RAMJET Engine Controller Design Using Quantitative Feedback Theory (QFT)

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***Abstract*—This paper proposes a Quantitative Feedback Theory (QFT) based robust controller design strategy for controlling the thrust of RAMJET engine. The 2-Degree of Freedom (2-DOF) QFT design approach comprises the loopshaping based controller synthesis and prefilter synthesis. The proposed QFT strategy satisfies the desired design specifications i.e., robust tracking and stability margin across all the chosen operating points (Mach and Angle of Attack). The efficacy of the proposed QFT based thrust control for RAMJET engine is compared with the contemporary H$_\infty$ technique.**

Keywords: Ramjet, QFT, Nonlinear, Robust control, Engine control

I. Introduction

Modern supersonic aerospace vehicles are powered by either solid propellant-based rocket motors or Ramjet engines. While solid propellant-based rocket motors are simple in construction, they must carry oxidizer along with fuel, resulting in lower specific impulse compared to air-breathing engines ([8]). On the other hand, Ramjet engines are air-breathing engines without any moving parts such as compressors and turbines. Due to their simple construction and efficient operation in the supersonic regime ([2], [3]), Ramjet engines are the preferred choice.

At a given operating condition, the Ramjet engine can be considered a Single-Input Single-Output (SISO) plant where the fuel mass flow rate primarily governs the thrust produced ([1]). The operating conditions of a Ramjet Engine are typically distinguished by the Angle-of-Attack (α) and the Mach number of the aerospace vehicle. At a given operating point, the relationship between fuel mass flow rate and thrust production is characterized as a first-order transfer function with a gain and a time constant ([1]). If we consider the dynamics of the fuel pump as a second-order dynamic system ([2], [3]) , the complete transfer function from thrust command to achieved thrust can be modeled as a third-order system.

The wide range of operating points (Mach and α combinations) typically requires a gain scheduling-based approach, which can be laborious to design. Originally, the controllers developed using H$_\infty$ robust control theory in [1], [4] and [6] for each operating point resulted in fifth-order controllers. Since the controller order exceeded the plant order (third-order in this case), the auther in [1] has used the balance reduction technique to reduce controller order for gain scheduling & real-time implementation feasiblity([5]) . In the present work, QFT is proposed as a controller design methodology for Ramjet Engine thrust control. QFT is known for its transparent & systematic design procedure, which reveals plant or design specification limitations at the design stage itself, and allows for the selection of a single, minimum-order controller to meet design requirements.

This paper is divided into three main parts. The first part discusses the problem definition and associated terminology. The second part describes the QFT design procedure. The third part presents time and frequency domain results with a fair comparison to results already obtained in the literature.

II. Problem Definition

The Ramjet Engine transfer function at a given operating point (Altitude, Mach, α) can be approximated as a linear first-order transfer function ([1]) , given by:

$$H_{rj}(s) = \frac{F(s)}{\dot{m}_{fuel}(s)} = \frac{G_{rj}}{\tau s + 1} \quad (1)$$

where G_{rj} and τ are constants that depend on the operating point as well as the fuel-to-air ratio.

At 200 m altitude, the obtained parameter values for the above equation are given in Table I.

TABLE I: Ramjet Engine Model Parameters at 200m Altitude

M	α [Deg]	G_{rj}	τ
2.8	0.0	14845.58	0.0742
2.8	2.5	14845.11	0.0743
2.8	5.5	14859.97	0.0794
3.0	0.0	14475.43	0.065
3.0	2.5	14474.98	0.0653
3.0	5.5	14453.53	0.066
3.2	0.0	14102.94	0.057
3.2	2.5	14102.5	0.0575
3.2	5.5	14099.79	0.0578
3.4	0.0	13730.86	0.05
3.4	2.5	13730.45	0.0508
3.4	5.5	13727.86	0.0518

The fuel pump model can be represented as a second-order transfer function ([2], [3]) as follows:

$$H_{fss}(s) = \frac{\dot{m}_{fuel}(s)}{i(s)} = \frac{K_c\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (2)$$

where $K_c = 2.4$, $\omega_n = 1507.96$, and $\zeta = 0.5$ are the low-frequency DC gain, natural frequency, and damping ratio of the fuel pump transfer function, respectively, and $i(s)$ represents current as the control input.

Finally, the plant transfer function used for controller design is given by:

$$H_{pl}(s) = H_{rj}(s) \times H_{fss}(s) \tag{3}$$

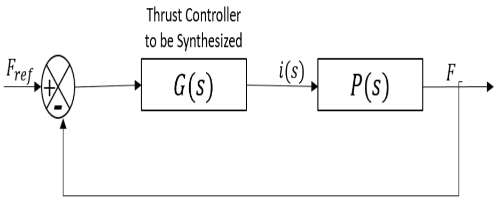

Fig. 1: Block diagram of the thrust control system

III. QFT Design Procedure

A. Thrust Control System Requirements

For our controller design problem, stability and reference tracking are taken as the main design criteria. Similar to any other control system, the stability criteria are:

- Gain Margin ≥ 5 dB
- Phase Margin $\geq 45°$

For reference tracking requirements, the time domain specifications are:

- Overshoot $\leq 10\%$
- Settling time ≤ 0.4 sec

B. Plant Definition

As mentioned in the previous section, different transfer functions are obtained for the ramjet engine at various operating points (Mach number and Angle-Of-Attack) from Table I. For a chosen frequency template ([7]), the plant template is computed and plotted on the Nichols plot, as shown in Fig. 2. The choice of nominal plant, although unimportant, is the plant with Mach number 2.8 and Angle of Attack 0.0°

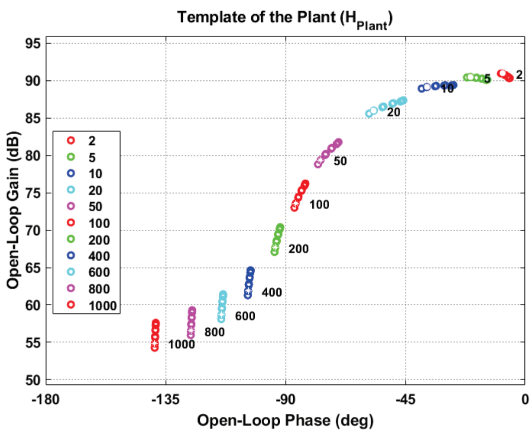

Fig. 2: Template of the Plant (H_{plant})

C. QFT Design Specifications

To meet the Ramjet thrust control system design specifications, all design requirements need to be translated into suitable frequency domain constraints.

1) Stability Specifications: The stability specifications for QFT design are as follows ([7]) :

$$\left| \frac{P(j\omega)G(j\omega)}{1 + P(j\omega)G(j\omega)} \right| \leq W_s \tag{4}$$

for $\omega \in [2, 5, 10, 20, 50, 100, 200, 400, 600, 800, 1000]$ rad/s

Gain margin for the given W_s is:

$$GM = 20 \log_{10} \left(1 + \frac{1}{W_s}\right) \text{ in dB} \tag{5}$$

Phase margin for the given W_s is:

$$PM = 180 - 2\left(\frac{180}{\pi}\right) \cos^{-1}\left(\frac{0.5}{W_s}\right) \text{ in Deg} \tag{6}$$

By choosing $W_s = 1.2$, we obtain the following stability margins:

$$GM \geq 5.2 \text{ dB}, \quad PM \geq 49.24° \tag{7}$$

which are in line with our design specifications.

2) Reference Tracking Specifications: The reference tracking specifications for QFT design are as follows ([7]) :

$$\delta_{low}(\omega) < \left| F(j\omega) \frac{P(j\omega)G(j\omega)}{1 + P(j\omega)G(j\omega)} \right| \leq \delta_{up}(\omega) \tag{8}$$

for $\omega \in [2, 5, 10, 20]$ rad/s

$$\delta_{low}(s) = \frac{1}{[(\frac{s}{a_L}) + 1]^2}; a_L = 20; \tag{9}$$

$$\begin{aligned} \delta_{up}(s) &= \frac{(\frac{s}{a_U}) + 1}{[(\frac{s}{\omega_n})^2 + (2\zeta\omega_n s) + 1]^2}; a_U = 9.5; \zeta = 0.9; \\ \omega_n &= \frac{1.25 a_U}{\zeta} \end{aligned} \tag{10}$$

The step response for the chosen reference tracking bounds is shown in Fig. 3.

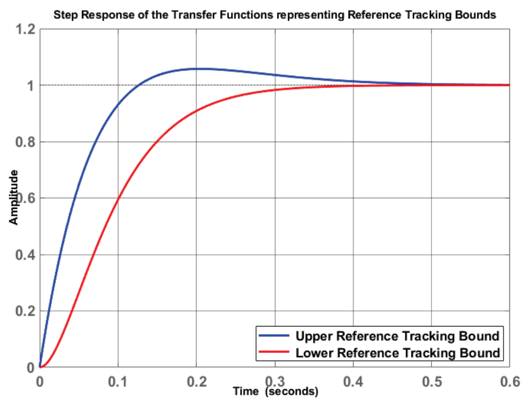

Fig. 3: Step Response of the Transfer Functions Representing Reference Tracking Bounds

D. QFT Bounds

Equations 4 and 8 need to be converted into quadratic inequalities. These quadratic inequalities are solved and represented as bounds on the Nichols plot. To carry out this process, a MATLAB-based QFTCT toolbox is used. The stability bounds and reference tracking bounds are found, and their intersection is taken at each frequency, as shown in Fig. 4, Fig. 5, and Fig. 6, respectively. These bounds represent the plant model with uncertainty and specifications satisfied at each frequency of interest.

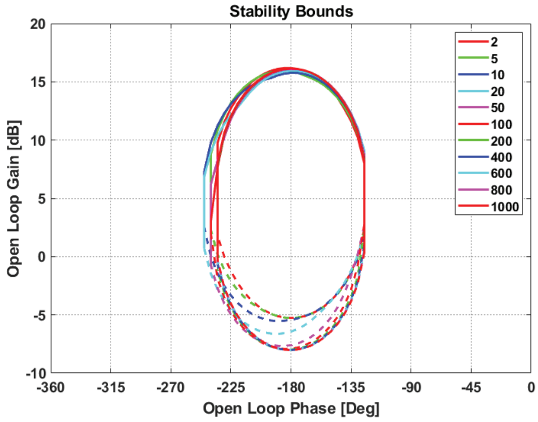

Fig. 4: Stability Bounds

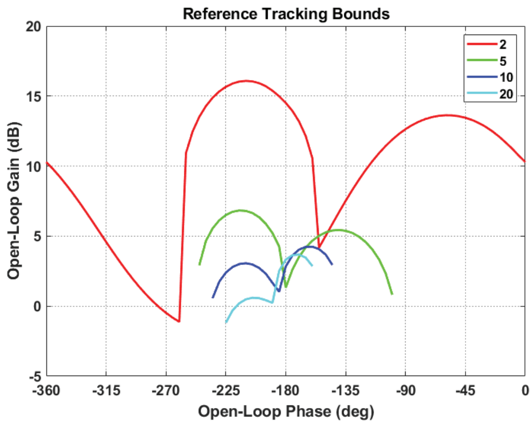

Fig. 5: Reference Tracking Bounds

E. Controller Design (Loop Shaping)

To satisfy all design specifications with different plant variations, all intersections of bounds shown in Fig. 7 need to be satisfied by the nominal plant. Loop shaping is carried out by adding poles and zeros until the nominal loop point lies above the solid line and below the dashed line at each frequency of interest. A first order controller which can be repersented in PI controller form with $K_p = 2.858 \cdot 10^{-5}, K_i = 0.0004859$ and satisfy the design requirements is as given below:

$$G_c(s) = 2.858 \cdot 10^{-5} + \frac{0.0004859}{s} \tag{11}$$

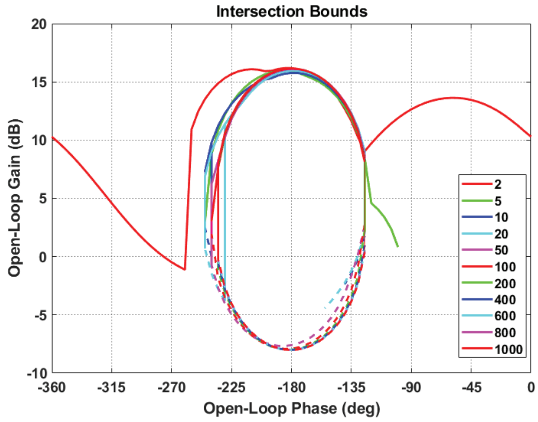

Fig. 6: Intersection Bounds

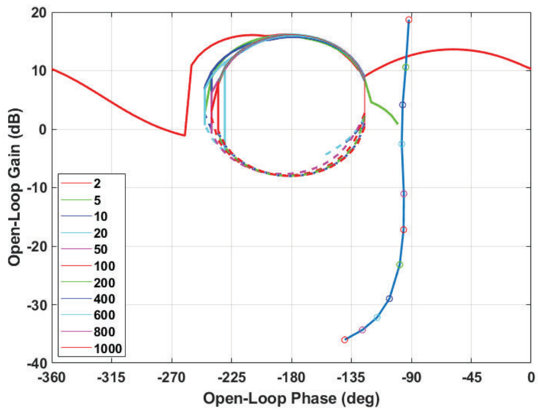

Fig. 7: Loop Shaping

IV. Results

A. Time Domain

Time domain simulations are carried out to check the performance of the QFT controller for tracking specifications. It can be seen in Fig. 8 that the tracking specifications defined in terms of settling time and overshoot are satisfied by a single QFT controller at all operating points.

Furthermore, a comparison of time domain results obtained using a gain-scheduled H_∞ controller in [1] has been carried out and tabulated in Table II. The comparison shows that the the QFT controller performed better in terms of settling time with insignificant difference in % overshoot. Moreover, the QFT controller results are obtained at the advantage of a single robust controller which doesn't require gain sechduling alongwith minimum order of controller(first order in this case). In the Table II, τ_s is settling time.

B. Frequency Domain

To check the stability specifications with the QFT controller, a Bode plot at all 12 different operating points is plotted. From Fig. 9, it can be seen that a single QFT controller satisfies both phase margin and gain margin specifications at all operating points. A comparison of stability specifications with gain-scheduled H_∞ controllers is done and tabulated

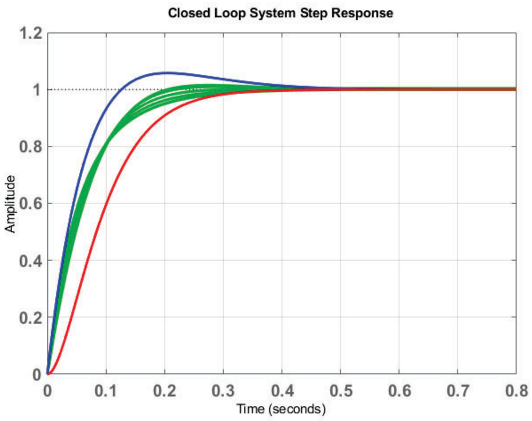

Fig. 8: Closed-Loop System Step Response

TABLE II: Comparison of Time Domain Performance

Mach	α [Deg]	H_∞ Controller		QFT Controller	
		τ_s (sec)	Overshoot (%)	τ_s (sec)	Overshoot (%
2.8	0.0	0.32660	0.00	0.18492	0.90
2.8	2.5	0.32675	0.00	0.18479	0.92
2.8	5.5	0.32663	0.00	0.17875	1.54
3.0	0.0	0.32669	0.00	0.20910	0.10
3.0	2.5	0.32677	0.00	0.20827	0.11
3.0	5.5	0.32673	0.00	0.20665	0.15
3.2	0.0	0.32674	0.00	0.24304	0.00
3.2	2.5	0.32667	0.00	0.24109	0.00
3.2	5.5	0.32674	0.00	0.23996	0.00
3.4	0.0	0.32674	0.00	0.27673	0.00
3.4	2.5	0.32686	0.00	0.27377	0.00
3.4	5.5	0.32473	0.00	0.27003	0.00

in Table III. The stability margins obtained using the QFT controller are of the practically same order as those of the gain-scheduled H_∞ controller.

TABLE III: Comparison of Frequency Domain Performance

Mach	α [Deg]	H_∞ Controller		QFT Controller	
		GM (dB)	PM (Deg)	GM (dB)	PM (Deg)
2.8	0.0	41.94	89.80	40.80	82.78
2.8	2.5	41.93	89.83	40.81	82.74
2.8	5.5	41.94	89.80	41.37	80.87
3.0	0.0	41.93	89.84	39.88	86.53
3.0	2.5	41.93	89.85	39.92	86.40
3.0	5.5	41.93	89.84	40.02	86.10
3.2	0.0	41.92	89.87	38.98	90.27
3.2	2.5	41.92	89.86	39.05	90.02
3.2	5.5	41.92	89.87	39.10	89.87
3.4	0.0	41.90	89.89	38.09	93.98
3.4	2.5	41.90	89.90	38.22	93.54
3.4	5.5	41.99	89.64	38.39	92.99

V. Conclusion

Plant parameter variation is the biggest concern for control design experts. For simplicity and trustworthiness, gain scheduling has been the design choice in the past. However, gain scheduling generally comes at the cost of computational overhead and can be laborious to design, as it does not account for other concerns such as controller order. Through this paper, we have proposed QFT as a design tool for Ramjet Engine thrust control. QFT, being a systematic and

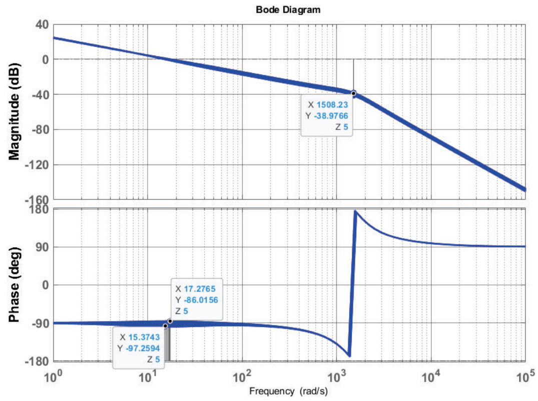

Fig. 9: Bode Plot at Different Operating Points

transparent methodology, can reveal system limitations at the design stage.

Moreover, QFT design results in minimum order, a single controller that satisfies all design specifications for all operating points. The time domain domain results obtained via QFT are slightly better in terms of settling time whereas % overshoot and frequency domain result are practically of same order as compared to the gain-scheduled H_∞ controller discussed in already published literature.

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