Design and Development of Interacting Multiple Model for State Estimation of Very High Speed Maneuvering Target

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***Abstract*— The main aim of any Area Defense Weapon System (ADWS) is to engage the incoming tactical missiles by neutralizing them with the available interceptors in a layered defense system. The performance of the layered defense system (either exo or endo atmospheric interception) critically depends upon the incoming target state estimation. For a Very High Speed - Maneuvering Reentry Vehicle (VHS-MaRV), the state estimation is difficult compared to a normal ballistic missile due to the change in flight dynamics of the MaRV. So the accurate state estimation with minimum estimation error and lag is essential to track a randomly maneuvering target. This can be achieved by using an adaptive estimator called the Interacting Multiple Model (IMM) filter which is robust enough to handle flight uncertainties in a real-world scenario, where the measurements from sensors are noisy. The design of IMM consists of (a) the selection of number of filter models (b) type of filter models and (c) selection of filter parameters for each model. In this article, the results of two selected filter models which are mostly suitable for VHS-MaRV are discussed and shown. The designed IMM algorithm is tested with five different VHS-MaRV trajectories and compared.**

Keywords—Interacting Multiple Model, Extended Kalman filter, Mode Probability, VHS-MaRV, State Estimation, Maneuvering Index, Latex

I. Introduction

The advancements in the missile technology leads to the development of very high speed as well as evasive targets. The estimation of kinematic states such as position, velocity and acceleration of such targets along any inertial frame of reference using limited available sensor data is difficult but mandatory for successful neutralization of these incoming threats. The early detection sensors such as ground radar sensors provide target information in the form of target's range, azimuth and elevation angles (Rm, Am, Em) which are highly polluted with Gaussian random noise. From these noisy measurements, estimation of position and velocity can be achieved with acceptable tracking noise, but the estimation of target acceleration is challenging task to achieve. Also for VHS-MaRV, even the position and velocity components with required accuracy are difficult to achieve because, strong non linearities are involved in its system dynamics of measurement equations along with higher sensor noise. This leads to a challenging task for target state estimation. This problem of target state estimation can be solved with proper target model selection and adaptive filter tuning.

The state estimation of a target is a combination of linear systems and probability theory [1]. If the target is moving with a constant speed, the simple filtering algorithms like α-β filter and Kalman filter are sufficient. However, there are unpredictable changes in the target motion that cause nonlinearities, if the target is manoeuvring which may result in estimation error if we go with simple filtering algorithms. Successful tracking of a randomly manoeuvring target needs some sort of adaptive state estimation due to its highly time varying dynamics with significant uncertainty. So one single model can't represent the entire dynamics. The Extended Kalman Filter (EKF) can linearize the nonlinear dynamics, but relies on a fixed underlying process noise model. So, design of a multiple model algorithm is must here. The commonly used multiple model approaches are Generalized Pseudo Bayeseian-1 (GPB1), GPB2 and Interacting Multiple Model (IMM).

The IMM approach is popular due to its versatility and computational simplicity. If the manoeuvrability of the target is high, then the uncertainty will be high. The steady-state gain of a filter is expressed in terms the target manoeuvring index (λ). If $\lambda > 0.5$, the IMM estimation reduces the errors significantly over the other filtering algorithms [1]. So, the adaptive bandwidth or dynamic range capability of the IMM estimator becomes a significant factor in the estimation process. The adaptation via mode probability update helps the IMM estimator to keep the errors low during manoeuvres and benign motion intervals [1]. The switching behaviour between the models will be controlled by the underlying Markov chain [3]. In this article, the proposed IMM approach consists of two extended Kalman filters (i.e., Constant Acceleration and Constant Jerk Models). The current IMM filter has been designed for long range VHS - MaRV target. So, modelling of centrifugal and Coriolis acceleration is carried out considering the earth's rotation and curvature effect.

The current article is as follows. Section II discusses different frames used in the filters formulation. The proposed IMM filter algorithm will be described in Section III. The results will be presented in Section IV. Section V concludes the article.

II. REFERENCE FRAMES

In the present study, the measurements of target received from sensors are in Polar frame. The state kinematic models of target consisting of governing equations of model in presence of spherical and rotating earth have been evolved using VEN and ECEF frames. The corresponding direction cosine matrices are defined and used for converting the input/output coordinates to another coordinate reference frames.

A. Earth Centered Earth Fixed (ECEF) Frame

This is a coordinate system with origin at earth center, its Z-axis coincides with true north and perpendicular to XY plane. Its X_E and Y_E axes rotate about the earth's spin axis (Z_E). Units of X_E, Y_E, and Z_E are in meters.

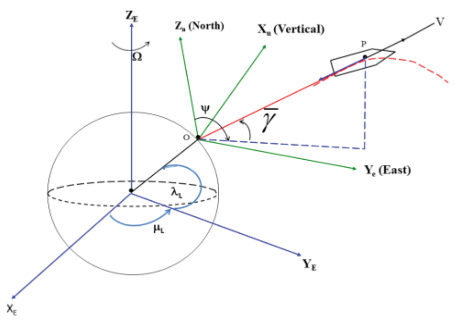

Fig. 1. *Representation of Target in ECEF & VEN Frame*

B. VEN Frame

The VEN (Vertical, East, North) frame is coordinate system with origin at O in Figure 1 at some point on the earth surface (here the sensor location). Z-axis is North (Z_n), X-axis is Vertical (X_u) and Y-axis is East (Y_e). The local origin is generally defined as (lat0 (degree), lon0 (degree), h0 (meter)).

C. Polar Frame

The polar frame is also called as Spherical Coordinate System (Local). Generally, the sensor will give the target measurements in spherical coordinates form: range (r), azimuth (θ) and elevation (φ). The elevation is the angle of target measured with respect to XY -Plane. Azimuth is the angle measured from X-axis. Units of r, θ, φ are meter, degree and degree respectively.

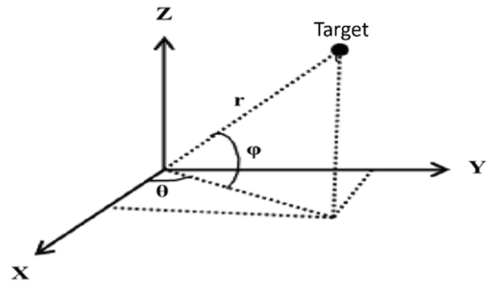

Fig. 2. *Representation of Target in Polar Frame*

III. IMM FILTER ALGORITHM

The proposed IMM filter in this article, includes the following two Extended Kalman Filter (EKF) Models: (i) Constant Acceleration (CA) Model (ii) Constant Jerk (CJ) Model. The IMM filter's block diagram is shown in figure 3. The raw measurements of the target received from the sensors, will be given as input to each model along with the previous step output (States and Covariance). The likelihood functions and innovations are calculated in each step and will be utilized while updating the state and covariance at current step. In present IMM model, homogeneous Markov process has been assumed. The VHS-MaRV in present article is treated as a point mass target with three degrees of freedom, its position in space. In the dynamical model, the rotational degrees of freedom due to body rates is neglected without loss of generality.

The randomly manoeuvring target sometimes will fly with zero latex, sometimes with high latex and also sometimes with very high latex. Apart from that the manoeuvring point will not be known apriori. So, intuitively it is felt that during no latex phase, Constant Velocity (CV) model, during soft manoeuvre CA model and during high manoeuvre CJ model are ideal. The aesthetics of IMM model is that it uses the information from all the models running in parallel via mode probability. So automatically proper model gets higher weightage and vice versa which makes the overall estimated states closest to optimal.

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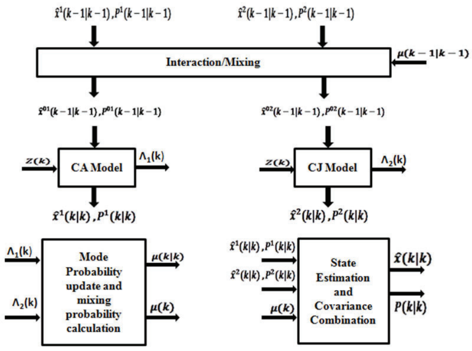

Fig. 3. *The IMM Flow*

Where

X = State estimate

P = Covariance matrix

Z = Sensor measurements

Λ_j = Likelihood value of model j

μ_j = Mode probability of model j

k = sample number

A. *IMM State Interaction / Mixing*

The difference between actual and dynamic model will be modelled as process noise in both the models described above. At sample time, based on measurements received from radars, probabilistic model switching and mixing of states and co variances takes place in IMM. So same sensor information has been processed by using IMM consisting 2 models. Here, the received radar measurements are in spherical coordinate system which is non-linear in nature. So, the Direction Cosine Matrices (DCMs) are used to convert the spherical coordinates into VEN frame and then to ECEF Frame and vice versa. This strategy makes the motion equations to be in linear system [2]. Here the MaRV is assumed as long range target, the modelling of centrifugal and Coriolis acceleration has carried out in each model, by considering the spherical and rotating earth.

B. *Constant Acceleration (CA) Model*

When the VHS-MaRV manoeuvres with nominal latex, the estimation error is less when the constant acceleration model is used [2,7]. The target state vector for CA model is,

$$X = [x\ v_x\ a_x\ \ y\ v_y\ a_y\ \ z\ v_z\ a_z]^T \quad (1)$$

Where (x,y,z), (v_x, v_y, v_z), (a_x, a_y, a_z) are target's position, velocity and acceleration components respectively.

The Q-matrix computation for each kinematic component of (position, velocity, acceleration) is

$$Q_k = Q_{wk} \begin{pmatrix} \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ \frac{T^4}{8} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^3}{6} & \frac{T^2}{2} & T \end{pmatrix} \quad (2)$$

Here Q_{wk} (k=x,y,z) is power spectral density of process noise. T is the sampling time. Q_k is a 9x9 matrix.

C. *Constant Jerk (CJ) Model*

When the VHS-MaRV manoeuvres with very high latex, it subjects to large acceleration due to drag and the earth's gravity. Therefore the rate of change of acceleration is considerable as the latex varies drastically [6,7]. Therefore, the constant jerk model is considered here. The target state vector is

$$X = [\ x\ \ v_x\ \ a_x\ \ j_x\ \ y\ \ v_y\ \ a_y\ \ j_y\ \ z\ \ v_z\ \ a_z\ \ j_z\ ]^T \quad (3)$$

Where (x,y,z), (v_x, v_y, v_z), (a_x, a_y, a_z), (j_x, j_y, j_z) are target's position, velocity, acceleration and jerk components respectively.

The Q-matrix computation for each kinematic component of (position, velocity, acceleration, jerk) is

$$Q_k = Q_{wk} \begin{pmatrix} \frac{T^7}{252} & \frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{20} \\ \frac{T^6}{72} & \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ \frac{T^5}{30} & \frac{T^4}{8} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^4}{20} & \frac{T^3}{6} & \frac{T^2}{2} & T \end{pmatrix} \quad (4)$$

Here Q_{wk} (k=x,y,z) is power spectral density of process noise. Q_k is a 12x12 matrix.

The state transition matrix (F-matrix) for both CA and CJ models is evaluated both numerically and analytically and the results are consistent in both the ways. The CJ model contains 12 states where as the CA model contains only 9 states as per equation (1). Asymmetric mixing occurs, while doing the state interaction & mixing as described in section 3.1. So the CA model will also be converted to 12 state model while implementation, by assuming the jerk components (j_x, j_y, j_z) as zeros, so that the IMM mixing will become symmetric [5].

While implementing the algorithm, it is observed that, the sampling data rate received from sensors is not always same and sometimes it is lesser than 10Hz (0.1 second). So the CA and CJ models are implemented to perform the step propagation for every 0.1 second internally and estimate the state variables. Also both the models are designed to handle the spurious data received from sensors. This will make the system more robust.

D. Measurement Model

The measurement model for the above two filters is modelled in both linear and non-linear system of equations. The results are consistent in both ways. The radar measurements are contaminated by measurement noise. The measurement noise covariance matrix (R-Matrix) is assumed with one σ standard deviation. The R-matrix and H-matrix are generated in both linear and non-linear modes and compared. The results are consistent.

E. Model Switching (Mode Jumping)

IMM estimation is a unique approach in which the state and covariance estimates from multiple models are combined according to the Markov Models for transition between target manoeuvre state spaces (here 2 models), also called as Model Switching. To implement Markov model , it is assumed that, at scan time there is a probability **'p_{ij}'** that the target will make transition from model state ($i \rightarrow j$) . These probabilities are assumed to be known apriori and can be represented in a state transition matrix. This is called as **Markov Mode Transition Probability** or **Mode Transition Probability Matrix** [4].

$$\pi = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{5}$$

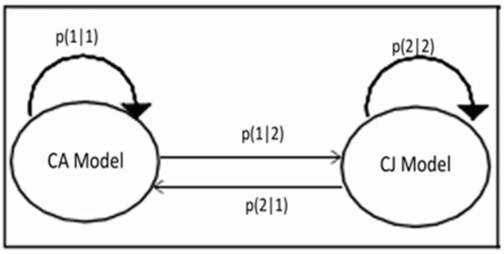

Fig. 4. *Model Switching in the proposed IMM Estimator*

Since the $\pi -$ is constant throughout the estimation, the IMM performance subsequently depends on time evolution of likelihood functions [λ_1, λ_2] and in turn that of [μ_1, μ_2] where μ_i is the average mode probability of model 'i' at each step. These will be used as weighted factors to combine the updated states and covariance for all individual filter models (i = 1,2) to obtain overall state estimates and covariance.

F. Normalized Innovation Sequence (NIS) Cost

Fine tuning of Q-matrix of each model has been carried out based on tracking Normalized Innovation Sequence (NIS) of the estimator. For an optimal state estimator, the innovations should be around zero mean and have uncertainty magnitude commensurate with the measurement covariance as yielded by filter. The NIS cost function is defined as

$$\mathbf{J_m} = \frac{1}{N}\sum_{k=1}^{N} v_k^T \, [H_k \, P_{k(-)} \, H_k^T + R]^{-1} \, v_k \tag{6}$$

where m is total number of measurement channels (here x,y and z. so m=3). v_k is measurement innovation (Nu) at k^{th} sample, N is total number of filter models (here N = 2).

Generally J_m should be around the total number of measurement channels. Therefore, in present context, the ideal J_m should be equal to 3. The tuning of Q-matrix of each model has been carried out based on NIS cost by making **Jm ≈ 3**.

IV. RESULTS

In this section the tracking performance of CA, CJ and IMM filters is discussed. The filter errors in position, velocity and acceleration along with one sigma bound is presented. The sampling rate is 10Hz and the radar provides the target data in spherical coordinate system. The sensor noise is assumed as σ_r=50 metres, σ_θ=1 milli radian,σ_φ=1 milli radian. The results are shown for a normalized sample trajectory with 4 manoeuvring cycles as shown in Figure 5. The individual filters along with IMM filter are also compared with one sigma for better analysis of estimation errors and model switching behaviour.

The error estimation in position x, y, z are compared among CA , CJ and IMM filtering models with true data with one sigma error and shown in figures 6 to 8. The position plots are normalized to 0 to 1 sec for time (x-axis) and -500 to +500 for error in x-ecef, y-ecef, z-ecef (y-axis).

The same comparison for velocity components is shown in figures 9 to 11. The velocity plots are normalized to 0 to 1 sec for time (x-axis) and -200 to +200 for error in vx-ecef, vy-ecef, vz-ecef (y-axis).

Due to space constraint the acceleration component plots are not shown here. The mode probabilities history is shown in figure 12. It can be observed from figure 12 that, during the non-manoeuvring phase, CA model is giving less estimation errors and during manoeuvring phase CJ model is giving less estimation errors. This switching between the models is done by IMM estimator based on the mode probability profile and behaved as expected.

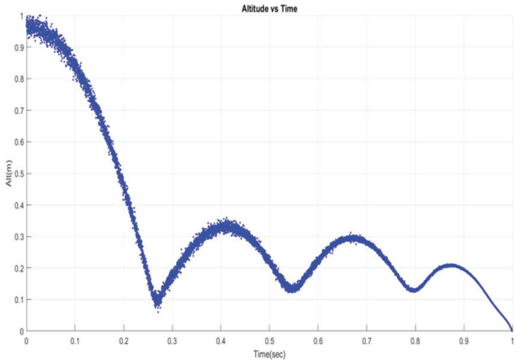

Fig. 5. *Altitude Vs Time*

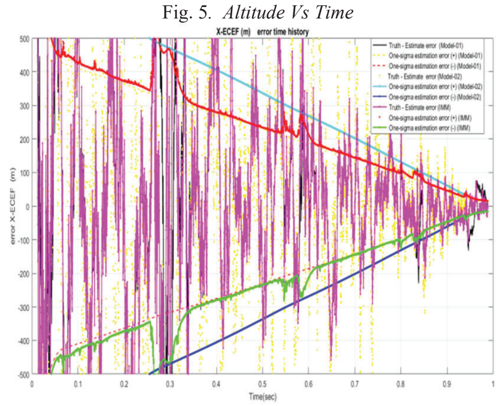

Fig. 6. *Comparison of Err Position X (ECEF) with one sigma err estimation w.r.t. Truth, CA, CJ and IMM*

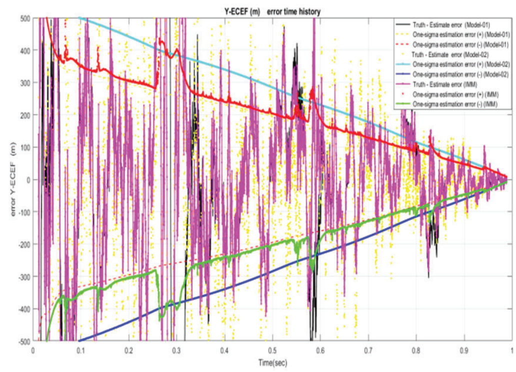

Fig. 7. *Comparison of Err Position Y (ECEF) with one sigma err estimation w.r.t. Truth, CA, CJ and IMM*

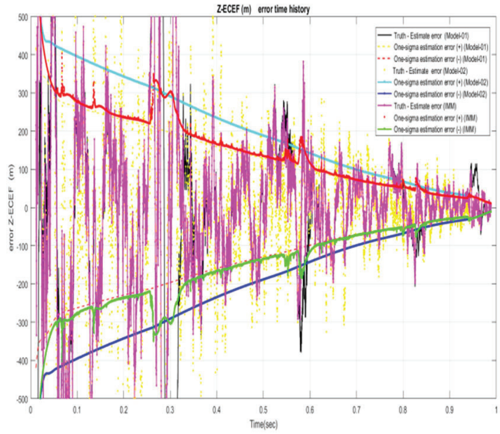

Fig. 8. *Comparison of Err Position Z (ECEF) with one sigma err estimation w.r.t. Truth, CA, CJ and IMM*

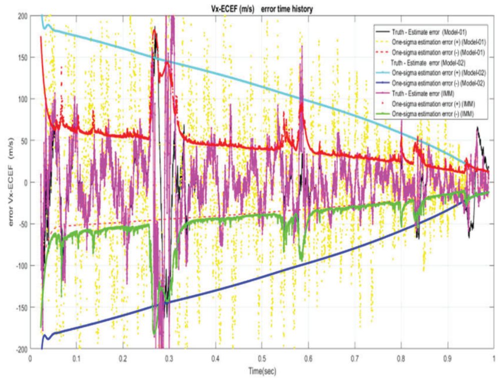

Fig. 9. *Comparison of Err Velocity X (ECEF) with one sigma err estimation w.r.t. Truth, CA, CJ and IMM*

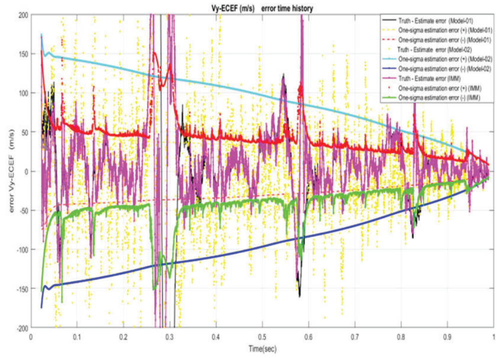

Fig. 10. *Comparison of Err Velocity Y (ECEF) with one sigma err estimation w.r.t. Truth, CA, CJ and IMM*

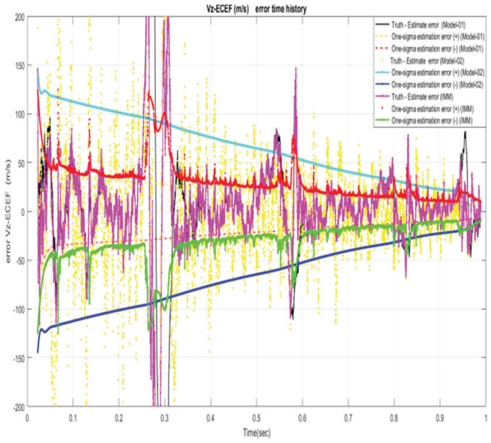

Fig. 11. *Comparison of Err Velocity Z (ECEF) with one sigma err estimation w.r.t. Truth, CA, CJ and IMM*

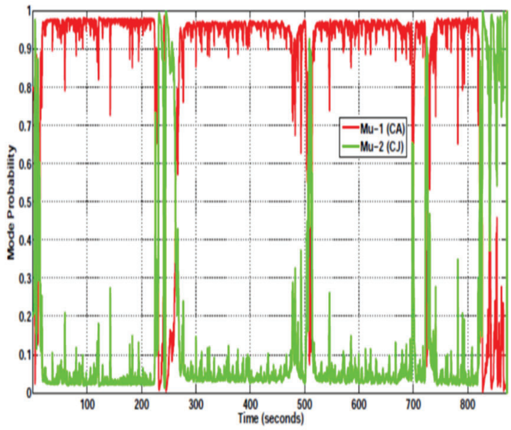

Fig. 12. *Time history of Mode Probabilities [Mu1, Mu2] in IMM Estimator*

V. CONCLUSION AND FUTURE WORK

Proper filter tuning is the key point for IMM to perform near optimally. Experience has shown that diagonal elements in Mode Transition Probability Matrix should be large and off diagonal elements should be small. Good initial choice of p_{ii} can be between (0.80 - 0.98) to start with. The lower (higher) value will yield less (more) peak error during manoeuvre but higher (lower) RMS error during the quiescent period which gives rise to higher (lower) bandwidth [3]. By fine tuning these parameters along with fixed μ_0 matrix elements, time history of $[\mu_1, \mu_2]$ will be used to detect the manoeuvre and model switching.

IMM estimates will be always better that those of (CA, CJ). How much better, depends on (μ_1, μ_2) time evolution. Best estimates are possible during manoeuvring zone with μ_2 and during non-manoeuvring zone with μ_1.

In all cases of position, velocity and acceleration estimates, the estimation error with respect to truth model is bounded by (one sigma) error. Only during manoeuvre locations, the error exceeds the (one sigma) bound. Also estimation error is minimum in position components and maximum in acceleration components. This is expected due to non- availability of Doppler (range rate) measurements. Therefore, the future scope is that, the filtering models can be extended with Doppler components to reduce the acceleration estimation errors.

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