Ground based passive orbit determination by TDOA/FDOA method to assist Space Situational Awareness

Devi Prasad Panda P
Scientist E, AOCS & MA,
Directorate of Special Projects, DRDO
Hyderabad, India
devi.prasad.dsp@gov.in

Kalpana Bandi.
Project Scientist C, AOCS & MA,
Directorate of Special Projects, DRDO
Hyderabad, India

doi: https://doi.org/10.37285/bsp.SACAD2025.38

***Abstract*—Satellite Orbit Determination in Low Earth Orbit between 300 km to 1000 km altitudes is crucial for Space Situational Awareness. A proposed method uses ground based passive Radio Frequency (RF) measurements of received signals to determine the position and velocity of the satellite. Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) methods are used to compute position and velocity of the satellite, which can subsequently be used to compute satellite orbital parameters. Effect of various receiver geometries are analyzed on Satellite Position and velocity fix accuracies using CRLB. Based on the errors produced by different geometries of receivers, optimal placements of receivers are suggested for better location fix. Placements of receiver at center in all configurations improving the location fix accuracies. Parametric study performed to finalize optimal distance between receivers and number of receivers. Effect of the satellite altitude on positional accuracies also analyzed.**

***Index Terms*—Passive RF geolocation, Space situational Awareness, TDOA, FDOA, CRLB.**

I. Introduction

Space situational awareness is of significant importance to Defence for the protection of the space systems and to observe maneuver activities of neighboring countries space systems. Traditional observing systems used for SSA such as ground based RADAR and Electro Optical systems can be very expensive to realize and hard to maintain. TDOA and FDOA techniques are well proven for location fixing of emitters. Space systems are bound to emit RF signals to its intended regular operations such as Telemetry, tracking and monitoring. The emitted RF signals of Satellite can be intercepted by another satellite or with ground system. By placing passive receivers on the ground it is possible to intercept the RF signals and information of the emitter orbit and status can be deduced.

Passive RF geolocation of satellite can be done with TDOA, FDOA or both together. TDOA method of geolocation is performed by taking differential time arrival of signal at multiple receiver ground stations. FDOA method of geolocation of satellite is calculated by taking differential frequency arrival of signal at multiple receiver ground stations. Space based emitter detection requires minimum four receivers to solve position and velocity.

Conventional SSA measurement hardware such as RADAR requires high power transmitters and Electro optical payloads requires higher aperture lens with high frame rate sensors to locate the satellites with high degree position accuracy. On the other hand, passive RF location fix requires receiver and antenna. The main drawback of passive RF geolocation is, this method can be used only for active emitters. So debris or any other satellites which are not emitting RF signals cant be detected. Passive RF systems can be used to reduce the SSA traffic load so that non RF emitting bodies around the earth can be detected by RADAR and EO systems. Passive RF systems location fix can be great benefit to SSA for active satellite orbit determination. Accuracy of position and velocity depends on various parameters like number of receivers, receiver geometry, distance between the geometry, emitter altitude and receiver noise.

II. TDOA and FDOA solution with array of receivers

The emitter or source position and velocities are estimated using an algebraic solution with TDOA and FDOA measurements from an array of receivers. The proposed method does not require initial solution guess and involves weighted least-squares minimization only [1]. The performance of scenario is analyzed by calculating the CRLB (Cramer Rao Lower Bound). Scenarios are considered by varying number of receivers and varying distance between receivers location with TDOA and FDOA measurements noises with considering receivers position errors. Range and range rate from emitter to receiver are the main inputs for the algorithm.

A. Basics of Algorithm

Consider three-dimensional (3-D) scenario where an array of M stationary receivers with sensor positions $S_i = [x_i, y_i, z_i]^T$ and velocities $\dot{S}_l = [\dot{x}_l, \dot{y}_l, \dot{z}_l]^T$, i = 1,2, M are known. By using TDOA/FDOA method the position $u = [x, y, z]^T$ and velocity $\dot{u} = [\dot{x}, \dot{y}, \dot{z}]^T$ of a moving source shall be computed [2]. The geo-location problem requires at least M = 4 receivers to produce three TDOAs and three FDOAs. Four or more number of receivers are used to solve the TDOA FDOA equation which are over determined equations. The distance between the source or emitter and the receiver i is

$$r_i = |u - s_i| = \sqrt{(u - s_i)^T (u - s_i)} \tag{1}$$

TDOA of a signal received by the receiver pair and 1st number reference receiver is t_{i1}, and if c is the signal propagation speed, the set of equations that relate the TDOAs and the source position is

$$r_{i1} = ct_{i1} = r_i - r_1 \tag{2}$$

Where r_{i1} is the range difference, and i = 2,3,..M upon rewriting (2) as $r_{i1} + r_1 = r_i$, squaring both sides and substituting (1) for $r_1{}^2 and r_i{}^2$, we arrive at a set of TDOA equations:

$$r_{i1}{}^2 + 2r_{i1}r_1 = s_i{}^T s_i - s_1{}^T s_1 - 2(s_i - s_1)^T u, \quad i = 2, 3, ...M \tag{3}$$

The intersection of the curves in equation (3) gives the emitter position estimate. The TDOA equations only allow the estimation of source position and not velocity. TDOA equations alone may not be sufficient to provide enough accuracy to the position estimate. FDOA measurements between receivers can provide velocity of the source as well as provide improved position accuracies. With range rate or FDOA measurements at different receivers, relative motion between the source and receivers can be computed and provides better accuracies for emitter position and velocity. The time derivative of equation (1) gives the relationship between the range rate and target location parameters:

$$\dot{r_i} = \frac{(\dot{u} - \dot{S_i})^T (u - s_i)}{r_i} \tag{4}$$

The FDOA between the receiver pair and 1st number reference receiver is proportional to range rate.

$$\dot{f_{i1}} = \frac{\dot{r_{i1}}}{c} \tag{5}$$

Let $r = [r_{21}, r_{31}, ..., r_{M1}]$ and $\dot{r} = [\dot{r}_{21}, \dot{r}_{31}, ..., \dot{r}_{M1}]$ be the vectors of range differences and range rate differences. Where c is the RF wave propagation speed.

B. CRLB:

The CRLB is known to be the best accuracy that an estimator can achieve. It is usually used to evaluate the performance of estimation methods. Let $\theta = [u^T \dot{u}^T]$ be a vector that contains the final source position and velocity estimates. The CRLB of θ considering TDOA, FDOA measurements noises without any receiver location errors is given as:

$$CRLB(\theta) = J^{-1} = [(\frac{\partial q(\theta)}{\partial \theta}) \ Q^{-1} \ (\frac{\partial q(\theta)}{\partial \theta})]^{-1} \tag{6}$$

where J is the Fisher information matrix and Q is the measurement noises matrix.

$$\frac{\partial q(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial r(\theta)}{\partial u} & \frac{\partial r(\theta)}{\partial \dot{u}} \\ \frac{\partial \dot{r}(\theta)}{\partial u} & \frac{\partial \dot{r}(\theta)}{\partial u} \end{bmatrix}$$

$$\frac{\partial r(\theta)}{\partial u} = \begin{bmatrix} \frac{(u-s_2)^T}{r_2} - \frac{(u-s_1)^T}{r_1} \\ . \\ . \\ \frac{(u-s_M)^T}{r_M} - \frac{(u-s_1)^T}{r_1}; \end{bmatrix} \quad \frac{\partial r(\theta)}{\partial \dot{u}} = 0_{(M-1)X3} \tag{7}$$

$$\frac{\partial \dot{r}(\theta)}{\partial u} = \begin{bmatrix} \frac{(u-s_2)^T \dot{r_2}}{r_2{}^2} - \frac{(u-s_1)^T \dot{r_1}}{r_1{}^2} - \frac{(\dot{u}-\dot{s}_2)^T}{r_2} - \frac{(\dot{u}-\dot{s}_1)^T}{r_1} \\ . \\ . \\ \frac{(u-s_M)^T \dot{r_M}}{r_M{}^2} - \frac{(u-s_1)^T \dot{r_1}}{r_1{}^2} - \frac{(\dot{u}-\dot{s}_M)^T}{r_M} - \frac{(\dot{u}-\dot{s}_1)^T}{r_1} \end{bmatrix}_{(M-1)X3}$$

$$\frac{\partial \dot{r}(\theta)}{\partial \dot{u}} = \frac{\partial r(\theta)}{\partial u} =; \qquad Q = \begin{bmatrix} Q_t & 0 \\ 0 & Q_f \end{bmatrix}$$

Here Q_t, Q_f are TDOA and FDOA (range, range rate) measurement noises whose diagonal elements are set with 1 and remaining elements set with 0.5 of measurements noise. $Q_t = \sigma_t{}^2 c^2$ for diagonal elements and $Q_t = 0.5\sigma_t{}^2 c^2$ for remaining elements, $Q_f = \frac{\sigma_f{}^2 c^2}{f_0{}^2}$ for diagonal elements and$Q_f = 0.5\frac{\sigma_f{}^2 c^2}{f_0{}^2}$ for remaining elements where σ_t is TDOA measurement noise, σ_f is FDOA measurement noise and f_0 is the received frequency.

III. Simulations

Number of receivers and geometry of the receiver placement plays an important role in location fix of emitter. In this simulation 4, 5 and 6 number of receivers on ground are considered for location fix of the satellite. Simulations are carried out by taking emitters at different altitudes from 300 km to 1000 km within the simultaneous visibility of all receivers placed on Ground as shown in the figure 1. Emitter velocities are assigned based on respective emitter altitude. Position of receivers is assumed with different configurations like straight line, parallelogram, trapezoid, square, rectangle, T-shape and Y shape are placed on earth. 300 to 1000 km altitude emitters considered to address the LEO satellite detection, as the LEO space is being crowded in recent times. Seven different geometrical configurations of four ground receivers and emitters at an altitude of 300 km are studied and shown in the figure 2.

Four receivers in different Geometry Configurations

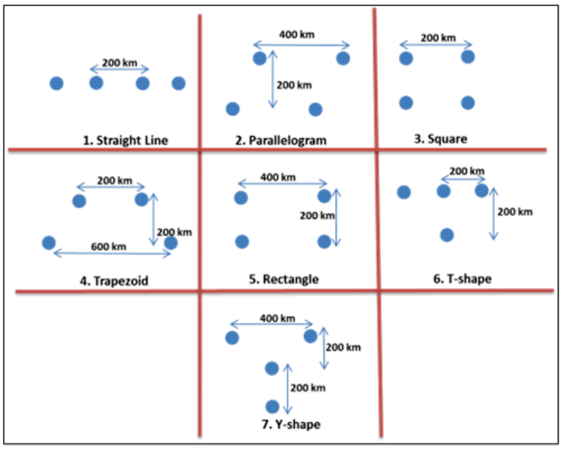

Fig. 2: Receivers geometry for Location fix

Location fix error results are shown for all 7 configurations in 3D, 2D for 300 km altitude satellites detection with 4 receivers are shown in the figure 2a to 2g.

It is observed from the figure 2a straight line configuration of receivers shown as yellow dots produces location fix errors < 1 km is 0.01% of common visibility area considered. In straight line configuration location fix error along line is high due to volume formed by the 4 receivers with emitter is nearly zero. Similarly, across perpendicular line location fix error is high due to TOA value of the two receivers are equal means

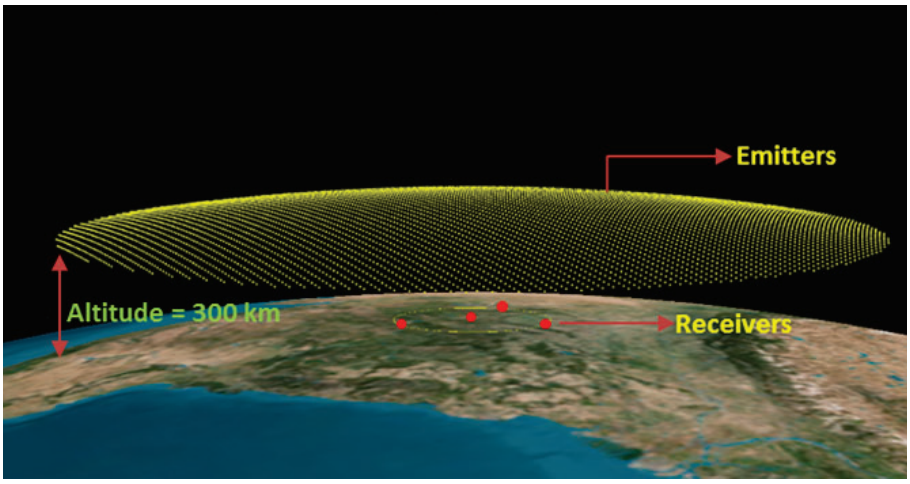

Fig. 1: Emitter and Receivers configuration

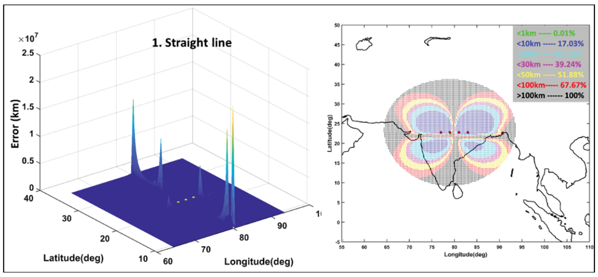

Fig2a: Error results for Straight line configuration with 4 receivers in 3D, 2D

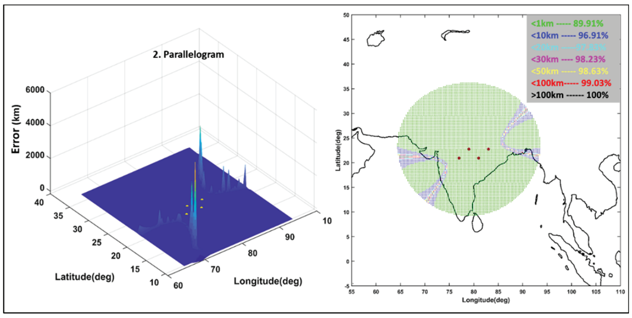

Fig2b: Error results for Parallelogram configuration with 4 receivers in 3D, 2D

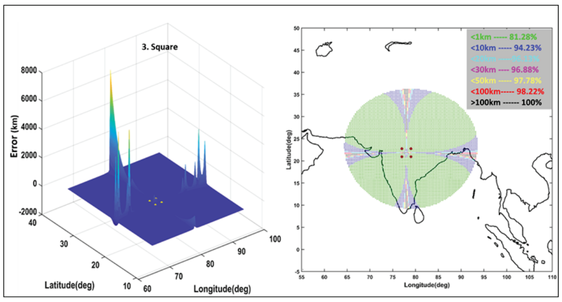

Fig2c: Error results for Square configuration with 4 receivers in 3D, 2D

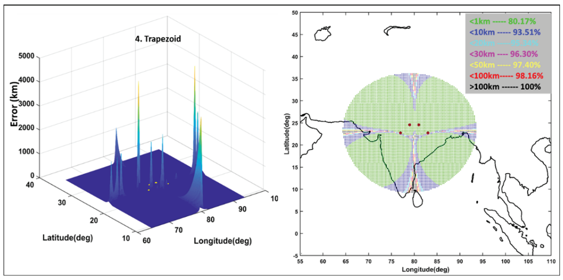

Fig2d: Error results for Trapezoid configuration with 4 receivers in 3D, 2D

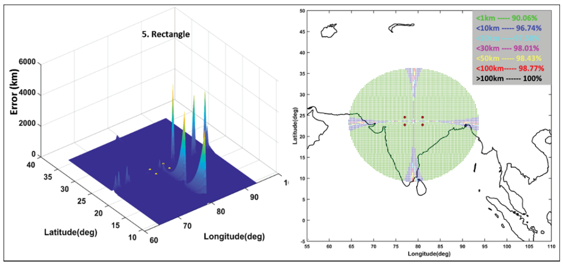

Fig2e: Error results for Rectangle configuration with 4 receivers in 3D, 2D

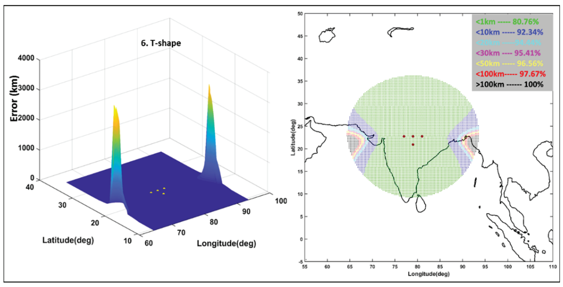

Fig2f: Error results for T-shape configuration with 4 receivers

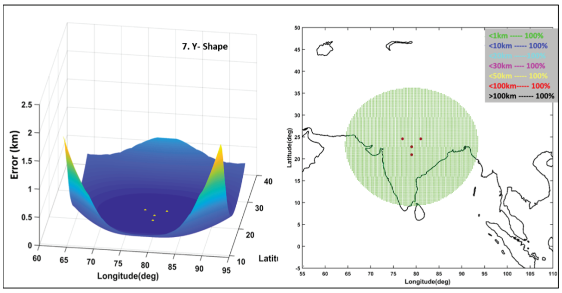

Fig2g: Error results for Y-shape configuration with 4 receivers

TABLE I: Position Error based on Receivers Configuration

Configuration	Error <1 Km(in %)
Straight Line	0.01
Parallelogram	89.91
Square	81.28
Trapezoid	80.17
Rectangle	90.06
T-shape	80.76
Y Shape	100

TDOA for both receivers are zero so equation cant be solved.

Parallelogram, Square, Trapezoid, and Rectangle receiver configurations produces location fix errors <1 km is 80 to 90 % of common visibility area considered. As the shapes changes from quadrilateral to T shape by moving one of the receiver to the diagonal intersection point of quadrilateral accuracies along one of the direction is improved at the same time in perpendicular direction error increases. T-shape configuration of receivers produces high error, the 3 receivers placed in line as the volume formed by those receivers with emitter is very less so error is high. Y shape configuration produces very good accuracies compared to all geometric shapes presented. When the receiver moved near to the centroid of the triangle formed by other three receivers configuration forms Y shape and it produces better TDOAs with all other receivers compared to other configurations.

Four rxs Geometry with & without center reference receiver

From the earlier study it is established that the receivers at center reference can improve the accuracies. In this section it is analyzed that the influence of center reference for four receiver configurations. The receiver configurations with and without center reference shown in the figure 3. Satellites at altitude 300 km with TDOA measurement error of 10 nsec and FDOA measurement error of 10 Hz are assumed for analysis. Configuration with center reference gives better results compared with without center reference as shown in the figures 4 and 5. Both position and velocity CRLB errors are improved by center reference configuration.

Effect of distance variation between four receivers for center reference geometry

Distance between the receivers plays an important role in location fix. Accuracy of the location fix improves by increasing the distances between the receivers. As the distance increases the volume formed by the receivers with emitter increases so the location fix accuracy improves. It is evident

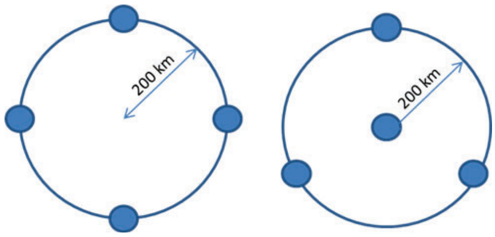

Fig 3 : Four receiver configuration without center and with center reference

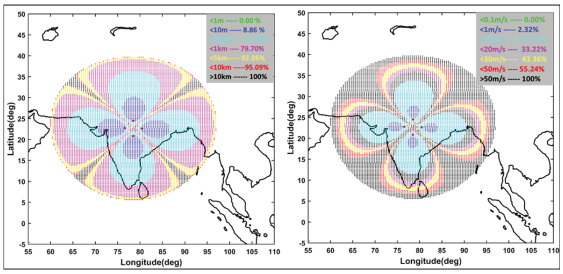

Fig 4 : Position and velocity CRLB error for 300 km altitude 10 ns 10 hz TDOA FDOA errors with 4 receivers without center configuration

form figure 6 that the distance between receivers varies from 10 km to 400 km the location fix accuracy improves and the range between receiver and emitter decreases or emitter elevation increases location fix accuracy improves. Two configurations are studied with 200 km and 400 km distance to analyze the effect of distance between the receivers on location fix accuracy. The analyzed configuration is shown in figure 7. Satellites at altitude 300 km with TDOA measurement error of 10 nsec and FDOA measurement error of 10 Hz are assumed for analysis. 200 km receiver configuration results are already shown in the figure 5. From figure 5 and 8 it has been established that 400 km receiver configuration produces location fix errors <100 m for 97% of combined visible area where as for 200 km <100 m for 49% of combined visible area. Altitude of the satellite to be location fix plays an important role in geo location accuracy. Satellite altitude increases the dependency of vectors from receivers to emitter increases so the location fix error increases. In other words as the distances increases from the receivers to emitter TDOA and FDOA produced by each pair of receiver and

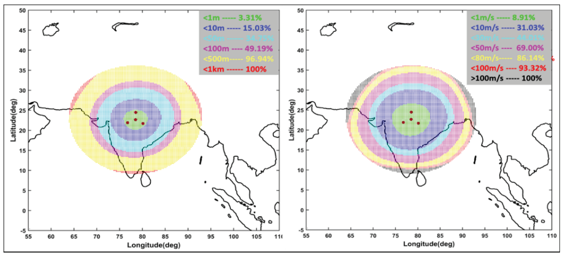

Fig 5 : Position and velocity CRLB error for 300 km altitude 10 ns 10 Hz TDOA FDOA errors with 4 receivers with center configuration

ISBN: 978-93-47311-06-2

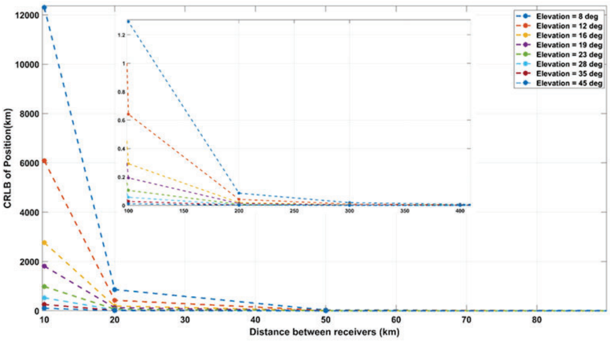

Fig 6 : Emitter position accuracy vs. receivers distance and emitter elevation with respect to central receiver

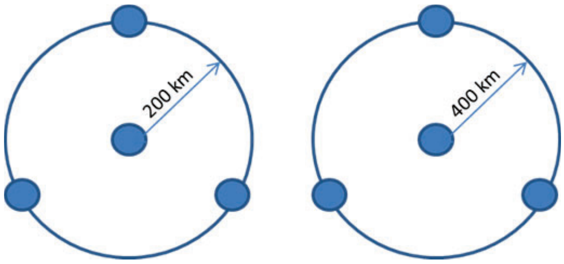

Fig 7 : Receiver configurations with 200 km and 400 km radius receiver positions around center receiver

emitter becomes similar and produces more errors in location fix. Two configurations are studied as shown in the figure 9, to analyze the effect of satellite altitude to be detected. Satellites at altitude 300 km and 1000km with TDOA measurement error of 10 nsec and FDOA measurement error of 10 Hz are assumed with four receivers in 200 km configuration. 200 km receiver configuration results for 300 km altitude results are already shown in the figure 5. 1000km satellite altitude results are shown in figure 9, produces higher errors <100 m for 33 % of combined visible area compared with 300 km altitude satellite location fix errors <100 m for 49 % combined visible area.

Effect of number of receivers on position and velocity accuracy

Number of receivers plays a vital role in location fix accuracy. Location fix accuracy improves as the receiver number increases to 5 or 6. Any one of the receiver failure

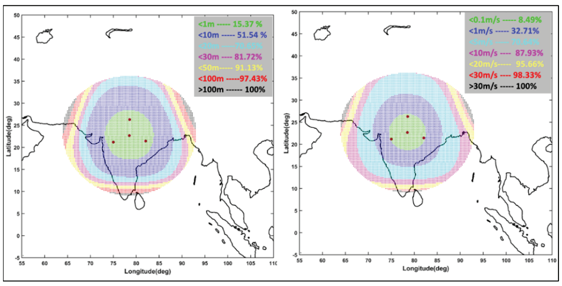

Fig 8 : Position and velocity CRLB for 400 km radius receiver position configuration

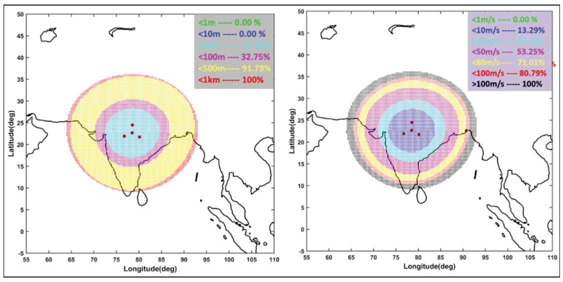

Fig 9 : Position and velocity error CRLB for 1000 km satellite altitude 10 nsec 10 Hz TDOA FDOA errors with 4 receivers

also can be addressed with increased number of receiver configuration. Increasing number of receivers can greatly improve the location accuracy of emitter but after certain number of receivers there will not considerable improvement in location fix accuracy. As shown in the figure 10, one can observe above six number of receivers, position accuracy is not improved. Three different configurations are studied and presented results to analyze the effect of number of receivers on location fix accuracy as shown in the figure 5, 9 and 10. Satellites at altitude 300km with TDOA measurement error of 10 nsec and FDOA measurement error of 10 Hz with four, five and six receivers in 200 km configuration assumed for analysis. 200 km receiver configuration results for 300 km altitude results are already shown in the figure 5. As shown in the figures 11, 12 and 5 receivers and 6 receivers configurations produce better location fix accuracies.

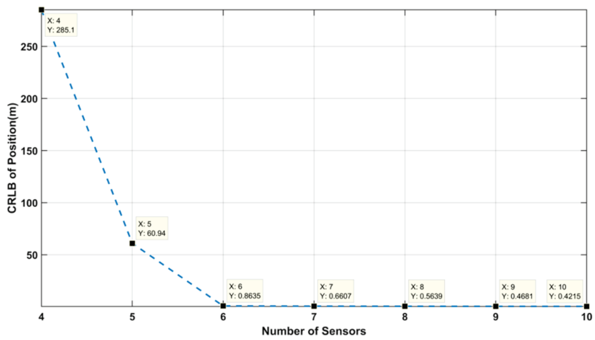

Fig 10 : Emitter location accuracy vs. number of receivers

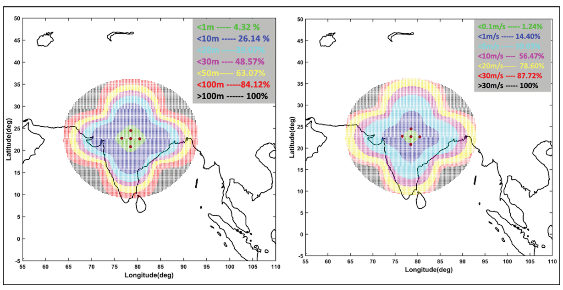

Fig 11 : Position and velocity error CRLB for 300 km satellite altitude 10 nsec 10 Hz TDOA FDOA errors with 5 receivers configuration

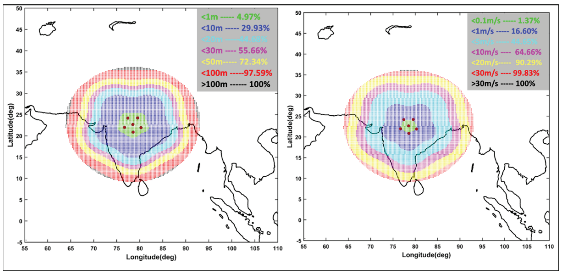

Fig 12 : Position and velocity error CRLB for 300 km satellite altitude 10 nsec 10 Hz TDOA FDOA errors with 6 receivers configuration

IV. Conclusions

Satellite location fix with Passive RF TDOA/FDOA technique provide actively emitting satellite orbit tracks data to SSA. Favorable conditions to produce better location fix are discussed elaborately. Different receiver configurations with 4 receivers in straight line, square, rectangle, trapezoid, parallelogram, T-shape and Y shape are presented. Y shape receiver configuration produces better Location fix accuracies compared with all other configurations. Four receiver configuration with and without Centre reference receiver analyzed and concluded that the presence center receiver produces better location fix in all configurations. Effect of receivers distance form center reference receiver analyzed and concluded that at 400 km distance location fix accuracy of less than 10 m can be achieved. Active emitter or Satellite altitudes effect on Location fix is analyzed and concluded that as altitude increases location error increases. Effect of number of receivers on location fix accuracies analyzed and concluded that up to 6 numbers the locations fix accuracies improving and further increase in number of receivers will not have considerable improvement.

References

[1] Ho, K. C., and Wenwei Xu. "An accurate algebraic solution for moving source location using TDOA and FDOA measurements." IEEE Transactions on Signal Processing 52, no. 9 (2004): 2453-2463.

[2] Ho, K. C., Xiaoning Lu, and La-or Kovavisaruch. "Source localization using TDOA and FDOA measurements in the presence of receiver location errors: Analysis and solution." IEEE Transactions on Signal Processing 55, no. 2 (2007): 684-696.

[3] Lei, Yan-Ping, Feng-Xun Gong, and Yan-Qiu Ma. "Optimal distribution for four-station TDOA location system." In 2010 3rd International Conference on Biomedical Engineering and Informatics, vol. 7, pp. 2858-2862. IEEE, 2010.