Extended State Observer-Theory, Advances and Applications

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Abstract— **Presence of uncertainties and disturbances which include unmodeled dynamics, external disturbance, system nonlinearities and parametric perturbations in practical engineering systems poses formidable challenge in robust control design for such systems. Design utilizing disturbance estimation and rejection provides one practical strategy that offers a viable solution. The Extended State Observer (ESO) has been widely used prominent technique which can enable estimation of the disturbance acting on the plant. This article reviews the theory of ESO, advances being taken place in its improvement and its applications in varied engineering applications.**

Keywords— Active disturbance rejection control, Uncertainty and disturbance estimation, Extended state observer, Robust control

I. INTRODUCTION

In general, controller performance gets adversely affected due to various reasons such as presence of parameter perturbations, system nonlinearities, unmodelled dynamics, external disturbances and measurement errors etc. Therefore, control design for such uncertain systems offers a significant challenge. One viable strategy for design of robust controller for uncertain systems is to estimate the effect of disturbances and uncertainties and reject the same by augmenting the controller designed for nominal system [1]. To this end, various disturbance estimation approaches such as classical Disturbance Observer (DOB) [2], Nonlinear Disturbance Observer (NDOB) [3], Unknown Input Observer (UIO) [4], Perturbation Observer (PO) [5], Time Delay Control, (TDC) [6], Equivalent Input Disturbance (EID) approach [7], the Uncertainty and Disturbance Estimator (UDE) [8] among others, have been proposed in the literature.

One prominent approach for estimation of disturbance is Extended State Observer (ESO) which forms an integral part of the Active Disturbance Rejection Control (ADRC) strategy proposed for robust control design [9]–[12].

The ESO treats effect of all components that adversely affect the plant such as system nonlinearities, parametric perturbations, un-modeled dynamics, and external disturbances as a composite disturbance that needs to be estimated. As it estimates this total disturbance as an augmented state, it is called as Extended State Observer. The ESO estimates effects of uncertainties and disturbances acting on the system along with the system states, making it possible to reject the disturbances. A performance comparison study of three advanced state observers namely High Gain Observer, ESO and Sliding Mode Observer is presented in [13] wherein the authors have shown that over all the ESO is superior in dealing with the uncertainties, disturbances and sensor noise.

The objective here is to present basic concept of the ESO, the advancements taken place in its improvement and its applications in varied engineering systems.

II. EXTENDED STATE OBSERVER

A. Concept of ESO

To illustrate the concept of ESO and for simplicity, consider a second order SISO nonlinear dynamics given by

$$\ddot{y} = a(y, \dot{y}, w) + bu \tag{1}$$

Where $a(.)$ is the plant and the disturbance dynamics, $w(t)$ represents an unknown external disturbance, u is the control input, and y is the measured output. Now assuming $b = b_o + \Delta b$ where b_o is the known available value of b while Δb is its uncertainty. Defining the net disturbance as $d \approx a(.) + \Delta bu$, (1) takes the form as

$$\ddot{y} = d + b_o u \tag{2}$$

It is straightforward to verify that the controller designed as

$$u = \frac{1}{b_o}(-d + k_1(y_r - y) + k_2(\dot{y}_r - \dot{y}) + \ddot{y}_r) \tag{3}$$

Where $y_r, \dot{y}_r$ and $\ddot{y}_r$ are the reference output and its derivatives respectively. Defining $e = y_r - y$, the closed-loop dynamics using the control law of (3) is

$$\ddot{e} + k_2\dot{e} + k_1 e = 0 \tag{4}$$

From (4), it is obvious that the tracking error in output converges to zero asymptotically. The controller, however, is not implementable as it needs the measurement/availability of the disturbance d as well as states of the plant. To address the issue, an ESO is formulated as follows:

The dynamics of Eq. (2) can be written in state-space model form as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = d + b_o u \quad (5)$$
$$y = x_1$$

Where x_i are the states. Treating d as an augmented state, i.e., $x_3 = d$, Eq. (5) is re-written as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 + b_o u$$
$$\dot{x}_3 = f$$
$$y = x_1$$
(6)

where f represents the derivative of the disturbance, i.e., $f = \dot{d}$ which is considered to be unknown but bounded function. However, owing to d being a component of state vector, a state observer can be designed for its estimation. Towards this, consider an observer formulated as

$$\dot{\hat{x}}_1 = \hat{x}_2 + \beta_1 g_1(e)$$
$$\dot{\hat{x}}_2 = \hat{x}_3 + \beta_2 g_2(e) + b_o u$$
$$\dot{\hat{x}}_3 = \beta_3 g_3(e)$$
$$\hat{y} = \hat{x}_1$$
(7)

Where $e = y - \hat{y} = x_1 - \hat{x}_1$ and $\hat{x}_3$ is the disturbance estimate. The quantities β_i are the observer gains whereas $g_i(i)$ are the appropriately designed nonlinear gain functions such that $e g_i(e) > 0, \forall e \neq 0$ and $g_i(0) = 0$. If $g_i(.)$and their parameters are selected appropriately, converegnce of the state estimates, $\hat{x}_i$, to their respective true values x_i, i.e $\hat{x}_i \to x_i$; $i = 1, 2, 3$ can be achieved. The selection of the nonlinear function forms an important aspect in the design of ESO. The general expression for these functions, selected heuristically based on experimental results [13] is

$$g_i(e, \alpha_i, \delta) = \begin{cases} |e|^{\alpha_i} sign(e), & |e| > \delta \\ \frac{e}{\delta^{1-\alpha_i}}, & |e| \leq \delta \end{cases} \quad i = 1,2,3$$
(8)

where $\delta > 0$.

It is obvious that the function $g_i(.)$ yields a linear interval near the origin. One important property of these functions is that for $0 < \alpha_i < 1$, $g_i(.)$ assumes a relatively high gain for small error and a smaller gain for large error. The parameter δ represents a small number and is used to limit the gain near the origin while defining the range of the error corresponding to high gain.

Having the estimate of states as well as disturbance, the control law of (3) takes a form as

$$u = \frac{1}{b_o}(-\hat{x}_3 + k_1(x_{1r} - \hat{x}_1) + k_2(x_{2r} - \hat{x}_2) + \ddot{x}_{1r})$$
(9)

Where $x_1 = y_r, x_1 = \dot{y}_r$ and $\ddot{x}_{1r} = \ddot{y}_r$ and is now implementable

B. Linear ESO

The observer given by Eq. (7) with the gains of Eq. (8) is designated as the nonlinear ESO (NESO) since as it uses nonlinear gain functions. However, if one considers $g_i(i) = e$, the ESO takes assumes the form of the conventional Luenberger observer and is referred to as the linear ESO (LESO). It is worth to note that the LESO essentially is a special case of the nonlinear ESO if one choses the NESO parameters, α_is, as unity.

In general, the NESO performs well in applications, however choice of the observer parameters for a given application can pose a difficulty. It also difficult to provide rigorous theoretical derivation for closed loop stability and observer error dynamics when an NESO is employed owing to its nonlinear structure. On the other hand, the LESO provides few advantages. Firstly, observer gains can be computed using well-established technique such as pole placement whereas the design variables in NESO are normally obtained by following a trial and error procedure. Next from hardware implementation point of view, the LESO is easy to implement as shown in [14].

C. Observer Error Dynamics

To establish observer error dynamics stability, the extended order system given by Eq. (6) is written as

$$\dot{x}_e = A_e x_e + B_e u + B_f f$$
$$y = C_e x_e \quad (10)$$

where x_e is the state vector of the extended order system, i.e., $x_e = [x_1\ x_2\ x_3]^T$ and

$$A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \ B_e = [0\ b_o\ 0]^T; \ B_{ef} = [0\ 0\ 0]^T;$$
$$C_e = [1\ 0\ 0] \quad (11)$$

Here it may be noted that the term f is present in the state equation. Since a linear ESO is used in the present treatment, it can be obtained by setting $g_i(i) = e$ in the nonlinear ESO (7). Towards this, the resulting observer dynamics is written as

$$\dot{\hat{x}}_e = A_e \hat{x}_e + B_e u + Le$$
$$\hat{y} = C_e \hat{x}_e \quad (12)$$

where $e = y - \hat{y} = x_1 - \hat{x}_1$ while $\hat{x}_e$ are the estimate of the extended state vector, x_e. The observer gain L vector is given by

$$L = [\beta_1\ \beta_2\ \beta_3]^T \quad (13)$$

Subtracting the state equation of (12) from Eq. (10) yields observer error dynamics as

$$\dot{e}_o = (A_e - LC_e)e_o + B_f f \quad (14)$$

where $e_o = x_e - \hat{x}_e$ is the state observation error vector. As (A_e, C_e) is observable, one can select the observer gain, L, and thus the ESO gains, β_i appropriately to ensure stability for the observer error dynamics with desired transient performance. As the poles of observer error dynamics given by Eq. (14) are placed at desired locations, it can be stated that with the assumption of bounded f, bounded input-bounded output stability for the linear system of Eq. (14) can be assured. Also for $f = 0$, i.e., when the derivative of disturbance is zero, the errors converge to zero asymptotically, i.e., $\lim_{t\to\infty} e_o(t) = 0$. Therefore, similar result is expected if the rate of change of the uncertainty is reasonably small.

Owing to its efficacy in estimation of disturbance, the ESO based ADRC methodology has attracted the attention of researchers and practitioners alike and the progress of its theoretical analysis are reviewed in [15]. Various achievements in design of control laws for uncertain nonlinear systems made possible by ADRC are discussed and the main objective in employing ADRC, that is to estimate the total disturbance accurately that influences the system output adversely has been brought out. The recent results in theoretical development of the ADRC-based control systems are also presented by the authors. Capabilities of the Extended State Observer for Estimating Uncertainties have been studied in [16] whereas frequency domain stability analysis of nonlinear active disturbance rejection control system is presented in [17]. Lastly survey on methods of increasing the efficiency of extended state disturbance observers can be found in [18].

III. ADVANCES IN ESO

A. Generalized Extended State Observer (GESO)

The standard ESO formulation presented in the last section is applicable only for systems that can be represented in integral chain form of Eq. (1). Also the disturbance is considered as matched, i.e., it appears in control input channel only. However, many practical systems do not meet these conditions. The dynamics may not be in the chain integral form and also the disturbances acting on the system may not be matched and hence the standard ESO formulation can not be used for such systems. Addressing this issue, efforts have been devoted to generalize the ESO formulation to enable its application for wider class of dynamical systems and to this end, the formulation of the Generalized ESO (GESO) has been proposed in [19]. The authors have proposed the formulation of GESO for systems which are not in integral chain form subject to uncertainties which are not matched without applying any co-ordinate transformation. By choosing the gain for disturbance compensation, k_d, appropriately, it is brought out that the effect of mismatched disturbances can be mitigated from the output channel.

As an example of the use of GESO, in [20], continuous time predictive control and GESO based pitch autopilot design is presented for a tail controlled missile. For a missile that is stabilized in roll position, the pitch dynamics is represented by

(15)
$$\begin{aligned} \dot{\alpha}(t) &= z_\alpha \alpha(t) + z_\delta \delta(t) + q(t) + d_\alpha \\ \dot{q}(t) &= m_\alpha \alpha(t) + m_q q(t) + m_\delta \delta(t) + d_q \\ \dot{\delta}(t) &= -\frac{1}{\tau}\delta(t) + \frac{1}{\tau}\delta_c(t) \\ a_z(t) &= V[z_\alpha \alpha(t) + z_\delta \delta(t)] \end{aligned} \tag{16}$$

where V, α, q, δ and δ_c are the velocity (m/s), angle of attack (rad), pitch rate (rad/s), and the actual and commanded control surface deflections (rad) respectively. The parameters $z_\alpha, z_\delta, m_\alpha, m_q$ and m_δ are the aerodynamic force and moment derivatives whereas d_α and d_q denote the effect of uncertainties and external disturbance in the respective channel.

Normal acceleration, a_z (m/s^2) as given by (16) is the output to be controlled. Since the missile dynamics is neither in a chain integral form not have matched disturbances, the GESO is used for estimating the state and disturbances and the estimated disturbances are then used to robustify an output tracking predictive controller designed for nominal system. Usefulness of GESO for states and disturbance estimation and to robustify the predictive controller in presence of parametric uncertainties, external disturbances, un-modeled dynamics, and measurement noise is validated by presenting simulation results.

B. Hybrid Extended State Observer (HyESO)

To cater for systems for which the traditional ESO framework is not applicable, GESO has been proposed as discussed in the last subsection. Progressing the work further for generalized systems, a formulation of Hybrid ESO has been presented in [21]. It is shown that in the presence of the multichannel lumped disturbances, the single GESO or EGESO method alone is not capable of providing the estimation of both (matched and mismatched) types of lumped disturbances. The shortcomings of the conventional ESO or GESO based control are addressed by hybridizing the ESO with GESO terming as Hybrid Extended State Observer (HyESO). The proposed HyESO is shown to be effective in estimation of multichannel lumped disturbances (i.e., matched and mismatched) for variety of practical systems. Simulation results and comparative analysis are presented to establish effectiveness of the proposed formulation.

C. Improved Extended State Observer (IESO)

In an ESO based control strategy, the ESO is used to estimate plant states as well as the total disturbance acting on the system. The estimated disturbance is then used to cancel out its effect by augmenting a nominal controller. Obviously satisfactory disturbance compensation needs faster convergence and higher accuracy for the ESO, which may cause the ESO to have a higher gains. However, the higher gains can adversely affect the ESO based control system performance owing to amplification of measurement noise resulting due to high gains of ESO. Addressing the issue, a formulation of Improved ESO (IESO) has been proposed in [22]. In this work, the authors analysed disturbance estimation and noise suppression performance of a traditional ESO in frequency domain and based on it, a novel formulation of the ESO is proposed to segregate the disturbance estimation and state estimation. They proposed a new parameter to adjust the disturbance estimation accuracy instead of using the ESO bandwidth for the purpose resulting in having relatively smaller observer gains. The authors have established that the efficacy in disturbance-estimation is primarily decided by this parameter. Further a low-pass filter is introduced in the observer to mitigate the high frequency components of measurement noise. Results on application to motion control problem and magnetic levitation system are presented to highlight the efficacy of the proposed approach.

D. Cascaded Extended State Observer (CESO)

In [23], a new paradigm to redesign high-gain ESO is proposed in order to improve its performance in the presence of measurement noise. It attempts to address directly the shortcomings associated with the trade-off between estimation accuracy and convergence speed and sensitivity to noise existing in the conventional ESO. The presented formulation is based on splitting of the unknown total disturbance into a pre-decided number of components, each representing certain frequency range signal, and estimation of the split parts by means of a set of cascaded observers. In contrary to the standard implementation of ESO based control designs wherein a single ESO is employed, the proposed approach allows use of higher bandwidths for observer for signal estimation with lesser sensor noise. As the ESOs are used in a cascaded manner, it is designated as cascaded ESO (CESO) which is effective in offering estimate that is fast and accurate while reducing sensor noise amplification and keeping the implementation simple.

E. Higher Order Extended State Observer (HESO)

As the performance of control systems designed using ESO mostly are dictated by the state and disturbance estimation accuracy, it is vital to ensure that they are estimated as correctly as possible. In general, for a traditional ESO, findings show that LESO provides asymptotic convergence for the state and disturbance estimation for the case of constant disturbances only as is evident from (14). In order to achieve asymptotic convergence in cases with time-varying disturbances, higher order ESO (HESO) has been proposed in [24]. It is shown that HESO designed with m augmented states results into observer error dynamics that is asymptotically stable implying that the estimated states converge to their true values asymptotically when the disturbance is a polynomial in time such as $d(t) = d_0 + d_1 t + d_2 t^2 + \cdots d_{m-1} t^{m-1}$, i.e., when the $m - th$ order derivative of $d(t)$ is zero. Performance analysis of HESO for sinusoidal disturbances is presented in [25] wherein the authors have shown that the higher order ESO provides improved tracking of fast-varying sinusoidal disturbances if bandwidth of ESO is selected significantly larger than disturbance frequency while ensuring that it is sufficiently smaller than unmodeled high frequency dynamics.

Increasing order of ESO although offers improvement, the asymptotic convergence cannot be obtained for periodic disturbances such as sinusoids. Further, high-order ESO reduces stability margins and robustness to unmodeled high frequency dynamics.

F. Resonant Extended State Observer (RESO)

As brought out earlier, the conventional ESO does not offer asymptotic convergence when the disturbance is periodic in nature. Addressing the issue, a formulation of modified ESO with integrated sinusoidal disturbance model, designated as Resonant ESO (RESO) is proposed in [26]. The proposed formulation achieves asymptotic convergence for state estimation for periodic disturbances whose resonant frequency is known. The findings presented by authors shows that the control design with RESO offers improved performance in tracking of sinusoid reference and suppressing of sinusoid disturbance when compared with the results using the controllers based on traditional ESO/GESO techniques whenever the reference signal frequency and frequency of disturbance lie within certain band in the vicinity of the nominal one. Further, it is brought out that the proposed structure achieves comparable performance on stability and robustness. Lastly, the authors have presented a realistic method for tuning of parameter that is optimal for the proposed controller based on the RESO.

G. Adaptive Extended State Observer (AESO)

In conventional linear ESO formulation, the observer gains are computed by employing the pole placement approach and are kept usually constant. In [27], an improvement in this has been proposed by setting the observer gains as time varying. Designating the resultant ESO as Adaptive Extended State Observer (AESO), the authors show that the proposed approach significantly expands applications of ESOs to many practical systems.

The AESO is formulated as a linear time-varying dynamics that includes the strength of LESO formulation with well-established theoretical framework and effective performance of the traditional nonlinear ESO (NESO). The observer gains which are time-varying are obtained by transforming the AESO error dynamics into a standard phase-variable form. Time-varying PD-eigenvalues are then assigned for the canonical system based on differential algebraic spectral theory. Results on stability and bounds of estimation error of the AESO are given for unknown disturbances. The results also provide practical guidelines for PD-eigenvalues assignments. The efficacy of the technique has been validated through two representative applications and comparison analysis is conducted among AESO, LESO, and NESO.

IV. APPLICATIONS

Owing to its effectiveness in estimation of state and disturbances in an integrated manner the ESO has been used to address vexed control design problems like systems involving time delay, control of non-minimum phase systems, and fault tolerant design etc. Apart from this, the approach has been extensively used in the design of robust controllers for varied engineering applications.

A. Systems with Time Delay

In [28], the authors have proposed ESO based control design for systems affected with time delays. Dynamical systems affected by time delay in input are treated as higher-order systems having no time delay in the input. The effect of error arising due to approximating the real system as well as other uncertainties are seen as a total disturbance which is estimated by means of the ESO to compensate.

Following the similar line of thinking, an application of ESO in extending the operating range of linear controller by means of ESO is presented in [29]. The higher order terms that are ignored while linearizing the nonlinear dynamical systems are considered as disturbance and the same is observed using ESO to compensate by augmenting the nominal linear controller. The authors have demonstrated the concept through an application to planar pendulum system.

B. Fault Tolerant Control

An ESO based fault diagnosis and fault-tolerant control design is proposed in [30] for systems affected by random disturbance and incipient actuator faults. Treating the effect of incipient fault as well as the disturbance as an additional state, the ESO enables diagnosis of incipient fault without considering the residual. An optimal fault-tolerant and dynamic self-healing control is then achieved through estimation of the incipient faults and disturbance. To bring out the effectiveness and reliability of the presented approach, simulation results are presented. Similarly, an ESO-based fault-tolerant antidisturbance control for air-breathing hypersonic vehicles with variable geometry inlet is presented in [31].

C. System Applications

Since the effects of disturbances and uncertainties apart from the system states are estimated by the ESO in an unified manner, nullifying the influence of disturbance in output as well as meeting the necessity of system states are achieved. Owing to its usefulness and effectiveness in design of robust control systems, controllers designed using ESO have found vast applications in varied fields of engineering systems have been reported in the literature. Applications in Unmanned Aerial Vehicles (UAVs) [32], gun control system [33], hydraulic system [34], [35], hypersonic vehicles [36], [37], piezoelectric smart structures [38], robotics [39], missile Guidance [40]–[42], power electronics [43], spacecraft control [44]–[46], marine vehicles [47], missile autopilot [48], quadrotor trajectory tracking [49], [50], MEMS gyroscope [51], airship trajectory tracking [52], transonic flutter suppression [53], lateral acceleration estimation of maneuverable ballistic targets [54], online estimation of helicopter mass and center of gravity location [55], spacecraft formation system [56], control of swing nozzle of anti-aircraft missile [57], fault tolerant control for air-breathing supersonic missiles [58], obstacle avoidance in quadrotor [59], launch vehicle attitude control [60] are some applications to mention.

V. CONCLUSION

Extended State Observer is an important tool in designing of robust controllers for practical engineering systems as it can provide estimate of the total disturbance comprising the effects of parametric perturbations, unmeasurable disturbances, system nonlinearities and un-modeled dynamics. Apart from the disturbance estimation, the system states as well are estimated in a unified manner enabling implementation of state feedback controllers. This paper attempted to review the basic concept of ESO along with the advances in improving the conventional ESO. Application in varied engineering fields have also been brought out to highlight the usefulness of the observer.

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