Ground and Airborne RF Emitter Geo-location determination by Space-Based Receivers using TDOA and FDOA Iso-Surfaces

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***Abstract*—RF emitter location is crucial information obtained from electronic reconnaissance systems. This study proposes using Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) iso-surfaces to determine the emitter location relative to the receivers position. The iso-surfaces are plotted on a geographic map with earth surface constraint, the intersection point determines the emitter location. TDOA/FDOA iso surfaces curves were plotted with measurement noises on earth surface, their intersection points area determines the emitter's location. Two or three receivers are required for RF emitter geolocation with known altitude. Geo-location of Airborne RF emitter with an unknown altitude was determined using with four receivers. Vehicle velocity is estimated with FDOA equation using estimated emitter position from TDOA curves. The geolocation accuracy is improved using different earth surface models such as spherical, WGS-84, and Digital Elevation Models.**

***Index Terms*—TDOA, FDOA, ISO curves, WGS84, DEM**

I. Introduction

RF emitter geolocation relies on techniques such as Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) to accurately determine the location of emitters. These techniques involve complex mathematical calculations based on arrival time and frequency differences observed at multiple receiver locations.

One advantage of TDOA and FDOA techniques is their ability to handle emitters at different altitudes. With two receivers both TDOA and FDOA are required to calculate emitter position of known altitude. Introducing a third receiver enables position calculation using TDOA alone, assuming the altitude is provided. This altitude flexibility makes TDOA and FDOA techniques applicable to ground-based and airborne emitter geolocation.

Furthermore, we explore a four receiver configuration using the TDOA FDOA technique, which creates an over-determined system for position calculation. This technique eliminates the need for emitter altitude as an input. By analyzing TDOA and FDOA measurements from four receivers, we can accurately determine position of the stationary emitter. For non-stationary emitters, first position is calculated with TDOA measurements. With calculated position, velocity of moving emitter is calculated by using lateration algorithm [1]. This capability allows for tracking moving emitters and studying their dynamics.

To improve the accuracy of RF emitter geolocation on the Earth's surface, we consider different models representing the Earth's shape and elevation data. Models such as the spherical model, the widely used World Geodetic System 1984 (WGS-84) model, and Digital Elevation Models (DEMs) are analyzed. With these models various accuracies can be employed. The WGS-84 model provides a more accurate representation of the Earth's shape by considering its ellipsoidal nature. Integrating DEMs, which captures terrain variations, further enhances the accuracy, especially in complex terrains.

In this paper, we delve into the mathematical foundations and practical implementation of TDOA and FDOA techniques. We explore the conversion of TDOA and FDOA equations into ISO curves for precise visualization of emitter locations on a geographic map. Additionally, we investigate the utilization of different models, including the spherical, WGS-84, and DEM-based models, to enhance the accuracy of emitter location estimates.

By advancing our understanding of these techniques and models, we aim to improve the precision and reliability of RF emitter geolocation systems. This progress will contribute to better situational awareness and decision-making across various domains.

A. Scenario

In ECEF (Earth-Centered, Earth-Fixed) coordinates, the positions of the emitter u(x,y,z) and the four satellite receivers at a specific instant are denoted as $S_1(x_1, y_1, z_1)$, $S_2(x_2, y_2, z_2)$, $S_3(x_3, y_3, z_3)$, $S_4(x_4, y_4, z_4)$, and velocities are denoted as $\dot{u}(\dot{x}, \dot{y}, \dot{z})$, $\dot{S}_1(\dot{x_1}, \dot{y_1}, \dot{z_1})$, $\dot{S}_2(\dot{x_2}, \dot{y_2}, \dot{z_2})$, $\dot{S}_3(\dot{x_3}, \dot{y_3}, \dot{z_3})$ and $\dot{S}_4(\dot{x_4}, \dot{y_4}, \dot{z_4})$. The configuration of receivers placement considered for analysis is shown in below figure 1, it may change

The TDOAs (Time Difference of Arrival) are computed by subtracting the TOA (Time of Arrival) of the received signal at the satellites from the reference satellite (satellite 1). The TDOA for each satellite, denoted as $TDOA_i$, is represented as:

$TDOA_i = TOA_i - TOA_1 = \Delta t_i (for i = 2, 3, 4)$

The range from each satellite to the emitter is calculated using the ECEF coordinates. The range between the emitter and satellite i, denoted as r_i, is given by:

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} (for i = 1, 2, 3, 4) \quad (1)$$

The TDOAs represent the differences in range with respect to the reference satellite (satellite 1), which can be expressed as:

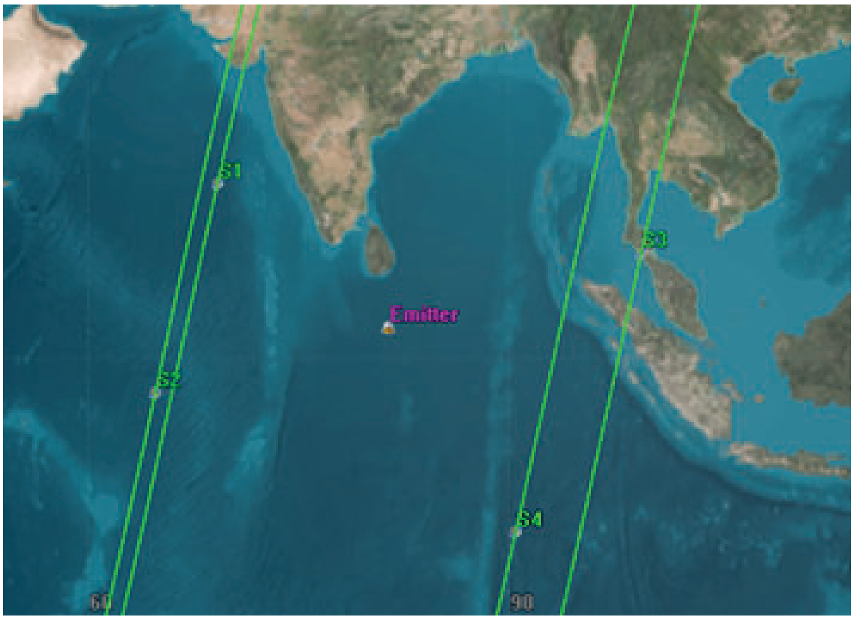

Fig. 1: Receivers placement configuration.

$$r_i - r_1 = \Delta r_i = c\Delta t_i (fori = 1, 2, 3, 4) \quad (2)$$

Here, c represents the speed of the electromagnetic wave.

The FDOAs (Frequency Difference of Arrival) are computed by subtracting the frequency received at the satellites from the reference satellite (satellite 1). The FDOA for each pair of satellites, denoted as $FDOA_i$, is represented as:

$$FDOA_i = f_i - f_1 = \frac{f_0}{c}(\dot{r}_i - \dot{r}_1)(fori = 2, 3, 4) \quad (3)$$

By taking the time derivative of range equation (1) gives range rate as

$$\dot{r}_i = \frac{(u - s_i)^T(\dot{u} - \dot{s}_i)}{r_i}(fori = 1, 2, 3, 4) \quad (4)$$

Here, f_0 represents the transmitter frequency. In all FDOA localization scenarios, f_0 is considered as mean of received frequencies [3], in this analysis we considered f_0 as known value. This study assumes that the receivers in the electronic reconnaissance system are represented by satellites positioned at an altitude of approximately 750 km which is the mean value of LEO altitude. In order to find the TDOA and FDOA values for emitter and receiver pairs, emitter position is taken as input. Errors or measurement noises are introduced to the calculated TDOA and FDOA values.

TDOA iso curves are drawn by considering receiver positions and TDOA values and solving equation 2 for (x, y, z) values, that satisfies the respective TDOA values. FDOA iso curves are drawn by considering receiver positions, velocities and FDOA values, assuming emitter velocity as zero and solving equation 3 for (x, y, z) values that satisfies the respective FDOA values.

To solve emitter location in 3d at least 3 equations are needed. In case of 2TDOA and 2FDOA, earth surface constraint is considered to solve emitter location. Earth surface equation depends on the shape of the earth we considered.

Spherical Earth:

$$x^2 + y^2 + z^2 = R^2$$

Where x, y, z are the emitter coordinates; R = mean radius of spherical Earth (R = 6371.009)

Wgs84 Earth:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Where a = Earth radius at equator (a = 6378.137); b = a; c = Earth radius at Poles (c = 6356.7523)

DEM Earth: we considered DEM height for the analysis and calculated coordinates (x, y, z). These coordinates are used to draw iso curves.

II. Simulations:

A. *TDOA FDOA iso surface generation:*

The process of generating iso-surfaces for Time Difference of Arrival (TDOA) involves using the positions of receivers and corresponding TDOA values. By simulating the TDOA/FDOA equations in MATLAB with the provided TDOA/FDOA values, an iso-surface is obtained, which represents the possible location of the emitter. In the scenario depicted in the figure 2, two receivers are assumed to be located at an altitude of 750 km. The receivers position and TDOA values are used as input to generate the iso-surface, and the emitter's location is determined based on this ISO surface. When spherical earth surface constraint is considered, the intersection of the TDOA iso-surface with the Earth's surface takes the form of a circle and the emitter is located on this curved surface, as shown in the figure 2.

The generation of FDOA (Frequency Difference of Arrival) isosurfaces involves considering the positions and velocities of receivers, along with their corresponding FDOA values. In this process, it is assumed that the satellite receivers are located at an altitude of 750 km. The emitter is assumed to

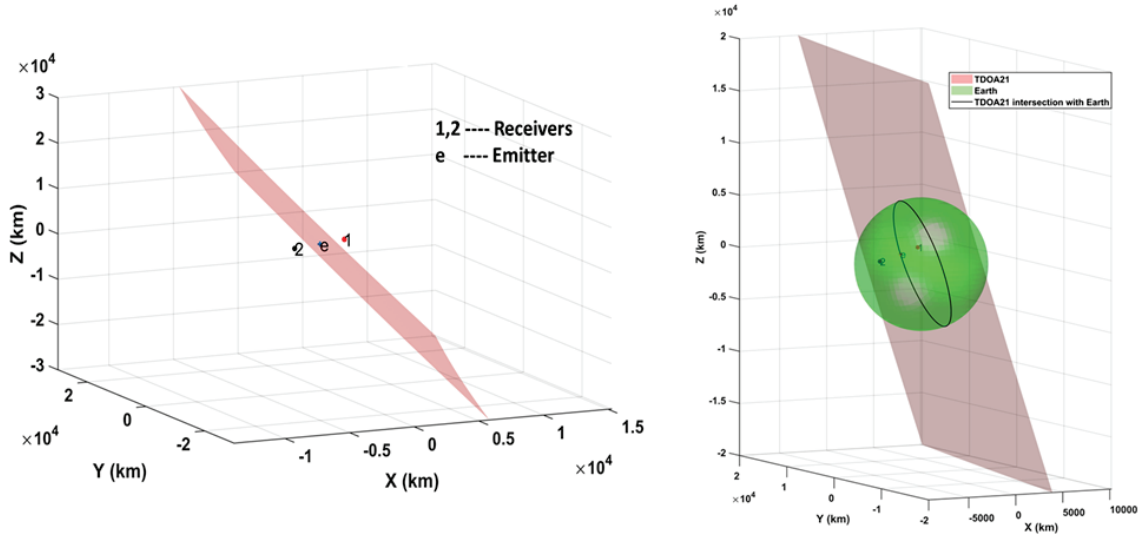

Fig. 2: TDOA ISO SURFACE.

be positioned on the ground, and its FDOA values are used as inputs for the FDOA equation. By applying the FDOA equation with the provided values, an isosurface is obtained, representing the possible location of the emitter.

In the depicted scenario figure 3, two receivers are assumed to be positioned at a 750 km altitude, and their velocities are determined based on their altitude. The emitter is assumed to be stationary, and the FDOA values are used as input to generate the FDOA isosurface. When spherical Earth constraint is considered, the intersection of the FDOA isosurface with the Earth's surface forms a curve, and the emitter is located on this curved surface, as shown in the figure 3.

B. 3 receiver TDOA iso curve method for RF emitter geo location:

In 3 receivers TDOA configuration with each pair of receivers, an iso-surface is generated. When considering the constraint of the Earth's spherical shape, these iso-surfaces intersection with the earth form circles that represent possible locations of the emitter. In the figure 4, three receivers are used, resulting in three pairs of iso surfaces and therefore three circles with earth surface. The intersection of these circles determines the precise location of the emitter. In real-world scenarios, TDOA measurements from each pair receiver often contain noise and errors. To simulate this, we introduce a constant bias representing the time error (10 nsec), which can have both positive and negative values. With the inclusion of this noise, each receiver pair produces a band instead of a precise circle, indicating the range within which the emitter could be located. In the figure 4, the three receivers pair produces overlapping bands, and the intersection area of these bands on the Earth's surface represents the possible location of the emitter.

C. 3 receiver FDOA iso curve method for RF emitter geo location:

In 3 receiver FDOA (Frequency Difference of Arrival) configuration, an iso-surface is generated for each pair of receivers. Taking into account the spherical shape of the Earth, these iso-surfaces intersection with earth form distinct curves that represent potential locations of the emitter. In the figure 5 provided, illustrates the use of three receivers, resulting in three pairs and, consequently, three iso curves. By determining the intersection points of these curves with earth, the precise location of the emitter can be found. In real-world scenarios, FDOA measurements obtained from each receiver pair often contain noise and errors. To account for this, we introduce a constant bias that represents the frequency error (10 Hz). This bias can be positive or negative values. As a result, instead of a precise curve, each receiver pair iso surface intersection with earth generates a band that denotes the range within which the emitter could be located. In the figure 5, the overlapping bands produced by the three pairs of receivers with earth indicate the area on the Earth's surface where the emitter is likely to be located.

By plotting the TDOA and FDOA measurements, along with the introduced errors or noises, on a three dimensional space, we can achieve a more precise estimation of the emitter's location without using earth constraint. In this scenario, three pairs of receivers are used, and the TDOA and FDOA values are obtained. Additionally, errors are incorporated into the measurements, such as a 10 ns TDOA error and a 10 Hz FDOA error. Using this data, the emitter's location can be determined by identifying the combined area of intersection iso curves in 3d space. By considering the overlapping regions derived from the TDOA and FDOA iso curves, accounting for the introduced errors, a more accurate estimation of the emitter's location is obtained. The figure 6 provided depicts this combined area of iso intersection curves, indicating the potential location of the emitter considering the given TDOA and FDOA measurements, along with their respective errors.

D. TDAO FDOA with 2 receiver configuration:

Using a two-receiver configuration, we can calculate the position of the emitter of known altitude. By generating TDOA and FDOA iso curves intersection with the Earth

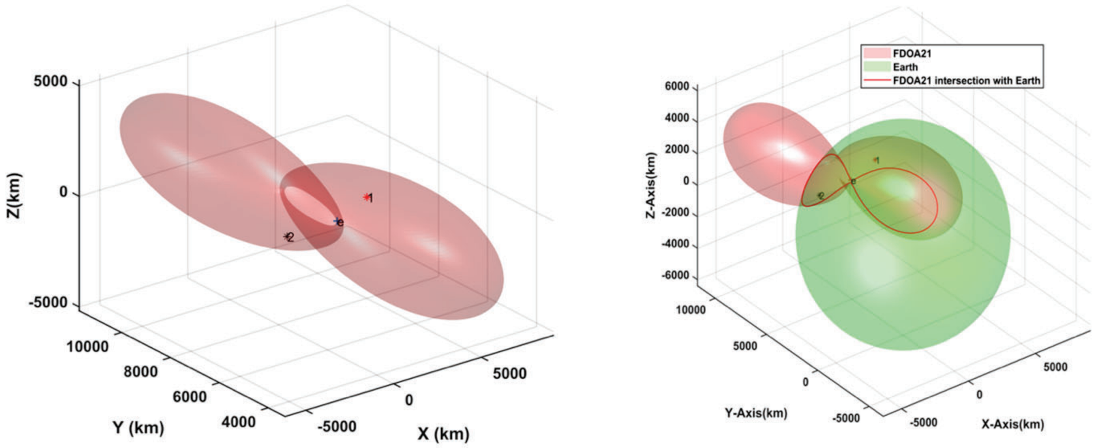

Fig. 3: FDOA ISO SURFACE.

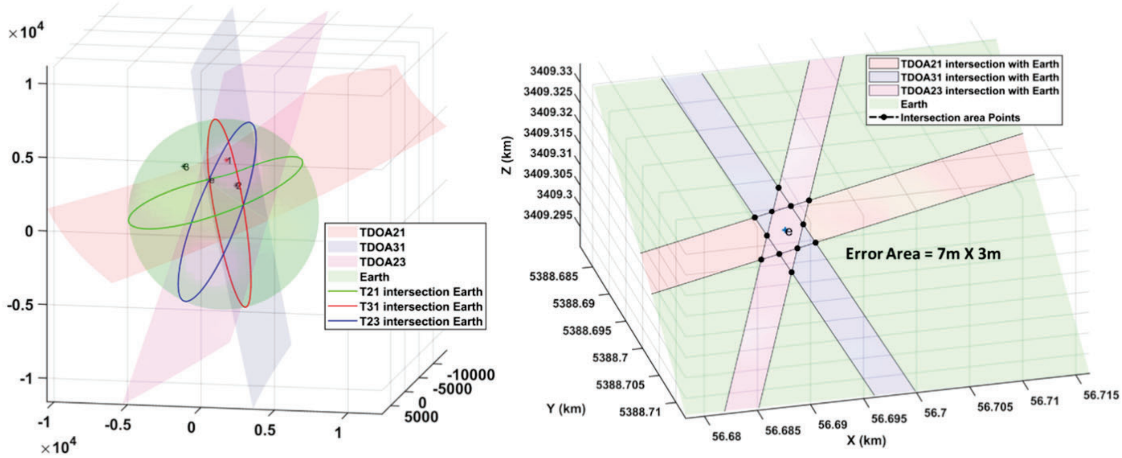

Fig. 4: 3 receiver TDOA configuration.

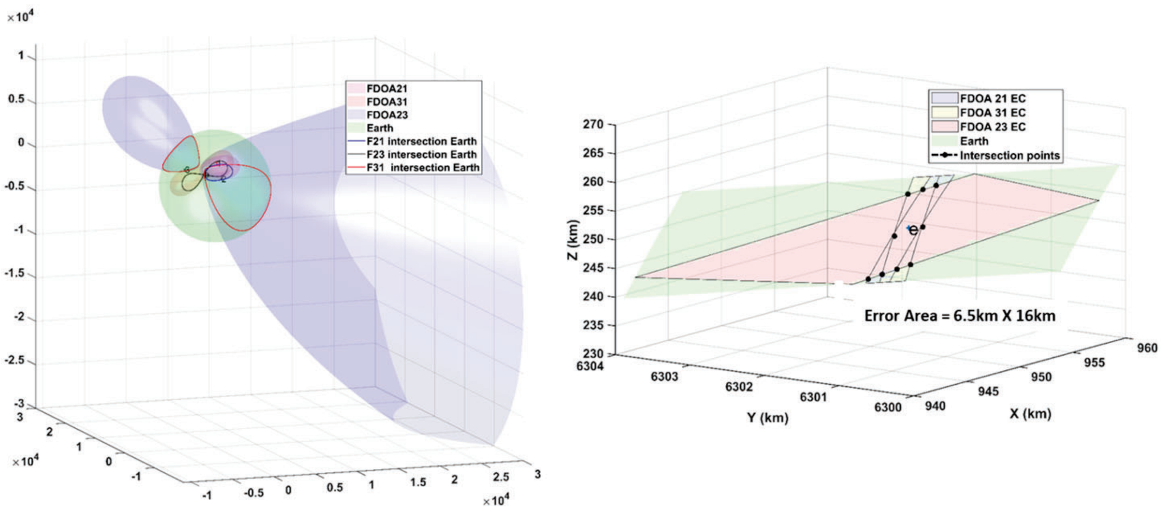

Fig. 5: 3 receiver FDOA configuration .

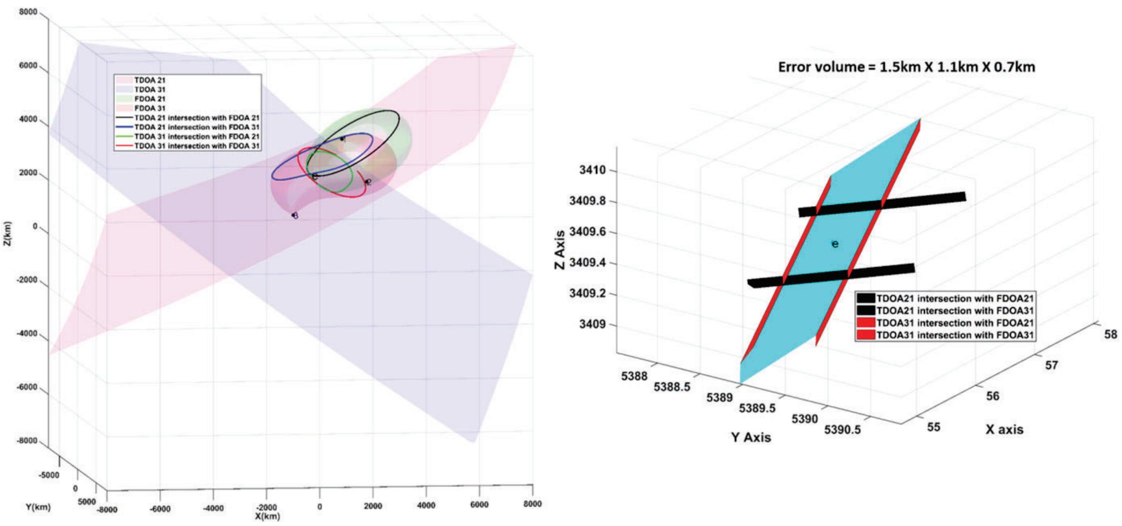

Fig. 6: TDOA FDOA with 3 receiver configuration

as shown in the figure 7, we determine potential emitter position. To account for measurement errors, we introduced errors into the TDOA and FDOA measurements. Plotting the combined intersection area with errors on a geographic map, we can observe that the emitter is located within this area as shown in the figure 7. This approach enables a more accurate estimation of the emitter's position by considering both the TDOA and FDOA iso curves intersection with the errors present in the measurements.

E. Combined four receivers TDOA and FDOA iso surface method for emitter geo location:

Using the TDOA method with four receivers, we can accurately determine the geographical location of the emitter without requiring knowledge of its altitude. By utilizing the four receivers, we can create six receiver pairs, with one receiver serving as the reference and the three pairs used to generate TDOA measurements. By selecting one receiver as the reference, we input the TDOA values for the three pairs into the system. This process generates three iso-surfaces, representing possible emitter locations. In the figure 8 provided, the intersection lines between these iso-surfaces are depicted, indicating the location of the emitter.

To account for measurement errors, we introduced additional errors into the TDOA measurements. The intersection volume of these iso-surfaces with incorporated error on a map denotes the estimated position of the emitter. By considering the overlapping region of these curves, we obtain a more refined estimation of the emitter's location. By employing the 4 receiver FDOA method, we can accurately determine the geographical location of an emitter with unknown altitude, without considering the Earth's constraint. Using four receivers, we can form six receiver pairs; out of six pairs with one receiver acting as the reference we selected three pairs used to generate FDOA measurements. By selecting one receiver as the reference and using FDOA values as inputs, we can generate three iso-surfaces that represent potential locations of the emitter. The figure 9 illustrates the intersection curves between these iso-surfaces, which indicate the precise location of the emitter.

To account for measurement errors, we introduced additional errors into the FDOA measurements. The intersection area of these error-incorporated iso-surfaces on a map denotes the estimated position of the emitter. By considering the overlapping region, we obtain a more accurate estimation of the emitter's location. Once the emitter's position is known, we can utilize FDOA equations to determine its velocity. Thus, by utilizing a four-receiver configuration with iso-surfaces derived from both TDOA and FDOA measurements, we can determine both the position and velocity of the emitter.

Earth constraint: In the two-receiver and three-receiver TDOA-FDOA configurations, we initially assumed that the altitude of the emitter is known. However, this assumption can impact the accuracy of geolocation. We considered the Earth as spherical model and solved for the emitter's location. Unfortunately, this approach resulted in high geolocation errors, as depicted in the figure 10. To improve the accuracy, we then considered the Earth as an ellipsoid, specifically using the WGS-84 model. This consideration yielded better accuracy, as shown in the figure 10, compared to the spherical Earth assumption. Once we had an approximate area for the emitter's location, we incorporated a Digital Elevation Model (DEM) specific to that area. We plotted the TDOA intersection lines on the surface of the DEM and precisely determined the emitter's position. The figure 10 demonstrates this precise emitter location on the DEM surface.

By considering the realistic Earth shape using the WGS-84 model and incorporating DEM data, we can achieve enhanced geolocation accuracy compared to the spherical Earth assumption.

Conclusions:

The presented paper focuses on the utilization of Time Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) iso curves to determine the location of an RF

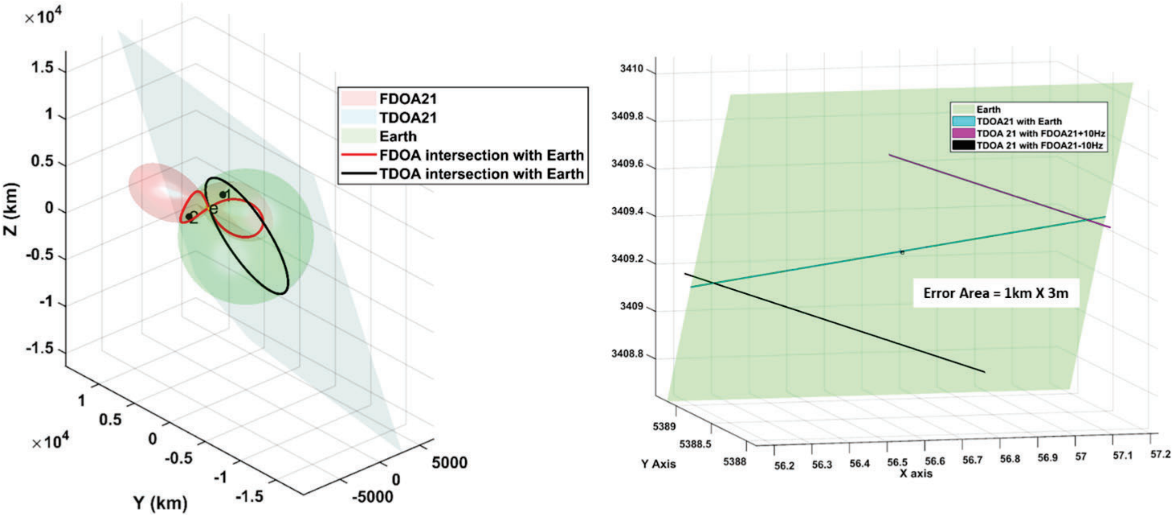

Fig. 7: Two receivers TDOA FDOA configuration

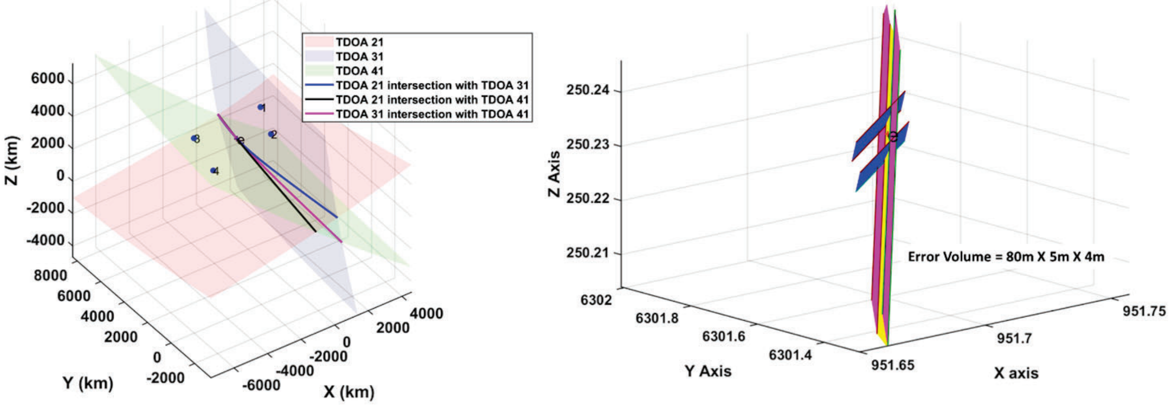

Fig. 8: 4 receivers TDOA configuration

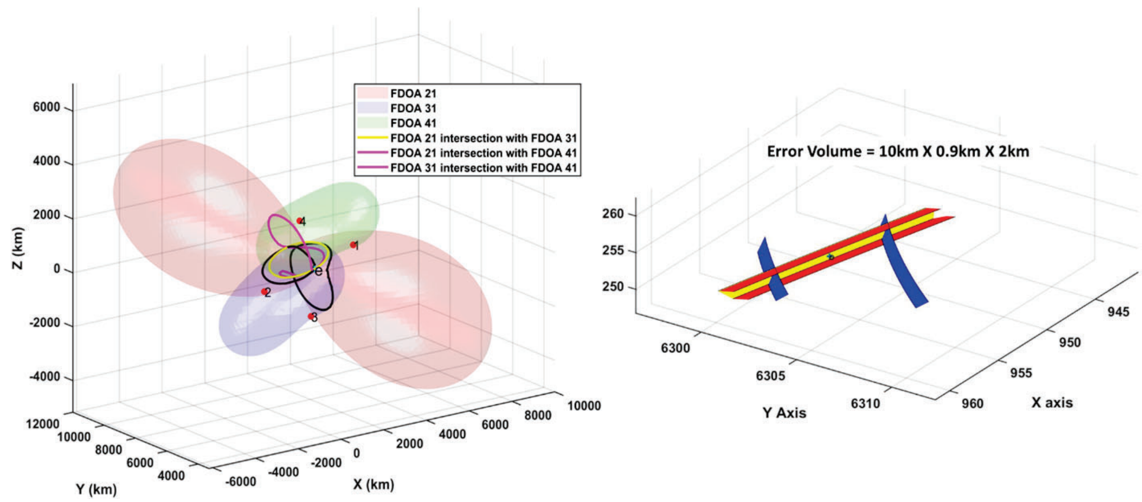

Fig. 9: 4 receivers FDOA configuration

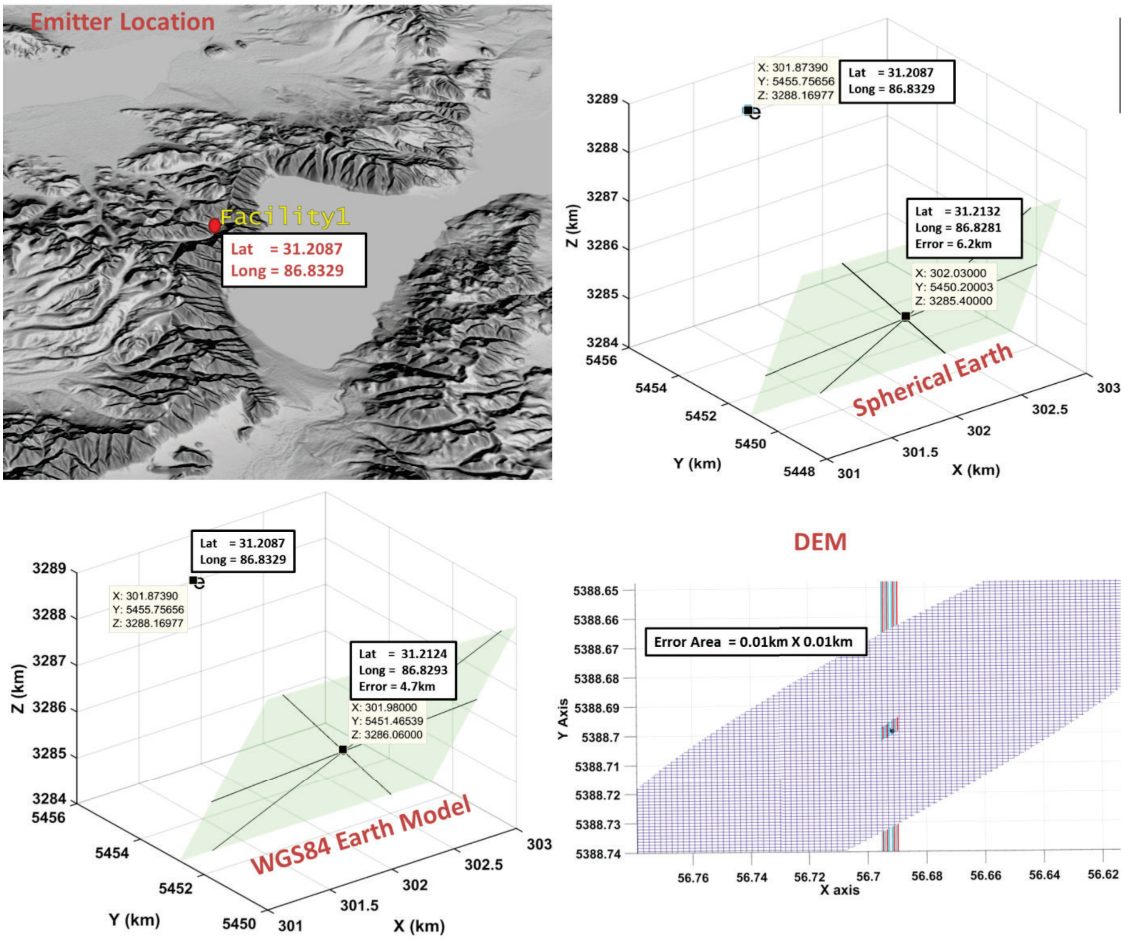

Fig. 10: Error accuracy with different Earth Models

emitter relative to receiver positions. The conclusions drawn from this research are as follows:

1) The TDOA/FDOA iso curves, plotted on a geo map while considering the earth surface constraint, provide valuable information for locating the emitter. The intersection of these curves represents the emitter's position.
2) The inclusion of measurement noises in the TDOA/FDOA curves, along with their intersection with the earth's surface, allows for more realistic estimation of the emitter's location.
3) The TDOA/FDOA iso surface technique, with two or three receivers, enables estimation of the emitter's location when the emitter's altitude is known.
4) By employing four receivers, the TDOA method can locate the emitter's position even when the altitude is unknown. Additionally, the FDOA equation can be used to estimate the velocity of the emitting vehicle.
5) The accuracy of the earth surface equation is enhanced by incorporating different models, such as the spherical model, WGS-84 model, and Digital Elevation Model, each with varying accuracies. This improves the accuracy of the emitter's location estimation.

In summary, the paper highlights the effectiveness of TDOA and FDOA iso curves for emitter localization, considering different receiver configurations and Earth surface models. These techniques offer valuable insights for military intelligence and related applications.

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