of present day ET namely 1) System model, 2) Measurement model, 3) Redundant data 4) Initial estimate of the unknown parameters 5) Cost function based on least square were all provided by Gauss. The cost function to be minimized by the criterion of least squares was also proposed by him. Later in the year 1960 the time domain description of the system in state space was handled by Kalman [6] to provide the well known Kalman filter (KF) solution to separate the noise and obtain the signal or a linear system. The filter further showed that batch processing least square technique can be replaced by sequential processing of the measurement data to update the state. Later it was extended for nonlinear system as EKF by Kalman and Busy in 1961 ([7]). It was not known to many that soon after EKF was invented, the enthusiasm was followed by depression since the filter design required proper choice of parameters namely $\mathbf{X}_0$, $\mathbf{P}_0$, $\mathbf{Q}$ and $\mathbf{R}$ as **Filter Tuning**. Here $(\mathbf{X}_0, \mathbf{P}_0)$ are initial state and error covariance and $(\mathbf{Q}, \mathbf{R})$ are Process and measurement noise covariance. EKF is a solution to an optimization problem using cost function. But in most present day work involving EKF the cost function is completely forgotten and one obtains some formal, or notational solution by choosing manually above filter tuning elements. It is this aspect that led to the efforts for adaptive methods of filter tuning. By the time Gelb's book [4] got published one may say that most issues of Kalman filtering including nonlinear filtering was by and large handled reasonably well except for the tuning of the filter parameters. Even today no generic method for filter tuning exists and is an open area of research.

B. Derivative Free Estimators

EKF is probably the most widely used estimation algorithm for nonlinear systems. Butafter nineties there have been researches about new efficient nonlinear filtering techniques in which the nonlinear filters generalize elegantly to nonlinear systems without the burdensome linearization steps. These derivative free estimators are UKF and CKF. They are being briefly described now.

1) UKF Estimator: Unscented Transformation (UT) was developed by Julier, Uhlmann and Durrant-Whyte [8] to propagate the mean and covariance information through nonlinear transformation. Here a set of samples (Sigma Points) are used to parameterize the mean and covariance of a (not necessarily Gaussian) probability distribution. In their paper they have compared the performance of this new filter against an EKF for a nonlinear tracking problem. Through numerical simulation they have shown that mean position error and $(one\ \sigma)$ estimation error are less than that of EKF.

2) CKF Estimator: It has been developed by Arasaratnam and Haykin (2009) where they present a new nonlinear filter for state estimation. The heart of the CKF is a spherical-radial cubature rule, which makes it possible to numerically compute multivariate moment integrals encountered in the nonlinear Bayesian filter. They used CKF for tracking a maneuvering aircraft. The result of this demonstration demonstrate the improved performance of the CKF over conventional nonlinear filters [9]. Application of CKF on practical aerospace problems is still an open area of research for scientific community.

C. Interacting Multiple Models

EKF has been designed to track a maneuvering target based on three different models such as Constant Velocity (CV), Constant Acceleration (CA) and Constant Jerk (CJ) models ([10]–[13]). In the mean time Kirubarajan and Bar-Shalom (2003) compared performance of EKF with IMM estimator for single-target tracking. They have shown through simulation that it performs significantly better than EKF in certain cases while tracking maneuvering target.

Intuitively one can think IMM as generic system model consisting of different system models with different process noise in parallel. Depending on the detected level of target maneuver system output should be either the best estimate or the weighted average of estimates from different models. This is an adaptive estimator ([14]–[18]). The individual estimators in IMM can be (EKF,UKF, DDF) with different models according to designer's choice.

III. Literature Survey on Radar Based RV Tracking

Research on missile and aircraft tracking using EKF started late sixties after the success of Apolo Space Mission. In presnt paper we will carry out literature surveny specific to Reentry Vehicle (RV) ans Maneuverring Reentry Vehicle (MaRV). Complete research on this topic are discussed in following four phases.

A. Survey of Radar Based RV Tracking

Here we will discuss **different models** evolved over years along with **filter tuning**. Total research papers are categorized in three phases based on years of publication till date.

1) Phase-1 (1970-2000): Here noisy radar measurements mean (R_m, A_m, E_m) sampled at time Δt. The first research paper on radar tracker for tracking RV has been reported by Mehera (1971) during early seventies. This research reports that EKF mechanized along radar coordinate performs better than other filters such as IEKF, and second order EKF in cartesian frame. From noisy measurements of phased array radar, estimated states are position, velocity components along with ballistic coefficient (β) [19] . Siouris *et al.* (1997) studied the same radar tracking problem to track an incoming ballistic RV which contains uncertainty in modeling and noise in both dynamics and measurements. He reported that a new Extended Interval Kalman Filter (EIKF) performs better than EKF while solving uncertain and nonlinear ballistic RV tracking problem. The radar measurements are from phased array radar. They also considered states to be position, velocity components and ballistic coefficient β [20] . Chang *et al.* (1977) examined several versions of EKF to estimate the position, velocity, and other key parameters such as maneuvering coefficients associated with MaRv from radar measurements. In this context, they have studied three state models . They are a) CA Model, b)Constant Drag Model, c) Constant Drag and Lift. Later Kameda *et al.* (1997) proposed a nonlinear filter design algorithm using multiple maneuvering models for tracking MaRV as continuation of Chang's research. The MaRV gets influenced by aerodynamic forces and experience nonlinear motion. When tracking such targets, they used EKF for different models as filter bank. They proposed a tracking filter based on these multiple maneuvering model using IMM. Their results show that tracking with multiple models using IMM improves impact prediction over traditional tracking methods [21], [22] . Here α is inverse of ballistic coefficient β (the ratio of vehicle mass to effective drag area). In this context β indicates an increase in the lift and γ is revolving due to lift. Total acceleration are summation of acceleration due to a) gravity, b) centrifugal force and the coriolis force and c) (α, β, γ) [22] .

In between some researchers started modeling plant consisting of only position and velocity components as state variable with target acceleration components as control input. At first Chan *et al.* (1979) estimated position and velocity components in two-dimension along with input estimation. The available measurements are position alone of radar.

Later Bogler (1987) reported solution of same problem. His solution is divided up into three distinct phases a) Maneuver detection b) Compensation of KF state estimation from the previous target maneuver c)Adjustment of KF parameters in anticipation of future target maneuvers. Later as extension of this research Lee *et al.* (1999) estimated trajectory of ballistic RV in real time along with online input estimation. This proposed approach used EKF for state estimation along with an innovative recursive input estimator to estimate the target acceleration. They have validated the estimation approach with actual radar tracked flight test data. The control inputs to the system are acceleration components due to aerodynamics. They compensated the model uncertainty by process noise which also gets estimated adaptively [23]–[26] . Excellent treatment on algorithmic presentation of input estimation with clarity is available in Barshalom *et al.* ([13] , pp 427-440). The state models in all case studies are cartesian coordinate in three dimension at radar site. Different state models used discussed till now are depicted in Table I .

2) Phase-2 (2001-2010): Next Cooperman (2002) reported IMM based first ballistic RV tracking algorithm. He used point mass dynamics of RV along East North Vertical (ENV) frame from spherical and rotating earth. He used three models for IMM. These models are a) Boost phase model b) Coast phase ballistic model c) Re-entry model as cost phase along with β [27] . Farina (2002) also studied ballistic RV tracking in the reentry phase by processing radar measurements. He assumed flat earth and state model was in two-dimension with ballistic coefficient and gravity. He compared the estimation accuracy of EKF, UKF, the statistical linearization filter and particle filter (PF). The simulation results favored the EKF. This conclusion is valid when β is known apriori [28] . Later Li *et al.* (2003, 2005, 2010) carried out uptodate survey on a) Different models for tracking MaRV 2) Survey of Multiple models and Filter c) Different motions of ballistic target [29], [30], [30] . This is a very important review paper. Minvielle (2003) reported decades long research since sixties and possible future research areas in RV tracking from available radar measurements. In the paper he discussed evolution of estimation techniques as EKF, Iterated EKF (IEKF), UKF and IMM. IMM estimates are best because here model switching takes place between a bank of models adapted to a different phase. About plant model he projected the better accuracy of RV with respect to spherical rotating earth. In his survey it became clear that previous researchers used initially fixed gain $\alpha - \beta$-tracker. Then they switched over to uncoupled 9 state model using position, velocity and acceleration components. But although the ballistic coefficient β can be estimated, its estimation is rather noisy because these KFs are not able to have enough process knowledge about the ballistic re-entry motion [31] . Thus, their estimation of the deceleration component gets fully affected by the measurement noise, without any smoothing. So later they switched over to 7 states model which includes estimation of β as another augmented state variable [32], [33] . Subsequently Farrell (2008) used IMM filter to track Tactical Ballistic Missile (TBM) in real time whose dynamics is not known apriori. Different state models used were a) Constant Axial Force: valid during both ascend phase with thrust and drag and reentry phase with drag, b) Ballistic Acceleration: for exo-atmospheric phase consists of gravitational, coriolis and centripetal forces c) Autocorrelated Acceleration model for compensating process noise according to Singer's model [10] . and the total acceleration can be computed from following Newton's law. The proposed IMM filter is shown to yield consistent state estimates throughout the entire TBM trajectory, which includes a dual-stage boost during launch [34] . Different state models discussed herein are depicted in Table II .

3) Phase-3 (2011-2023): Chen *et al.* (2009) used EKF and UKF mechanization to track a MaRV using the state model consisting of $(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta)$ at radar site in cartesian frame. They tracked the maneuvering RV assuming flat earth by using a self-adaptive filtering method. Based on control of two parameters (q_1, q_2) they modeled the process noise of target dynamics and ballistic coefficient. From the simulations, they have concluded that self-adaptive filters have a better robustness for tracking maneuverable ballistic missile targets on reentry than the conventional filters. Also UKF tracking performance is better than EKF and computational complexity of UKF is approximately 2.7 times as that by EKF [35] . Kim *et al.* (2013) tracked an RV using IMM based method for Ballistic Missile Defence (BMD). Without the priori information like aerodynamic parameters, it is hard to make the near perfect judicious filter dynamical state model of the hostile missile. So they suggested the mode probability revision method in this paper. With this method, the IMM mode set is adaptively changed and tracking performance improves. In the simulation, it has 6 modes and each has its own ballistic coefficient as its mode probability parameter. The estimated ballistic coefficient β is calculated by using weighted average of (β_i, μ_i). Here weightage factor is the mode probability μ_i. They have mechanized IMM with UKF based estimator [36] . Moon *et al.* (2012) and Kim *et al.* (2013) studies the tracking of RV in order to predict its impact point on the ground. Here they proposed β-model based IMM-EKF estimator (β-IMM-EKF) for precise tracking of RV. To evaluate the performance, other ballistic coefficient model based filters such as (γ-augmented filter, $\gamma - \gamma$ bootstrapped filter [37]) were compared and assessed with the proposed β-IMM-EKF for precise tracking of RV. They used rotating spherical earth with point mass equations in ECEF frame with standard radar measurements. In present β-IMM-EKF model, they selected three hypotheses or modes based on variation in β-values as $(\beta_1, \beta_2, \beta_3)$ as $(800, 1200, 1600)$ kg/m^2. A distinct feature of the IMM algorithm is the combination process where the estimated mean and covariance are produced through the computation of mode probabilities. Mode probability means the probability where each filter dynamic model matches the target dynamic model. Later they improved the β-IMM-EKF model to Variable-Structured Interacting Multiple Model $(\beta - VSIMM)$ which gives the best tracking performance. But β-VSIMM consisting of $(\beta_1, \beta_2, \beta_3)$ as $(1000, 20000, 40000)$ kg/m^2 gave rise to best tracking performance it terms of position and velocity tracking error [38], [39] . Next Fan, Zhu and Bai (2016) reported a tracking algorithm for tracking Hypersonic Glide Vehicle (HGV) maneuver based on Modified Aerodynamic Model (MAM) using CKF from radar track measurement data. To establish the state equation, the MAM for HGV tracking has been proposed by using additive white noise to model the rates of change of two control variables along pitch and yaw. The plant is spherical rotating earth with states $(V, \theta, \sigma, r, \lambda, \phi)$ as HGV speed, flight path angle, azimuth angle measured from north clockwise, radial distance from earth, longitude and latitude. (L, D) are L and D are lift acceleration and drag acceleration respectively. They are functions of attack of angle and Mach number (α, M). The control variables are angle of attack and bank angle (α, ν). For the ease of comparison several multiple model algorithms based on CKF [9] have been presented including IMM algorithm, Adaptive Grid Interacting Multiple Model (AGIMM) algorithm and Hybrid Grid Multiple Model (HGMM) algorithm. Now let us discuss these different models briefly. The different models are

- **MAM Model:**Here the control input to the state model is (α, ν) assumed as constant Gaussian Markov (GaM) process which has to be estimated as augmented state.
- **HGMM Model:** The HGMM is an IMM tracking algorithms with three-model IMM algorithm and nine-model IMM algorithm based on aerodynamic equations and radar measurement equations.
- **AGIMM Model:** In order to solve real-world problems the use of only a small number of models is not good enough but the use of more models does not necessarily improve the performance and increases the computational burden considerably. Thus, the concept of Variable Structure Multiple Model (VSMM) estimation is proposed and several VSMM algorithms are developed by researchers. An effective VSMM algorithm is to make adaptive the grid of the parameters that characterize the possible modes. In AGIMM tracking algorithm a coarse grid relating about attack angle is set up initially, and then the grid is adjusted recursively.

Their simulation results indicate that the proposed tracking algorithm based on MAM has the best tracking performance with the best accuracy and least computational cost among all tracking algorithms in the paper. The proposed algorithm is cost-effective for HGV tracking [40] . CKF has been mechanized for all state models. Later Sun and Xin (2017) reported a new adaptive CKF for state estimation of a hypersonic entry vehicle in Mars atmosphere. Here the states are (altitude of planet over earth, geocentric longitude, geocentric latitude, velocity, flight path angle, heading angle) as $(h, \theta, \lambda, V, \gamma, \xi)$ and the measurements are $(q, \theta, \lambda, \gamma,)$ contaminated by gaussian noise. The plant is rotating spherical model. The other parameters in plant equations are $\left(\beta, \frac{L}{D}, \right)$ [41] . Recently Hough (2017) carried out reentry maneuver estimation of using nonlinear Markov Acceleration models of MaRV. Nonlinear Markov acceleration models with variable process noise has been implemented in an IEKF for reentry maneuver estimation. Drag and two lift acceleration components have been modeled by nonlinear, first-order differential equations whose parameters are functions of position, velocity, and acceleration. The paper depicts close match between estimated and true acceleration components. The plant has been modeled assuming spherical rotating earth. This aerodynamic model is similar to the same as of proposed by Kim *et al.* and Dwivedi *et al.* ([42] , [43]). But extra content in this research is more involved modeling of (a_D, a_N, a_P) as nonlinear Markov acceleration process [44] . These state models used discussed in this paragraph are depicted in Table III . Very recently Zhang *et al.* (2018) has carried out state estimation of HGV based on coupled dynamic model with assumption of flat earth with RAE measirements with noise assumed as $(500\, m, 10\, milli\, rad, 10\, milli\, rad)$ with a sampling time of 100 milli second. They have carried out three coupled dynamic models of state estimation. These models are mainly based on detailed modeling of aerodynamic acceleration acting on HGV.

1) **Spiral Model:** The key idea of spiral model (SM) is to assume the HGV undergoing the spiral motion based on the moment equilibrium ([42] , [43]).
2) **Bank Model:** Like the SM, the Bank Model (BM) also takes the correlation between (α_t, α_c) into consideration. The difference is that the latter uses the bank angle ϕ to maneuver the HGV.
3) **Guide Model:** The Guide Model (GM) no longer simply treats unknown aerodynamic parameters as the random processes, but models them by changing rule of guidance variables (α, ϕ). For lack of information of the HGV control system, we can assume that variation of (α, ϕ) obeys the first-order time delay processes as [44]

They have compared the performance of all these models through detailed simulation study. The results show that the proposed models perform excellently. The GM produces the best tracking performance and the BM is the second best. However SM does not outperform other model significantly [45] . These state models used discussed in this paragraph are depicted in Table IV .

B. Survey of Seeker Based RV Tracking

on this topic based on open literature which are scanty. Bhattacharyya *et al.* (2008) modeled realistic RF seeker dynamics to generate correlated non-gaussian and non-stationary noise. They generated realistic range, range rate, gimbal angles and LOS rates measured by seeker contaminated by above realistic noise. Ghosh *et al.* (2011) used acceleration model and jerk model ([10] , [11] , [12]) for kinematic state estimation of RV using EKF using these measurements. This is Direct Acceleration (DA) model. The novelty claimed by the authors is estimation of ratio of air density and ballistic coefficient (γ) and its time derivatives using a separate Kalman filter (KF) $(\gamma - filter)$ which utilizes pseudo measurements of γ computed from the velocity and acceleration estimated by the EKF at each time step. The parameter $\gamma = \dfrac{\rho}{\beta}$ and its derivatives estimated by this model are in turn, used for the estimation of position, velocity, acceleration and jerk in the EKF. The use of the pseudo measurements of (γ) makes the algorithms inherently adaptive to variations of the ballistic coefficient and air density during reentry. This method also models the non-stationary and correlated noise of the RF seeker for estimating the target acceleration and the relative kinematics for endo-atmospheric engagement [37], [46], [47] . In parallel Roy *et al.* (2013) and Saha *et al.* (2014) focused on **Filter Tuning** of EKF for target tracking application. It is an well known fact that **Filter Tuning** as proper selection of $(\mathbf{P}_0, \mathbf{Q}, \mathbf{R})$ is context dependant and notoriously difficult problem!. This problem has addressed by them. They defined a robustness metric and a sensitivity metric which has been used to determine a suitable combination of the filter tuning parameters of EKF for getting the near-optimal estimates. The characteristics of these metrics have been used to predict the root-mean-squared error (RMSE) performances in a 2-D falling body problem. They proposed a generic method for initial choice of the filter tuning parameters $(\mathbf{P}_0, \mathbf{Q}, \mathbf{R})$. The RMSE performances are then obtained for a range of variation of the most critical tuning parameter, namely the filter process noise covariance [48], [49] . This an important study carried out by them relevant to tracking application. Very recently Aastha *et al.* (2022) carried out tracking and interception of spiraling reentry using EKF, UKF, New Sigma-point Kalman filter (NSKF) and CKF. Their study recommends NSKF with Proportional Navigation (PN) Guidance is the best for solving the spiraling target tracking and interception problem [50] . Vaddi *et al.* (2007) proposed EKF for estimating the states and parameters of a ballistic target undergoing spiraling motion. In the formulation, they considered gravitational and aerodynamic forces in the formulation. Spiraling target motions were modeled by a second-order oscillator dynamics modulating the aerodynamic normal force, together with aerodynamic drag and the acceleration due to gravity. Here the state variables consist of target position (x_t, y_t, z_t) , velocity components $(\dot{x}_t, \dot{y}_t, \dot{z}_t)$ along with modified drag coefficient D_c , oscillator states $\left(Z_1, \dot{Z}_1\right)$ and spiraling frequency ω. The formulation is applicable both of radar and seeker measurements. They validated estimator performance with integrated guidance-control (IGC) law to intercept a spiraling reentry target successfully. Kim *et al.* (2008) from the same group of researchers, enhanced their estimator model to handle motion of certain ballistic RVs reentering the atmosphere, which undergo violent spiraling motions induced by aerodynamic resonance between roll and yaw/pitch modes. This case is for a maneuvering RV. Successful interception of such a spiraling maneuvering RV is critically dependent on the performance of the target state estimator. Strong nonlinearities involved in the system dynamics and measurement equations together with sensor noise make this a challenging estimation task. So they have enhanced the state model to $(x_t, y_t, z_t, \dot{x}_t, \dot{y}_t, \dot{z}_t, Z_1, Z_2, D_1, \omega)$. Model being kinematic, following grouping of the variables are carried out to reduce the number of unknowns. Here (D_1, Z_1, Z_2) are Inverse of ballistic coefficient, maneuvering coefficient along yaw and pitch plane. Target mass and Reference area are (m, S). The variable ω is the rolling frequency, assumed to be equal to the steady state roll resonance frequency This is how they modeled the kinematic state dynamical model of maneuvering RV inclusive of aerodynamic forces. They have compared performance of the EKF, UKF and PF designed for this estimation problem. Later Kim *et al.* (2010) focused their research on developing filter motion models which can reproduce a wide variety of target motions which include intelligent evasive maneuvers. The proposed filtering technique reported to improve tracking performance for a maneuvering target undergoing unpredictable motions, which would eventually reduce the miss distance of the ballistic target interception. They demonstrated the feasibility of the developed motion models through numerical tracking simulations, and compared their enhanced performance over conventional filter motion models. The target models considered are White Noise Acceleration, Wiener Process Acceleration, Ballistic Reentry Vehicle, Spiraling and Generalized Snap model. Subsequent research of Kim *et al.* (2010) proposed ballistic target tracking under glint noise. This is because seeker sensor measurements corrupted by range-dependent glint noise which is non-gaussian in nature. They reported applicability of a PF to present problem enhances tracking accuracy compared with that of the EKF. Finally Kim *et al.* (2012) and Ohlmeyer *et al.* (2014) validated the robustness of tracking performance of complete enhanced model for tracking maneuvering RV through MC analysis [42], [51]–[55] . Dwivedi *et al.* (2014) used this generalized state estimation approach for accurately estimating the states of incoming high-speed targets successfully [43] . A key advantage of this nine-state problem formulation is that it is very much generic and can capture spiraling as well as pure ballistic motion of targets without any change of the target model and the tuning parameters. Evolution of research on seeker filter design is depicted in Table V

IV. Design of Radar and Seeker Estimators in Missile Complex

Requirement of design of Radar Estimator came nearabout in 2000 for Akash Missile to track Rajendra Radar. Similarly the requirement of design of seeker estimator came during the same time for Air-to-air and Ballistic Missile Defence (BMD) applications. Since then the research carried out both cases till date is discussed below. It is worth to say that the folowing radar and seeker estimators have been validated through high fedility 6 DOF simulation Monte Carlo (MC) studies and successful flight trials.

A. *Design of Radar Estimator*

State estimation of evader as well as pusuer with high tracking accuracy plays vital role in achieving miss-distance performance of RADAR based command guided surface to air missile (SAM) system. So for this purpose EKF based tracking algorithm for both pursuer and evader has been developed and later compatible IMM estimator based on CV, CA and CJ models has been implemented and flight tested [56] . Here Praveen *et al.* (2008) brings out how an appreciable improvement in miss distance performance of a medium range SAM system has been achieved using an IMM tracking algorithm for monmaneuvering and maneuvering targets.

B. *Design of Seeker Estimator*

In due course of time RF gimbaled Seeker hardware technology got matured for Air-to-ar, SAM anf BMD applibations. So it became the need of the day to design a filter to process the seeker noisy measurements. It is worth to mention at this juncture that lot of unclassified literature on target tracking using seeker are very scanty. Generic state model for radar and seeker estimator is available in

[57] (pp. 68-75, pp 442-443). Later Hablani (1998) developed a second order filter to estimate the state variables from angles only measurements of seeker for proportional navigation (PN) guidance to intercept an exo-atmospheric interceptor [58] . Ekstrand (2001) developed two-state tracking filters for seeker applications. He only discussed estimation of LOS rates and equations of motion for the LOS along yaw and pitch channels [59] . Based on these research papers, first seeker estimator design for Beyond Visual Range Air to Air Missile (BVRAAM) application was carried out by Ananthasayanam *et al.* (2004). Later they extended same design for AD application ([60], [61]). The software aspects of seeker technology consists of a) Seeker modeling to generate the seeker measurements b) Estimation of kinematic state variables from noisy seeker measurements in realtime. Onboard RF seeker measurements are pursuer-evader relative range, range rate, gimbal angles and bore-sight error contaminated by realistic noise. The gimbal angles and line-of-sight rates in the gimbal frame are generally contaminated by time-correlated non-Gaussian noise. EKF is generally used for estimation of kinematic state variables based on these seeker measurements. Thus, a seeker estimator with white Gaussian assumption has been found to be adequate to handle the measurements even in present algorithm and has been successfully flight tested for all applications. For BMD applicatiion seeker measurements are relative range, range rate, Gimbal angles and LOS rates along (yaw, pitch) plane. Here the LOS rates are contaminated by correlated noise due to glint, radar cross section fluctuation and thermal noise effect. Also there is as aperiodic data loss in line of sight rate measurements due to eclipsing effect of radar transmitted signal. The Gimbal angles having the integrated effect of LOS rates, are contaminated by colored noise. The range and range rates are assumed to be contaminated by Gaussian white noise. Based on the realistic non Gaussian measurements, pursuer-evader relative position, relative velocity and target acceleration components along inertial cartesian frame have been estimated by using EKF. Here state equations are in Cartesian and measurements are in Polar (CP) frame. Later the same problem has been solved with both state equations and measurements along Polar (PP) frame. The estimators were EKF and UKF. Even later same problem was solved using both EKF and UKF with state equations in Polar and measurements in Polar frame along Gimbal axes (PPG) which makes the measurements to be linear with state variables. So the same problem was formulated along (CP, PP, PPG) frames and was solved by EKF and UKF. Through Monte Carlo (MC) simulation all methodologies of estimator mechanization was found to be statistically consistent. Complete work was carried out in open loop by Ananthasayanam *et al.* [60]–[63] . In parallel the RF seeker software development activity continued more for BMD application. Here target being non-maneuvering endo and exo atmospheric ballistic RV, State Dependent Coefficient (SDC) formulation for ballistic target has been used. Research carried out using ths state model is being discussed now. Dwivedi *et al.* (2006) carried out accurate state estimation of non-maneuvering RV using SDC formulation from noisy seeker measurements using EKF with to reduce onboard computational burden. Also they have estimated ballistic coefficient β along with state variables. In parallel Bhale *et al.* (2006) validated consistency of above formulation by mechanizing DD formulation. Also they estimated β more accurately to estimate target acceleration. Later they improved the model to quaternion error free estimation of state variables using EKF, UKF and DD formulation. This new formulation became quite effective in estimating the target states accurately and thereby improving the mission performance substantially even in presence of large navigation errors in the attitude quaternion. Later they further generalized the approach for more accurate state estimation of a generic RV under spiraling as well as pure ballistic motion without any change of the target model and the tuning parameters. Also they developed a new nonlinear model predictive zero-effort-miss based guidance algorithm to achieve near zero-effort-miss with estimated states as inputs form this generic estimator. This information is then used for computing the necessary lateral acceleration. Extensive six-degrees-of-freedom (6 DOF) simulation experiments, which include noisy seeker measurements, a nonlinear dynamic inversion based autopilot for the interceptor along with appropriate actuator model with limiters and sensor models showed near-zero miss distance. So state variables in final model to estimate the states of ballistic or maneuvering reentry are relative position, relative velocity components between pursuer and evader, ballistic coefficient, maneuvering coefficient Z_0 and barrel roll frequency ω of the target. Complete research are documented by the authors in unclassified literature [43], [64]–[68] .

V. Conclusion and Future of RV tracking

In present review paper the authors have discussed radar and seeker based EKF and IMM estimators for tracking RV and MaRV. Scope exists in design IMM estimator to estimate directly the ballistic coefficients based on Dwivedi *et al.* [43] . Also in IMM transition probabilities are fixed based on the prior information which may be inaccurate apriori. To solve this problem, a Bayesian based online correction function has been proposed by [69] in their paper. They have adaptively adjusted the transition probabilities. Through this process state estimation error of maneuvering target reduces. So based on this paper there is enough scope to improve RV and MaRV tracking performance. Very recently Li *et al.* proposed a learning based method for vehicle state estimation. They combined model based EKF with Gaussian process regression (GPR). By establishing a dynamic model of the vehicle and describing the errors using machine learning techniques, the sources of state estimation errors were analyzed. Furthermore, by considering the model uncertainty, an error prediction model was developed through GPR learning from real-world vehicle data. Combined with EKF, the proposed method enables high-precision online estimation of vehicle state [70] . So plenty of scope exists on reducing state estimation using Machine Learning (ML) technique. This rode map based on ML is wide open for state estimation in future.

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TABLE I: **Dynamical Models of Radar Based RV Tracking from the Measurements** (R, A, E) (Section III-A)

Estimator	State Variables	Model Feature
EKF, IEKF , 2nd-order EKF [19]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta)^T$	RV kinematic equations for position, velocity components, Constant ballistic coefficient with additive noise.
EIKF [20]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta, \beta_0, \beta_1, \rho/\beta)^T$	RV kinematic equations for position, velocity components, ballistic coefficient second order model, relation between density and ballistic coefficient driven by white noise.
EKF [71]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z})^T$	MaRV CA Model (Polynomial Filter). White noise driven position, velocity and acceleration components .
	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \alpha)^T$	RV Constant Drag Model (BRV Filter). White noise driven position, velocity components and ballistic coefficient.
	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \alpha, \beta, \gamma)^T$	MaRV Constant Drag, Lift Model (MaRV Filter). Consists of (position, velocity) components with noisy (ballistic, maneuver) coefficients. Adaptive filtering carried out after maneuver detection.
EKF, IMM [21], [22]	CA, BRV, MaRV Models	Multiple Maneuver Model (M^3-RV Filter). Fusion, smoothing of all elemental models output using IMM
KF with Input Estimation [23]	$\mathbf{X} = (x, y, \dot{x}, \dot{y})^T$, $\mathbf{U} = (0, 0, \ddot{x}, \ddot{y})^T$	State model consists of position and velocity components (2 D). Input is acceleration component. Jointly states and inputs are estimated based on output residual match and maneuver detection adaptively.
KF with Input Estimation [24]	$\mathbf{X} = (r, \dot{r}, \ddot{r})^T$, $\mathbf{U} = (0, 0, u)^T$	It is 1 D tracking problem consisting of (position, velocity, acceleration) components. A bank of KFs with different inputs are propagated in parallel. By hypothesis testing, adaptively input estimation is carried out and best model is selected.
EKF with input estimation [25], [26]	$\mathbf{X} = (x, y, z, \dot{x}, \dot{y}, \dot{z})^T$, $\mathbf{U} = (0, 0, 0, \ddot{x}, \ddot{y}, \ddot{z})^T$, constant β.	State model of position, velocity and acceleration components as input. Jointly states, inputs are estimated adaptively based on output residual match and maneuver detection. Flat non rotating earth assumed.

TABLE II: **Dynamical Models of Radar Based RV Tracking from the Measurements (R,A,E) (Continued)** (Section III-A)

Estimator	State Variables	Model Feature
EKF, IMM [27]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z})^T$	Boost phase tracking. Model consists of kinematic position, velocity and acceleration components driven by white noise.
	$(x, y, z, \dot{x}, \dot{y}, \dot{z})^T$	Ballistic phase (exo-atmospheric) tracking. Model consisting of position and velocity components consisting of gravity and Coriolis terms. All states are driven by white noise.
	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, 1/\beta)^T$	Reentry (endo-atmospheric) phase tracking. Model consisting of states of ballistic phase along with inverse of ballistic coefficient driven by white noise. Simulation has been carried out along ENV frame. Spherical and rotating earth assumed.
EKF, CADET, UKF, PF [28]	$(x, y, \dot{x}, \dot{y})^T$	Tracking RV in 2-D frame work based on radar measurements. Input is g with additive noise. β is parameter of model assumed known apriori. Simulation favours EKF.
EKF, IEKF, UKF [32], [33]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta)^T$	Tracking RV in 3-D frame work based on radar measurements. β is estimated and all states are driven by additive white noise. Spherical and rotating earth assumed.
EKF, IMM [27], [32], [33]	Different state models (boost, ballistic,reentry phase) [27] .	IMM tracking performance is best. Spherical and rotating earth assumed.
EKF, IMM [34]	$\left(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta, \dot{\beta}\right)^T$	Tracking RV in spherical earth model consisting of gravitational, Coriolis and centripetal components. CAF model to accommodate both boosting and reentry dynamics
	$(x, y, z, \dot{x}, \dot{y}, \dot{z})^T$	BA model to incorporates gravitational, Coriolis and centripetal forces during exo-atmospheric phase.
	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z})^T$	Standard AAS [10] model driven by white noise. It accommodates transition between different phases , possibility of reentry tumbling and off-nominal trajectories as failure mode.

TABLE III: **Dynamical Models of Radar Based RV Tracking from the Measurements (R,A,E) (Continued)** (Section III-A)

Estimator	State Variables	Model Feature
EKF, UKF [35]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta)^T$	Tracking MaRV using self-adaptive filtering by maneuver detection and adjust process noise covariance. Reported UKF tracking better than EKF.
IMM-UKF [36]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta)^T$	Tracking RV with IMM. Total modes were six with varying β within given bound. Final estimate is weighted average of all modes.
β-IMM-EKF [38], [39]	$(x, y, z, \dot{x}, \dot{y}, \dot{z})^T$	Tracking of RV (β-model based IMM-EKF). Two cases based on different values of β as β-IMM1 and β-IMM2 studied. Later evolved β-VSIMM for best tracking performance (rotating spherical earth, ECEF frame).
IMM-CKF [40]	$(V, \theta, \sigma, r, \lambda, \phi, \alpha, \nu)^T$	Tracking of maneuvering HGV under spherical rotating earth using MAM model driven by white noise. Best tracking performance.
	$(V, \theta, \sigma, r, \lambda, \alpha)^T$ Model1 $(V, \theta, \sigma, r, \lambda)^T$ Model2	HGIMM has a) Model1 with three models (fixed ν) b) Model2 with nine models (fixed (α, ν)). States are driven by white noise.
	$(V, \theta, \sigma, r, \lambda, \alpha)^T$	AGIMM consisting of VSMM algorithms.**Q** is estimated adaptively.
ACKF [41]	$\left(h, \theta, \lambda, V, \gamma, \xi, \beta, \frac{L}{D}\right)^T$	Tracking HGV in Mars. Measurements are $(q, \theta, \lambda, \gamma,)$. State model is driven by white noise to handle $\left(\beta, \frac{L}{D}, \rho\right)$ uncertainties. Rotating spherical planet model used.
IEKF [44]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, K_D, K_N, K_P)^T$	Tracking MaRV (Nonlinear Markov Acceleration models) to track (a_D, a_N, a_P). Adaptation is based on maneuver detection logic.

TABLE IV: **Dynamical Models of Radar Based RV Tracking from the Measurements (R,A,E) (Continued)** (Section III-A)

Estimator	State Variables	Model Feature
EKF [43], [45], [51], [71]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \alpha_v, \alpha_t, \alpha_c)^T$	Tracking MaRV with MM (MaRV Model). Kinematic model consisting of position, velocity components along with drag, side force and normal force parameters driven by white noise.
EKF [43], [45], [51]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \alpha_v, \alpha_t, \alpha_c, \omega)^T$	Tracking MaRV with SM (Spiral Model). Kinematic model consisting of position, velocity components along with along with drag, side force and normal force parameters and oscillation frequency driven by white noise.
EKF [45], [52]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \alpha_v, \alpha_l, k)^T$	Tracking MaRV with SM (Spiral Model). Kinematic model consisting of position, velocity components along with white noise triven Drag and resultant lift parameters, Bank angle.
EKF [44], [45]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \alpha_v, \alpha_t, \alpha_c)^T$	Tracking MaRV with GM. Kinematic model inclusive of drag, side force and normal force parameters driven by white noise.

REFERENCES

[1] Ohlmeyer E J and Phillips C A. Generalized vector explicit guidance. *Journal of Guidance Control and Dynamics*, 29:261–268, 2006. http://doi.org/10.2514/1.14956.

[2] Zarchan P. *Tactical and Strategic Missile Guidance*. AIAA Inc., sixth edition, 2012.

[3] Zarchan Paul. Tracking and intercepting spiraling ballistic missiles. *IEEE Position Location and Navigation Symposium Proceeding*, 2000.

[4] Gelb A. *Applied Optimal Estimation*. MIT Press, Cambridge, MA, first edition, 1974.

[5] Simon D. *Optimal State Estimation: Kalman, H_∞ and Nonlinear approaches*. Wiley Inter science, John Willey and Sons Inc., first edition, 2006, pp. 212-223.

[6] Kalman R E. A new approach to linear filtering and prediction problems. *Transaction of ASME, Journal of Basic Engineering*, 82:34–45, 1960.

[7] Kalman R E and Bucy R S. New results in linear filtering and prediction theory. *Transaction of ASME, Journal of Basic Engineering*, 83D:95–108, 1961.

[8] Julier S, Uhlmann J K, and Durrant-Whyte H F. New method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transaction on Automatic Control*, 45(3), March 2000.

[9] Arasaratnam I and Haykin S. Cubature kalman filters. *IEEE Transactions on Automatic Control*, 54:1254–1269, 2009. http://doi.org/10.1109/TAC.2009.2019800.

[10] Singer R A. Estimating optimal tracking filter performance for manned maneuvering targets. *IEEE Transactions on Aerospace and Electronic Systems*, AES-6:0–483, 1970. http://doi.org/10.1109/taes.1970.310128.

[11] Mehrotra K and Mahapatra P R. A jerk model for tracking highly maneuvering targets. *IEEE Transactions on Aerospace and Electronic Systems*, 33:1094–1105, 1997. http://doi.org/10.1109/7.624345.

[12] Mahapatra P R and Mehrotra K. Mixed coordinate tracking of generalized maneuvering targets using acceleration and jerk models. *IEEE Transactions on Aerospace and Electronic Systems*, 36:992–1000, 2000. http://doi.org/10.1109/7.869519.

[13] Y Bar-Shalom and X R Li. *Estimation and Tracking, Principles, Techniques and Software*. Artech House, Boston, first edition, 1993, pp. 417-483.

[14] Bar-Shalom Y and Birmiwal K. Variable dimension filter for maneuvering target tracking. *IEEE Transactions on Aerospace and Electronic Systems*, AES-18:0–629, 1982. http://doi.org/10.1109/taes.1982.309274.

[15] Bar-Shalom Y, Chang K C, and Blom H A P. Tracking a maneuvering target using input estimation versus the interacting multiple model algorithm. *IEEE Transactions on Aerospace and Electronic Systems*, 25:296–300, 1989. http://doi.org/10.1109/7.18693.

TABLE V: **Dynamical Models of Seeker Based RV Tracking from Measurements along (gimbal, LOS) frame** (Section III-B)

Estimator	**State Variables**	**Model Feature**
EKF, UKF, DD [64], [65]	$(\Delta x, \Delta_y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, 1/\beta)^T$	Tracking ballistic RV with model of position, velocity components, β. Seeker measurements are along gimbal frame.
relative EKF, UKF, [43], [66]–[68]	$(\Delta x, \Delta_y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, 1/\beta, Z_0, \omega)^T$	Tracking MaRV with spiraling motion. Model consists of position, velocity components, β, Z_0 and ω driven by white noise. Seeker measurements are along gimbal frame.
		Estimator estimates (a_{tx}, a_{ty}, a_{tz}) with moderate accuracy. Helps to apply advanced guidance law to improve miss-distance performance.
EKF [37], [46], [47]	$(\Delta x, \Delta_y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, 1/\beta, \gamma)^T$	Tracking ballistic RV with Direct Acceleration (DA) model. Model consisting of position, velocity components, White noise driven β and γ. Seeker measurements are along gimbal frame.
EKF [48], [49]	$(x, y, z, \dot{x}, \dot{y}, \dot{z}, \beta)^T$	Tracking ballistic RV with a generic method for initial choice of the filter tuning parameters has been evolved. Seeker measurements are along gimbal frame.
EKF, UKF, NSKF, CKF [43], [50]	$(\Delta x, \Delta_y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, 1/\beta, Z_0, \omega)^T$	Tracking MaRV with spiraling motion, with NSKF performance is best. Seeker measurements are along gimbal frame.
EKF, UKF [51]	$\left(\Delta x, \Delta_y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, Z_1, \dot{Z}_1, D_1, \omega\right)^T$	Tracking MaRV with spiraling motion. Seeker measurements are along LOS frame.
EKF, UKF, PF [42], [52], [55]	$(\Delta x, \Delta_y, \Delta z, \Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}, Z_1, Z_2, D_1, \omega)^T$	Tracking spiraling MaRV, Seeker measurements along LOS frame.

[16] Cloutier J R, Lin C F, and Yang C. Enhanced variable dimension filter for maneuvering target tracking. *IEEE Transactions on Aerospace and Electronic Systems*, 29:786–797, 1993. http://doi.org/10.1109/7.220930.

[17] Mazor E, Averbuch A, Bar-Shalom Y, and Dayan J. Interacting multiple model methods in target tracking: a survey. *IEEE Transactions on Aerospace and Electronic Systems*, 34:103–123, 1998. http://doi.org/10.1109/7.640267.

[18] Kirubarajan T and Bar shalom Y. Kalman filter versus IMM estimator: when do we need the latter? *IEEE Transactions on Aerospace and Electronic Systems*, 39:1452–1457, 2003. http://doi.org/10.1109/TAES.2003.1261143.

[19] Mehra R. A comparison of several nonlinear filters for reentry vehicle tracking. *IEEE Transactions on Automatic Control*, 16:307–319, 1971. http://doi.org/10.1109/tac.1971.1099744.

[20] Siouris G M, Guanrong Chen, and Jianrong Wang. Tracking an incoming ballistic missile using an extended interval kalman filter. *IEEE Transactions on Aerospace and Electronic Systems*, 33:232–240, 1997. http://doi.org/10.1109/7.570753.

[21] Kameda H, Tsujimichi S, and Kosuge Y. Target tracking for maneuvering reentry vehicles using multiple maneuvering models. *(29-31) July, International Session Papers - Tokushima, Japan*, 1997.

[22] Hiroshi Kameda, Shingo Tsujimichi, and Yoshio Kosuge. Target tracking for maneuvering reentry vehicles using multiple maneuvering models. *Electronics and Communications in Japan (Part I: Communications))*, 83:84–92, 2000. http://doi.org/10.1002/1520-6424

[23] Chan Y T, Hu A G C, and Plant J B. A kalman filter based tracking scheme with input estimation. *IEEE Transactions on Aerospace and Electronic Systems*, AES-15:237–244, 1979. http://doi.org/10.1109/TAES.1979.308710.

[24] Bogler P L. Tracking a maneuvering target using input estimation. *IEEE Transactions on Aerospace and Electronic Systems*, AES-23:298–310, 1987.

[25] Lee Sou-Chen, Huang Yu-Chao, and Liu Cheng-Yu. Trajectory estimation for tactical ballistic missiles in terminal phase using on-line input estimator. *Proceeding of National Science Council*, 23:644–653, 1999.

[26] Lee Sou-Chen and Cheng-Yu Liu. Trajectory estimation of reentry vehicles by use of on-line input estimator. *Journal of Guidance Control and Dynamics*, 22:808–815, 1999. http://doi.org/10.2514/2.4482.

[27] Cooperman R L. Tactical ballistic missile tracking using the interacting multiple model algorithm. *Proceedings of the Fifth International Conference on Information Fusion, FUSION 2002, Annapolis, MD, USA*, July 2002. doi:10.1109/icif.2002.1020892.

[28] Farina A, Ristic B, and Benvenuti D. Tracking a ballistic target: comparison of several nonlinear filters. *IEEE Transactions on Aerospace and Electronic Systems*, 38:854–867, 2002. http://doi.org/10.1109/taes.2002.1039404.

[29] Rong Li X and Jilkov V P. Survey of maneuvering target tracking. part IMM. dynamic models. *IEEE Transactions on Aerospace and Electronic Systems*, 39:0–1364, 2003. http://doi.org/10.1109/taes.2003.1261132.

[30] Rong Li X and Jilkov V P. Survey of maneuvering target tracking. part V. multiple-model methods. *IEEE Transactions on Aerospace and Electronic Systems*, 41:0–1321, 2005. http://doi.org/10.1109/taes.2005.1561886.

[31] Lawton J A, Jesionowski R J, and Paul Zarchan. Comparison of four filtering options for a radar tracking problem. *Journal of Guidance Control and Dynamics*, 21:618–623, 1998. http://doi.org/10.2514/2.4280.

[32] Minvielle P. Tracking a ballistic re-entry vehicle with a sequential Monte-Carlo filter. *Proceedings, IEEE Aerospace Conference - Big Sky, MT, USA*, March 2002. doi:10.1109/aero.2002.103689.

[33] Minvielle P. Decades of improvements in re-entry ballistic vehicle tracking. *IEEE Aerospace and Electronic Systems Magazine*, 20:CF/1–CF14, 2005. http://doi.org/10.1109/maes.2005.1499296.

[34] Farrell W. Interacting multiple model filter for tactical ballistic missile tracking. *IEEE Transactions on Aerospace and Electronic Systems*, 44:418–426, 2008. http://doi.org/10.1109/TAES.2008.4560196.

[35] Chen Y, Cheng Z, and Wen Shuliang. Nonlinear filters for tracking maneuverable ballistic missile targets on reentry. *20-22 April, Guilin, China*, 2009.

[36] Kim S H and Choi H L. An IMM-based method for reentry-phase tracking of unknown ballistic missiles. *ICAS 20013, October 20-23, 2013, Kimdaejung Convention Center, Gwangju, Korea*, 2013.

[37] Ghosh S and Mukhopadhyay S. Tracking reentry ballistic targets using acceleration and jerk models. *IEEE Transactions on Aerospace and Electronic Systems*, 47:666–683, 2011. http://doi.org/10.1109/taes.2011.5705698.

[38] Moon K R, Kim T H, and Song T L. Comparison of ballistic-coefficient-based estimation algorithms for precise tracking of a re-entry vehicle and its impact point prediction. *Journal of Astronomy and Space Science*, 29:363–374, 2012. http://dx.doi.org/10.5140/JASS.2012.29.4.363.

[39] Kim T H, Moon K R, and Song T L. Variable-structured interacting multiple model algorithm for the ballistic coefficient estimation of a re-entry ballistic target. *International Journal of Control, Automation and Systems*, 11:1204–1213, 2013. http://doi.org/10.1007/s12555-013-0003-2.

[40] Fan Yu, Zhu W, and Bai G. A cost-effective tracking al-

gorithm for hypersonic glide vehicle maneuver based on modified aerodynamic model. *Applied Sciences*, 6:312, 2016. http://doi.org/10.3390/app6100312.

[41] Sun Tao and Xin Ming. Hypersonic entry vehicle state estimation using nonlinearity-based adaptive cubature kalman filters. *Acta Astronautica*, 134:221–230, 2017. http://doi.org/10.1016/j.actaastro.2017.01.036.

[42] Kim J, Vaddi S S, Menon P K, and Ohlmeyer E J. Comparison between nonlinear filtering techniques for spiraling ballistic missile state estimation. *IEEE Transactions on Aerospace and Electronic Systems*, 48:313–328, 2012. http://doi.org/10.1109/taes.2012.6129638.

[43] Dwivedi P N, Bhale P G, Bhattacharyya A, and Padhi R. Generalized state estimation and model predictive guidance for spiraling and ballistic targets. *Journal of Guidance Control and Dynamics*, 37:243–264, 2014. http://doi.org/10.2514/1.60075.

[44] Hough Michael E. Reentry maneuver estimation using nonlinear markov acceleration models. *Journal of Guidance Control and Dynamics*, pages 1–18, 2017. http://doi.org/10.2514/1.G002577.

[45] Zhang Kai1, Xiong Jiajun, and Fu Tingting. Coupled dynamic model of state estimation for hypersonic glide vehicle. *Journal of Systems Engineering and Electronics*, 29:12841292, December 2018. DOI: 10.21629/JSEE.2018.06.15.

[46] Bhattacharyya A K, Bhattacharya S, Garai T, and Mukhopadhyay S. Modeling of RF seeker dynamics and noise characteristics for estimator design in homing guidance applications. *December 8-10, Kharagpur, INDIA*, 2008.

[47] Ghosh S, Bhattacharyya A K, Mukhopadhyay S, and Routray A. Improved estimation of kinematic state of reentry ballistic targets with modeling of colored and nonstationary seeker noise. *IEEE Transactions on Aerospace and Electronic Systems*, 48:3186–3206, 2012. http://doi.org/10.1109/TAES.2012.6324691.

[48] Roy S, Ghosh R, and Goswami B. Comparison of cartesian and polar estimates of the EKF for different choices of the robustness and sensitivity metrics. *IFAC Conference Proceedings of Advances in Control and Optimization of Dynamical Systems, March 13-15, Kanpur, India*, 2014.

[49] Saha Manika, Ghosh Ratna, and Goswami Bhaswati. Robustness and sensitivity metrics for tuning the extended kalman filter. *IEEE Transactions on Instrumentation and Measurement*, 63:964–971, 2014. http://doi.org/10.1109/TIM.2013.2283151.

[50] Aaastha Dak and Rahul Radhakrishnan. Tracking and interception od a spiralling ballistic target on reeentry. *Open Access IFAC Paper*, pages 339–344, 2022.

[51] Vaddi S, Menon P K, and Ohlmeyer E. Target state estimation for integrated guidance-control of missiles. *Guidance, Navigation and Control Conference Proceeding, 20 - 23 August, Hilton Head, South Carolina, AIAA-2007-6838-CP*, 2007.

[52] Kim J, Vaddi S, Menon P K, and Ohlmeyer E. Comparison between three spiraling ballistic missile state estimators. *Guidance, Navigation and Control Conference Proceeding, 18 - 21 August, Honolulu, Hawaii, AIAA-2008-7459-CP*, 2008.

[53] Kim J, Menon P, and Ohlmeyer E. Motion models for use with the maneuvering ballistic missile tracking estimators. *Guidance, Navigation, and Control Conference Proceeding, 2 - 5 August, Toronto, Ontario Canada, AIAA-2010-7589-CP*, 2010.

[54] Kim Jinwhan, Tandale Monish, P K Menon, and Ohlmeyer Ernest. Particle filter for ballistic target tracking with glint noise. *Journal of Guidance Control and Dynamics*, 33:1918–1921, 2010. http://doi.org/10.2514/1.51000.

[55] Ohlmeyer E J and Menon P K. Tracking of spiraling reentry vehicles with varying frequency using a new target dynamic model. *Proceedings of the Institution of Mechanical Engineers Part G Journal of Aerospace Engineering*, 228:1235–1245, 2014. http://doi.org/10.1177/0954410013487296.

[56] Praveen Waugh, Ajit B Chaudhury, and R N Bhatterjee. Performance evaluation of (IMM) tracker in command guided (SAMM) system. *National Systems Conference, IIT-Roorkee*, 2008.

[57] Ching-Fang Lin. *Modern Navigation, Guidance, And Control Processing*, volume Two. American GNC Corporation, PTR Prentice Hall, New Jersey, first edition, 1991.

[58] Hablani Hari. Gaussian second-order filter for proportional navigation of exoatmospheric interceptors with angles-only measurements. *Guidance, Navigation and Control Conference Proceeding, AIAA-98-4217-CP*, 1998.

[59] Ekstrand B. Tracking filters and models for seeker applications. *IEEE Transactions on Aerospace and Electronic Systems*, 37:965–977, 2001. http://doi.org/10.1109/7.953250.

[60] Ananthasayanam M R, Sarkar A K, Bhattacharys A K, and Srivastava R. Estimation of LOS rates and angles using EKF from noisy seeker measurements. *International Conference on Signal Processing and Communications, 2004. SPCOM-04, IISc, Bangalore, India*, 2004. DOI: 10.1109/spcom.2004.1458515.

[61] Ananthasayanam M R, Sarkar A K, Bhattacharyaa, Tiwari P K, and Vohra Prashanth. Nonlinear observer state estimation from seeker measurements and seeker-radar measurements fusion. *Guidance, Navigation and Control Conference Proceeding, AIAA-2005-6066-CP*, 2005.

[62] Sarkar A K, Ananthasayanam M R, Srinivasan T, and Kar P K. Comparison of the radar and seeker modes of pursuer guidance. *Journal of Guidance Control and Dynamics*, 32:1912–1920, 2009. http://doi.org/10.2514/1.40404.

[63] Kar P K, Srinivasan T, Sarkar A K, and Padhi R. Real Time Performance of Pursuer Seeker Estimator. *AIAA Guidance, Navigation, and Control Conference and Exhibit, Totonto, Ontario, Canada*, AIAA 2010-7587, August 2010. DOI: 10.2514/6.2010-7587.

[64] Dwivedi P N, Bhale P G, Bhattacharyaa A, Tiwari P K, and Kumar Prem. Quick and accurate state estimation of RV from RF seeker measurements using EKF. *Guidance, Navigation and Control Conference Proceeding*, 2006.

[65] Bhale P G, Dwivedi P N, Kumar Prem, and Bhattacharyaa A. Estimation of ballistic coefficient of reentry vehicle with divided difference filtering using noisy RF seeker data. *Mumbai, India*, 2006.

[66] Dwivedi P, Bhale P, Bhattacharyya A, and Radhi R. State estimation of spiralling target with UKF with generalized sinusoidal target acceleration model. *Guidance, Navigation, and Control Conference Proceeding, 13 - 16 August, Minneapolis, Minnesota, AIAA-2012-4891-CP*, 2012.

[67] Dwivedi P, Bhale P, Bhattacharyya A, and Padhi R. Quaternion error free estimation of reentry ballistic targets. *IEEE Multi-Conference on Systems and Control Hyderabad, India, August 28-30*, 2013.

[68] Dwivedi P N, Bhale P, Bhattacharyya A, and Padhi R. Generalized estimation and predictive guidance for evasive targets. *IEEE Transactions on Aerospace and Electronic Systems*, 52:21111–2122, 2016. http://doi.org/10.1109/TAES.2016.140861.

[69] Guo Xie, Lanlan Sun, Tao Wen, Xinhong Hei, and Fucai Qian. Adaptive transition probability matrix-based parallel imm algorithm. *IEEE Transactions on Systems, Man and CYbernatics: SYSTEMS*, 51(5):2980 – 2989, May 2021.

[70] Qin Li, Hongwen He, Xiaokai Chen, and Jianping Gao. Learning-based vehicle state estimation using gaussian process regression combined with extended kalman filter. *Journal of the Franklin Institute*, 361(9), June 2024.

[71] Chang C B, Athans M, and Whiting R. On the state and parameter estimation for maneuvering reentry vehicles. *IEEE Transactions on Automatic Control*, 22:99–105, 1977. http://doi.org/10.1109/TAC.1977.1101412.