Design of Robust Controller for Electro-Pneumatic Actuator using Quantitative Feedback Theory

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***Abstract*—This paper purposes the Quantitative Feedback Theory (QFT) as a controller design methodology for Electro-Pneumatic actuators (EPAs). Main objective of this work is to demonstrate how a single, robust, minimum order controller can be designed using QFT for various plant parameter variations. The controller design problem is taken from an already published literature. The extensive simulation study is carried out to demonstrate the advantage of the proposed QFT based robust controller design measured in terms of better transient responce as well as tracking performance. The time domain and frequency domain results obtained from simulations are presented and discussed.**

Keywords: Electro-Pneumatic actuator, Non-linear control, QFT, Servo control, Robust control

I. Introduction

EPAs are the popular choice for medium weight category aerospace vehicle because of their high torque/weight ratio, compactness, robustness and speed of response etc. [2]. Aerospace vehicle control system designer generally gives the actuator specifications in terms of actuator dynamic performance such as rise time, % overshoot, -90 deg bandwidth etc. The controller designer not only addresses these design requirements but also cater for computational resource available, plant parameter variations and overall stability of the system.

The highly non-linear behavior, stringent controller design specifications and parameter variation of EPAs requires the robust control design technique. This paper purposes the QFT as controller design methodology for the EPAs. The QFT was first applied by Horowitz in 1963 for SISO plant, later extended to MIMO plants [3]–[5], is a systematic robust control design methodology which results in a single controller for all plant parameter variations and meets all design specifications.

The EPA linear model for this work is taken from [1]. To add the more practical aspect to this nominal model, plant parameter variations is added to 4 nos. parameters. The controller design problem for single plant using classical control tools is attempted in [1] which resulted into 10^{th} order controller (hereby referred as the Classical controller). In this paper controller for a EPA is designed using QFT. The controller design using QFT resulted into a sixth order controller which satisfies almost all design specifications for all plant variations. Further, the time and frequency domain results obtained using the QFT controller marks the significant improvement in terms of rise time, % overshoot, settling time, reference tracking and autopilot phase contribution at the gain crossover frequency.

The rest of this this paper is divided into three parts. The first part describes the problem definition. The second part describes the formulation of QFT design problem. The third part discusses the time and frequency domain results with improvement over the controller designed in [1].

II. Problem definition

The open loop plant transfer function (servo transfer function) for this work is taken from [1] and is reproduced as below

$$P(s) = \frac{858116.91}{(s+1.18)(s^2+179.7s+41230)} \tag{1}$$

This nominal transfer can be represented in pole-zero-gain form as below

$$P(s) = \frac{k}{(s+\omega_p)(s^2+2\delta\omega_p s+\omega_n s^2)} \tag{2}$$

Where, $K = 858116.91$, $\omega_p = 1.18$rad/sec, $\delta = 0.442$, $\omega_n = 203$rad/sec. Given, uncertainties in system identification process adopted in [1] and to add more practical aspect to the problem, the parameter variation of $\pm5\%$ is added to K, ω_p, δ and ω_n.

The control system specification for the controller design as taken from [1] are reproduced in Table I.

TABLE I: Control system design specification for controller

Specifications	Value
Rise time	$\leq$ 60ms
Bandwidth (-90 deg BW)	$\geq$ 15 Hz
Gain Margin	$\geq$ 6 dB
Phase Margin	$\geq$ 45deg
Overshoot	$\leq$ 20%
Steady state error	$\leq$ 0.7deg

The controller design problem with added controller block as shown in Figure 1.

The Classical controller, which meets all the design requirement for the nominal plant is given by equation (3)

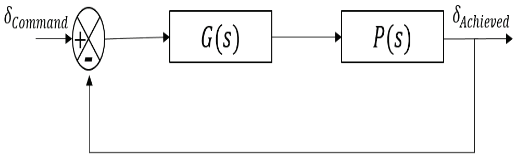

Fig. 1: Block diagram of EPA close loop control system

$$G_{\text{Classical}} = \frac{29990((s+71)(s+57)}{(s+1228)(s+116)} \times \frac{(s+16)(s^2+69.77s+41620)}{(s+107)(s+2)(s^2+724.8s+364800)} \times \frac{(s^2+0.6283s+39480)(s^2+1606000)}{(s^2+157.1s+394800)(s^2+633.3s+777700)} \quad (3)$$

The step response, bode plot of Open Loop Transfer Function (OLTF), reference tracking performance for 10Hz sinusoidal input of the Classical controller for nominal plant, is shown in Figure 2, Figure 3 and Figure 4 respectively.

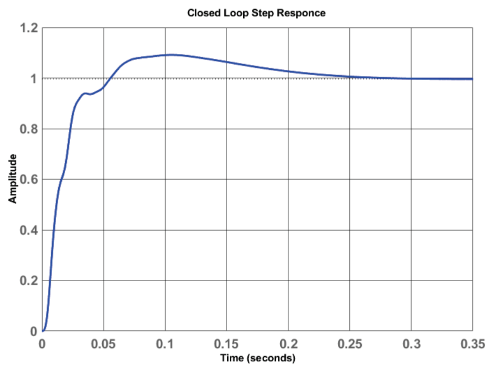

Fig. 2: Step response of the Classical controller for nominal plant

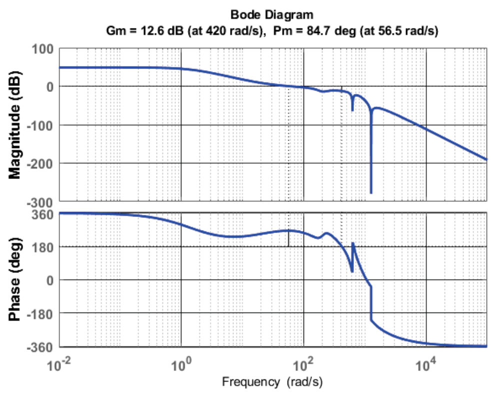

Fig. 3: Bode plot of OLTF with the Classical controller for nominal plant

The advantage / disadvantage, improvement of QFT controller over classical controller is discussed separately in results section.

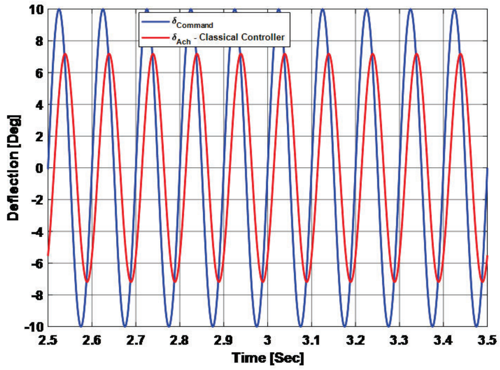

Fig. 4: Reference tracking performance of the Classical controller for 10Hz sinusoidal input

III. QFT design procedure

A. Plant template

For the different plant parameter variations, we will get different OLTF for the EPA model given by equation (1). Choosing ω as frequency template ([6]),

$$\text{Plant template} = P(j\omega)$$

Where,
$\omega \in [2, 4, 6, 8, 10, 12, 14, 16, 20, 24, 28, 32, 50, 100, 200]$rad/s.

The plant template is computed and plotted on Nicholas plot as shown in Figure 5. The nominal plant is taken as plant described using equation (1).

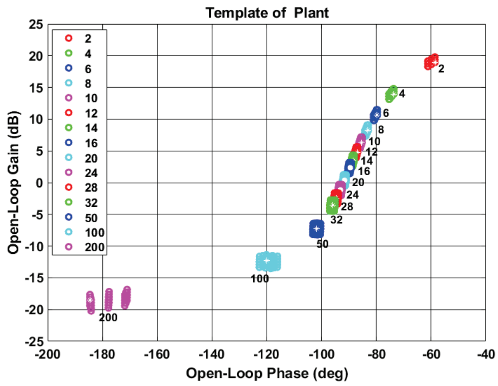

Fig. 5: Plant template for chosen frequency template

B. QFT Design Specifications

In order to meet all design specifications for all plant parameter variations, QFT requires the time and frequency domain specifications be translated into frequency domain inequality constraints.

1) Stability specification in QFT: The stability specifications for QFT controller can be specified in frequency domain as follows ([6])

$$\left| \frac{P(j\omega)G(j\omega)}{1+P(j\omega)G(j\omega)} \right| \leq W_s \quad (4)$$

for $\omega \in$ [2, 4, 6, 8, 10, 12, 14, 16, 20, 24, 28, 32, 50, 100, 200]rad/s.

Gain margin for the given W_s is:

$$GM = 20\log_{10}\left(1+\frac{1}{W_s}\right) \text{ in dB} \tag{5}$$

Phase margin for the given W_s is:

$$PM = 180 - 2\left(\frac{180}{\pi}\right)\cos^{-1}\left(\frac{0.5}{W_s}\right) \text{ in Deg} \tag{6}$$

By choosing $W_s = 1.05$, we obtain the following stability margins:

$$GM \geq 5.81 \text{ dB}, \quad PM \geq 56.87^\circ \tag{7}$$

Which are roughly close to with our design specifications given in Table I.

2) Time domain specifications in QFT: The time domain specifications (hereby referred as reference tracking bound) given in terms of rise time and % overshoot are specified using frequency domain inequality given as follow ([6])

$$\delta_{low}(\omega) < \left| F(j\omega)\frac{P(j\omega)G(j\omega)}{1+P(j\omega)G(j\omega)} \right| \leq \delta_{up}(\omega) \tag{8}$$

Where, $\delta_{low}(\omega)$ & $\delta_{up}(\omega)$ represents the desire time domain bounds for step command.

Choosing,

$$\delta_{low}(s) = \frac{1}{[(\frac{s}{a_L})+1]^2}; a_L = 62.5, \tag{9}$$

$$\delta_{up}(s) = \frac{(\frac{s}{a_U})+1}{[(\frac{s}{\omega_n})^2 + (2\zeta\omega_n s)+1]^2};$$
$$a_U = 33.75; \zeta = 0.82; \omega_n = \frac{1.25 a_U}{\zeta} \tag{10}$$

for $\omega \in [2, 4, 6, 8, 10, 12, 14, 16, 20, 24, 28, 32, 50]$ rad/s.

Step response for the reference tracking bounds is as shown in Figure 6.

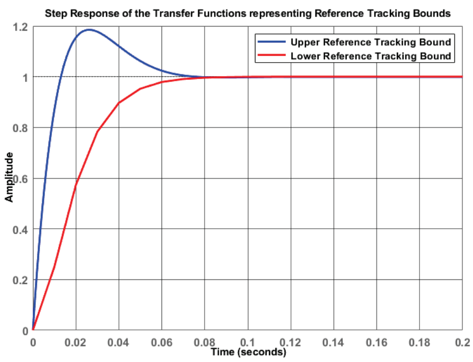

Fig. 6: Step response of the reference tracking bounds

3) Other specifications: The close loop bandwidth (-90deg BW) and steady state error specifications are checked for compliance after loop shaping is completed for stability and reference tracking bounds as there is no direct method is available to translated them into corresponding frequency domain inequalities.

4) QFT Bounds: The equations (4) and (8) are convert into quadratic inequalities. These quadratic inequalities are solved and represented as bounds on the Nichols plot. To carry out this process, Matlab based QFTCT toolbox is used. The stability bounds and reference tracking bounds are found and their intersection taken at each frequency of interest as shown in Figure 7, Figure 8 and Figure 9 respectively. These bounds represent the plant model with uncertainty and stability and reference tracking specifications satisfied at each frequency of interest.

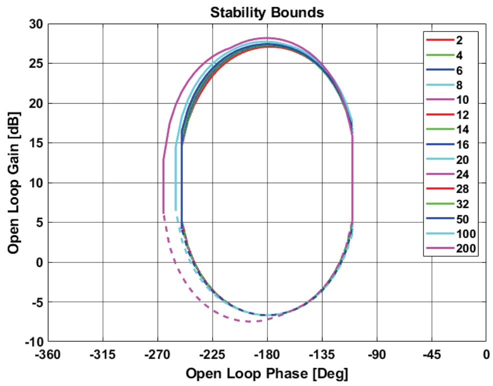

Fig. 7: QFT Stability bounds

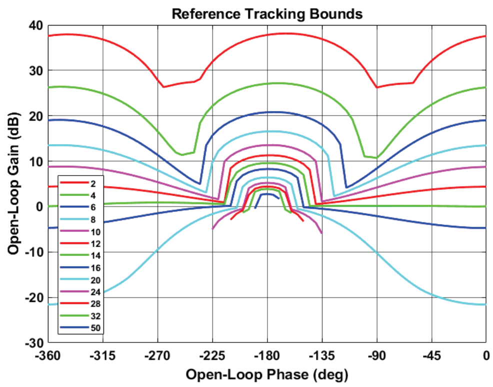

Fig. 8: QFT Reference tracking bounds

5) The QFT Controller Design: To deliver a single robust controller which meet all design specifications with all plant parameter variations, the intersection of bounds shown in Figure 9 need to be satisfy by nominal plant. Loop shaping is carried out using graphical tool provided by QFTCT toolbox until the nominal plant OLTF lies above the solid line and below dashed line at each frequency of interest. Further, to address practical aspect of PWM chattering at 100Hz and ensuring high roll off at higher frequency, a 2^{nd} order notch filter and a 3^{rd} order Chebyshev type-II low pass filter are taken as it is as initial controller components from [1]. Finally, a sixth order controller which satisfies all intersection of bounds is given below.

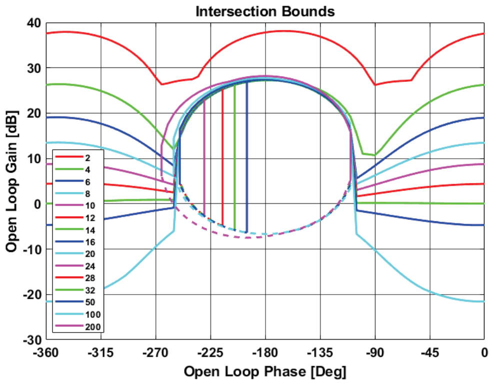

Fig. 9: Intersection of bounds to satisfy both stability as well as reference tracking specifications

$$G_{\text{QFT}} = \frac{1487.3(s+0.9)}{(s+1288)(s+0.375)} \times \frac{(s^2+1.606.10^6)(s^2+0.6283s+3.948.10^5)}{(s^2+633.3s+777.10^5)(s^2+157.1s+3.948.10^5)} \tag{11}$$

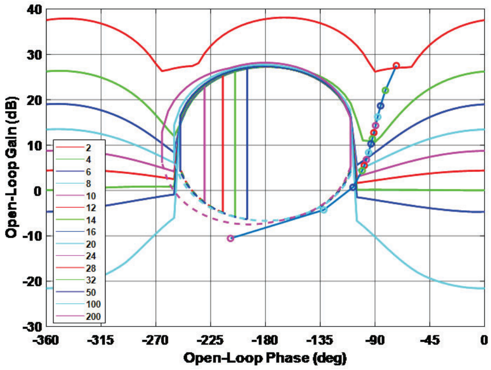

Fig. 10: Final QFT controller satisfying all QFT bounds

IV. Results

A. Time domain

The time domain simulations are carried out with the final QFT controller to confirm meeting the time domain specifications (Rise time, % overshoot and steady state error). The step response of the QFT and Classical controller for all plants is as shown in Figure 12.

The step response of the final QFT controller is analyzed and compared with design specification given in Table I. It is observed that the time domain specifications are satisfied for all plant parameter variations.

TABLE II: Time domain results using QFT controller for the nominal plant

Parameters	Specifications	Achieved
Rise time(ms)	≤ 60	17.2
Overshoot (%)	≤ 20	0.2331
Steady state error (deg)	≤ 0.7	0.2

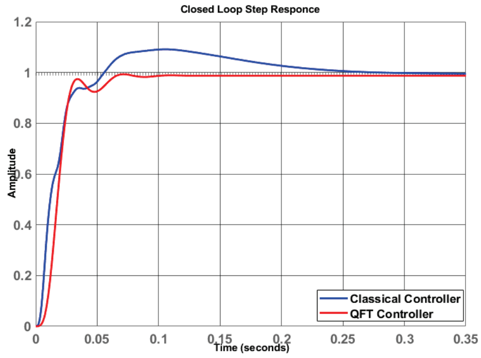

Fig. 11: Step response of QFT controller Vs the Classical controller for the nominal plant

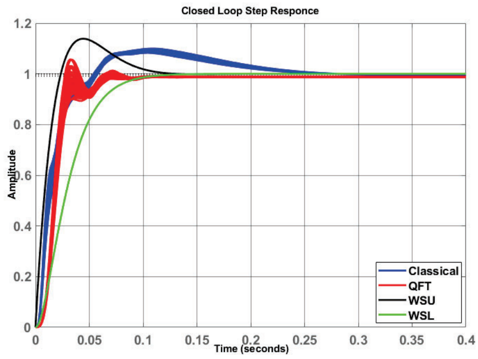

Fig. 12: Step response of the Classical controller and QFT controller for all plants

Further a relative comparison of time domain results obtained in [1] with QFT controller shows that the QFT controller marks significant improvement in rise time, % overshoot and settling time whereas albeit satisfying design specifications, but slightly inferior in terms of steady state error.

TABLE III: The QFT controller vs the Classical controller time domain results for the nominal plant

Parameter	Classical controller	QFT controller	% improvement
Rise Time (ms)	24	17.2	28.33
Overshoot (%)	14.5	0.2331	98.39
Settling time (ms)	219.6	60.8	72.31

TABLE IV: The QFT controller Vs the Classical controller time domain results for all plants

Parameter	Classical controller		QFT Controller	
	Min	Max	Min	Max
Rise Time (ms)	19.6	26.4	14.8	22.3
Overshoot (%)	8.70	10.41	0.0	6.64
Settling time (ms)	210.6	229.5	57.4	73.8

To gain little more insight into tracking performance of QFT controller and the Classical controller, time domain simulation is carried out for the sinusoidal input of 10Hz with 10deg amplitude as shown in Figure 13. It is observed

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that the QFT controller owing to its faster settling time marks significant improvement in controller's ability to track sinusoidal input up to 10Hz.

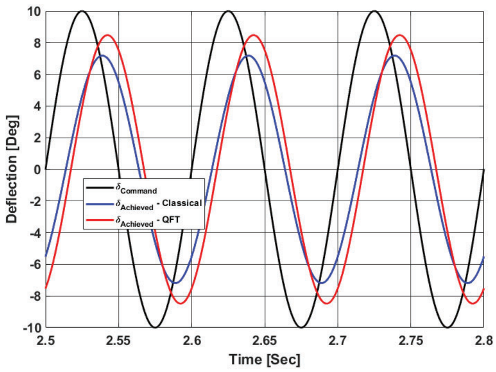

Fig. 13: The QFT controller Vs the Classical controller, reference tracking for sinusoidal input of 10Hz

B. Frequency domain

The QFT controller results were analysed in frequency domain to check the design specification compliance. The QFT controller Vs the classical controller OLTF bode plot for the nominal plant is shown in Figure 14.

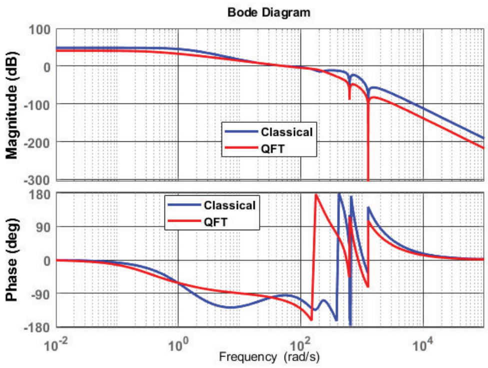

Fig. 14: QFT controller Vs the Classical controller OLTF bode plot for the nominal plant

The analysis of the QFT controller results indicates that it meets most of the stability and CLTF bandwidth specifications outlined in Table I, with the exception of not fully meeting the -90deg bandwidth requirement for certain plants.

TABLE V: The QFT controller frequency domain results, specification Vs achieved for the nominal plant

Parameter	Specification	Achieved
Gain margin (dB)	$\geq$ 6 dB	8.51 dB
Phase margin (deg)	$\geq$ 45 deg	70.1 deg
-90 deg Bandwidth (Hz)	$\geq$ 15 Hz	15.1 Hz

Practically, we know that actuator is element of inner most loop in aerospace vehicle autopilot design, therefore its phase contribution at autopilot gain crossover frequency is more important factor compared to -90deg bandwidth from

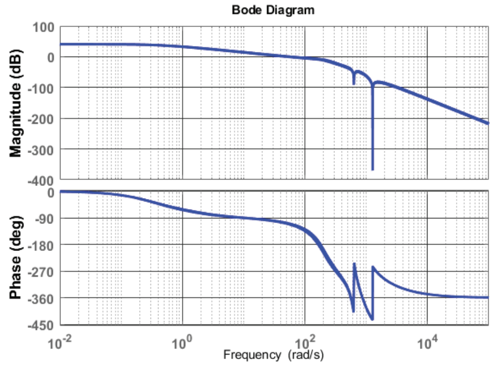

Fig. 15: OLTF Bode plot of the QFT Controller for all plants

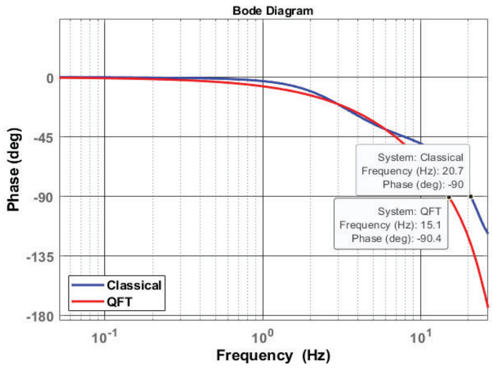

Fig. 16: QFT controller Vs the Classical controller CLTF -90 deg Bandwidth plot for the nominal plant

autopilot design point of view. Generally, the autopilot gain crossover should be less than one third of actuator bandwidth.

We have analysed the actuator phase lag contribution of the QFT controller Vs the Classical controller at autopilot gain crossover frequency (taken as 5Hz). Analysis shows that, owing to lower order controller design, the QFT controller contributes almost 2deg less phase as compare to the Classical controller for nominal plant. This saving in phase contribution is evident in all plants.

This improvement in phase contribution at autopilot gain crossover frequency for all plants suggest that shortfall in terms of -90deg bandwidth criterion for some of the plants can be safely ignored.

V. Conclusion

In this paper we have purposed the QFT as the controller design methodology for the EPAs. The QFT owing to its systematic and transparent design procedure resulted into a sixth order controller as compare to tenth order controller design in an already published literature. The time domain simulation shows that QFT controller marks significant improvement in rise time, %overshot, settling time and tracking performance for the sinusoidal input up to 10Hz.

QFT design process allows users to simultaneously see all control design constraints in single window. The graphical loop shaping process available in modern CAD tools further allows users to select minimalistic controller which satisfies

TABLE VI: The Classical controller vs the QFT controller frequency domain results for all plants

Parameter	Classical Controller		QFT Controller	
	Min	Max	Min	Max
Gain Margin (dB)	10.7	134	6.96	10.0
Phase margin (deg)	81.9	86.3	65.6	73.6
-90 deg Bandwidth (Hz)	19.11	22.45	13.7	16.4
Phase contribution at autopilot ω_{ge} (5 Hz)	-32.0	-38.6	-30.3	-36.6

all the bound at all frequency of interest. The lower order of QFT controller results into a cheaper computational resources.

Further phase lag contribution due to actuator at autopilot gain crossover frequency is also analyzed. We observed that at autopilot frequency of interest, the QFT controller contributed less phase lag which may significantly improve autopilot phase margins.

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