Aero-Heating Prediction Model for Hypersonic Vehicles

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Abstract: In the hypersonic flows during re-entry phase of missiles, the high kinetic energy of air molecules creates extremely high aerodynamic heating loads which constraints the hypersonic vehicle design and its trajectory. Approximate estimation of Aerodynamic heating is required for vehicle design and trajectory optimization. Aerodynamic heating can be accurately estimated using CFD tools but these requires high computational resources and processing time which make it an unsuitable option for design tasks requiring multiple iterations.

The proposed Aero-heating model estimates the heat flux using Empirical heat transfer correlations and thermodynamic relations instead of numerically solving the boundary layer partial differential equations. The temperature response of the vehicle is modelled using lumped capacitance approach and the differential equation is solved over the trajectory w.r.to time. The Proposed model provides approximate estimation of heat flux and temperature over the trajectory at fraction of the computation time and is much suitable for preliminary design and trajectory optimization tasks.

Introduction

Hypersonic flows are high Mach number flows usually occurring during re-entry phase of missiles and vehicles designed to attain high Mach speeds. In the hypersonic flow, the high kinetic energy of air molecules creates extremely high aerodynamic heating loads which constraints the trajectory selection and affects the hypersonic vehicle design. Aerodynamic heating can be accurately estimated using CFD tools but these requires high computational resources and processing time which make it an unsuitable option for trajectory optimization and preliminary design tasks which required repeated calculation of the aerodynamic properties.

Literature survey presents numerous engineering approaches to approximately predict the thermal heat loads and flow properties over the simplified vehicle geometries at hyper sonic flows. These approaches are both fast and computationally less expensive. The engineering methods begin with determination of inviscid flow field and surface pressure using the assumption of isentropic flow post shock expansion. Axis symmetric analogue of boundary layer equations are developed for 3D bodies and is solved along the inviscid streamline over the vehicle surface[1]. Although this approach is quite accurate but is still complex as it requires determination of Streamlines and solving of differential equation along the streamline[2].

The proposed model is a simpler method for heat flux and temperature estimation at hypersonic flows over a simplified geometry. The solution method is developed under the hypothesis of thin shock layer theory which helps to easily compute the inviscid flow field and boundary edge conditions around the vehicle after the shock expansion. Then empirical correlations are utilized to predict the heat flux over the surface (similar to prediction of heat flux over the flat plate in incompressible flows) using dimensionless numbers like Nusselt, Reynolds and Stanton number. The accurate modelling of the real gas behavior is required through the curve fit methods, since in hypersonic flow high temperature effects are significant and cannot be neglected. The Model is represented in Fig 1.

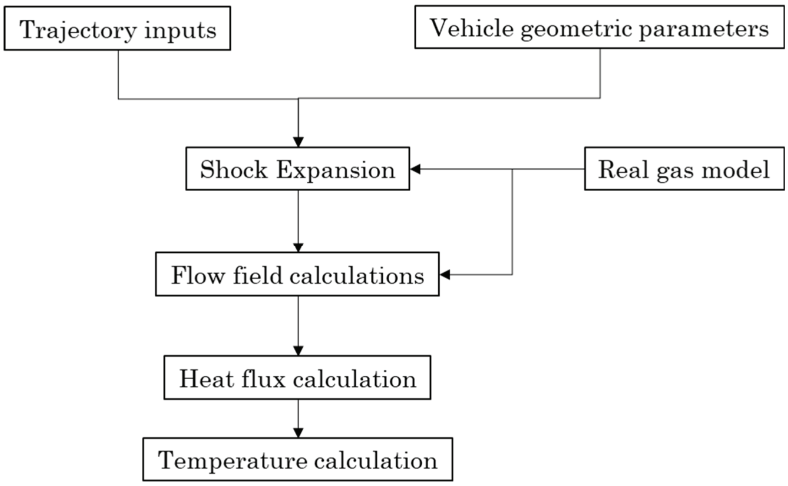

Fig 1 Aero-heating Model

Aero-Heating Model

The Aero-heating prediction begins with the identification of the dominant modes of heat transfer. In the hypersonic regime the aero heating is mainly caused due to the frictional effects of the air molecules with the surface which is governed by convective heat transfer. Also at elevated temperatures the body loses heat to the environment in the form of radiation heat transfer. To predict the convective heat transfer rate, the flow field around the vehicle needs to be specified and to calculate temperature. Following sub-models help to achieve the same. [3]

Shock Expansion Model

As the vehicle is moving at hypersonic speeds the flow field depends on the shock wave formed and aftershock properties. At hypersonic flows usually such shocks are extremely thin and across it large gradients of pressure, velocity and density exists. Normal shock, Oblique shock or Bow shock can be formed based on the geometry of the obstructing body. Applying the conservation equations across a normal shock provides the following three governing equations to calculate the aftershock properties [4] [5]:

1) Conservation of mass

$$\rho_\infty V_\infty = \rho_1 V_1$$

2) Conservation of momentum

$$P_\infty + \rho_\infty V_\infty^2 = P_1 + \rho_1 V_1^2$$

3) Conservation of energy

$$H_\infty + \frac{1}{2}V_\infty^2 = H_1 + \frac{1}{2}V_1^2$$

Solving the above three equations using ideal gas assumption gives the properties downstream of the shock wave as follows: -

$$M_1 = \frac{(\gamma-1)M_\infty^2 + 2}{2\gamma M_\infty^2 - (\gamma - 1)}$$

$$P_1 = \frac{1+\gamma M_\infty^2}{1+\gamma M_1^2} * P_1$$

$$T_1 = \frac{1+\frac{(\gamma-1)}{2}M_\infty^2}{1+\frac{(\gamma-1)}{2}M_1^2}$$

$$S_1 = Entropy(P_1, T_1)$$

The oblique shock formed at inclined flat surfaces is analyzed in similar way as normal shock by resolving the free stream and downstream velocity components normal and parallel to the formed shock wave. Bow shock is formed for blunt nose and its properties can be closely estimated using normal shock equations and properties downstream of bow shock can be calculated using isentropic expansion of normal shock aft properties for the local surface pressure.

Real Gas Model

In the hypersonic region above Mach 6, air molecules in the vicinity of the body store energy at the elevated temperatures created by the strong bow shock in the form of vibrational energy modes in addition to rotational and translational energy modes. The perfect gas law is no longer valid. This departure from perfect gas behavior is often referred to generically as real-gas effects which cannot be neglected. [3]

Real gas model consists of various sub-models to compute Temperature, Pressure, Density, Enthalpy, Entropy. An excellent set of polynomial correlations for Tabulated data of High temperature air properties was obtained by Tannehill and Mugge[6].They are used in the model for the real gas properties computations of a flow field.

Table 1 Coefficients of curve fits equation [6]

Density Range	Curve Range	k1	k2	k3	k4	k5	k6	k7
Y > -0.5	Z > 0.9	0.2358	-0.0433	1.1761	0.04649	0	-0.1437	0
	0.9 > Z >0.48	0.2740	0	1.0008	0	0	0	0
-4.5 < Y < -0.5	Z > 2.176	0.4182	-0.2521	0.7540	0.1445	0	0	0
	2.176 > Z > 1.478	1.0417	0.0419	0.4127	-0.0093	0	0	0
	1.478 > Z > 0.9165	0.4576	-0.0342	0.8191	0.0464	0	0	0
	0.9165 > Z > 0.48	0.3816	0.0012	0.9904	0	0	0	0
-7 < Y < -4.5	Z > 2.47	-45.0871	-9.0050	35.8685	6.7922	-6.7760	-0.0647	0.0253
	2.47 > Z > 1.79	1.0172	-0.0179	0.4735	0.0254	0	0	0
	1.79 > Z > 1.35	1.1140	0.0022	0.3518	0.0172	0	0	0
	1.35 > Z > 1	1.3992	0.1677	-0.1431	-0.1592	0	0	0
	1 > Z > 0.3	0.2718	0.0007	0.9901	-0.0049	0	0	0

Density Range	Curve Range	k8	k9	k10	k11	k12	k13	k14	k15
Y > -0.5	Z > 0.9	0	-1.3767	0.1604	1.0898	-0.0834	-0.2117	-10	-1.78
	0.9 > Z >0.48	0	0	0	0	0	0	0	0
-4.5 < Y < -0.5	Z > 2.176	0	-2.0001	-0.6390	0.7160	0.2064	0	-10	-2.4
	2.176 > Z > 1.478	0	-0.4340	-0.1969	0.2648	0.1005	0	-15	-1.778
	1.478 > Z > 0.9165	0	-0.0732	-0.1698	0.0432	0.1118	0	-15	-1.28
	0.9165 > Z > 0.48	0	0	0	0	0	0	0	0
-7 < Y < -4.5	Z > 2.47	-1.2737	0	0	0	0	0	0	0
	2.47 > Z > 1.79	0	-2.1737	-0.3347	0.8986	0.1273	0	-20	-2.22
	1.79 > Z > 1.35	0	-1.1509	-0.1735	0.6733	0.0883	0	-20	-1.56
	1.35 > Z > 1	0	-0.0276	-0.0907	0.3070	0.1216	0	-20	-1.17
	1 > Z > 0.3	0	0.9907	0.1751	-0.9824	-0.1592	0	-20	-0.88

Temperature correlation as a function of pressure and density is as follows:-

$$\log\left(\frac{T}{T_o}\right) = k_1 + k_2 Y + k_3 Z + k_4 YZ + k_5 Y^2 + k_6 Z^2 + k_7 ZY^2 + k_8 YZ^2 + \frac{k_9 + k_{10} Y + k_{11} Z + k_{12} YZ + k_{13} Z^2}{1 + \exp\left(k_{14}(Z + k_{15})\right)}$$

Where,

$$T_o = 288.16K$$

$$Y = \log\left(\frac{\rho}{1.225}\right)$$

$$X = \log\left(\frac{\rho}{1.0134\, e5}\right)$$

$$Z = X - Y$$

The coefficients of the equation are given in Table 1.

Flow field model

One of the properties of a hypersonic flow is it produces a shock which is very close to the body characterized by a thin shock layer. Flow field (Velocity, Pressure and temperature) around a body for a hypersonic flow can be approximately calculated easily using the thin shock layer theory assumption. Newton theory is valid under thin shock layer assumption and is one of the local inclination methods for flow field determination.

Newton assumed that the fluid particle loses its momentum normal to the flat plat upon incidence and tangential component is unaltered. It is utilized to compute velocity and pressure field. The tangential component gives the inviscid velocity field around the body. And the normal component gives the inviscid pressure field around the body as shown in Fig 2.[3]

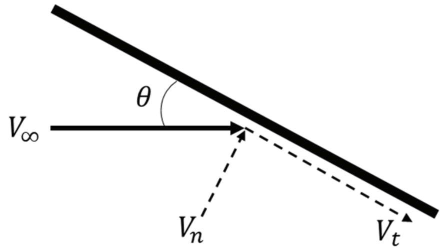

Fig 2 Flow around Flat Plate

Therefore: -

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2} = 2\sin^2\theta$$

This method is used to compute pressure in the blunt nose section of the missile.

Tangent wedge/cone methods are used for pressure computation of the conical / flat body section of the vehicle. The local surface pressure can be approximated with the pressure after oblique shock on the equivalent wedge or cone with angle equal to the local inclination angle as shown in Fig 3.[3]

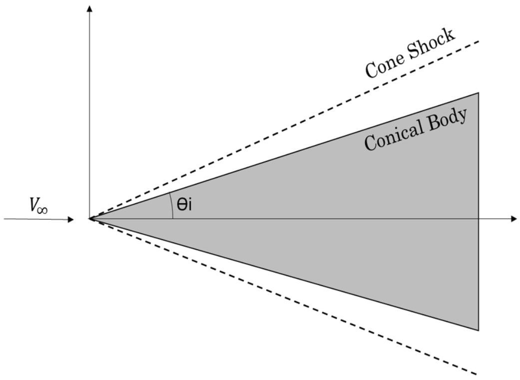

Fig 3 Tangent cone method

In order to describe correctly the viscous behaviour of the flow, we need a relation to describe properly the dependence of viscosity, thermal conductivity etc.wrt temperature. The classical Sutherland's law is valid for air in a range of several thousands of degrees and is appropriate for hypersonic flow calculations.[3]

The relations are as follows: -

$$\mu(T) = \mu_o\left(\frac{T}{T_o}\right)^{\frac{3}{2}}\left(\frac{T_o + 110}{T + 110}\right)$$
$$k(T) = k_o\left(\frac{T}{T_o}\right)^{\frac{3}{2}}\left(\frac{T_o + 110}{T + 110}\right)$$

Where, To = 288K and μ_o= 1.789e-5 Kg/ms

Heat Transfer Model

The empirical correlations offer a fast and approximate method to calculate heat flux instead of solving the boundary layer partial differential equations which is well suited for our application. The heat transfer due aerodynamic heating of the vehicle is modelled using first order linear differential equation under lumped mass assumption. Under lumped mass assumption the whole body is assumed to be present at uniform temperature and no temperature gradients are present within the body leading to no heat transfer within the body, only heat transfer between the body and the surrounding takes place. The lumped capacitance model of heat transfer provides average temperature of the surface of the

vehicle instead of the localized temperatures of the various sections.
The governing equation is as follows: -

$$(mC)_v \frac{dT_w}{dt} = \left(q"_{conv} - q"_{\mathrm{rad}}\right) * Area_v$$

The above differential equation shows the variation of vehicle wall temperature with time. Integrating the above equation with time along the trajectory provides the average temperature profile of the hypersonic vehicle.
The unknown in the equation, convective heat flux is calculated using the following empirical equation (similar to flat plate heating equations):

$$q"_{conv} = \rho_\infty V_\infty St\left(H_r - C_p.T_w\right)$$

Where, Stanton number is a Dimensionless number governing heat transfer and is a function of the aftershock flow field properties as follows: -

$$St = f(\rho^*, \mu^*, Re^*, Pr^*, V_e)$$

Simonides assembled different empirical correlations of Stanton number for different geometries into one single equation as follows. [7]

$$Re^* = \frac{\rho^*.V_e.L}{\mu^*}$$

$$Pr^* = \frac{\mu^*.C_p}{k^*} \approx 0.7$$

$$St = C\left(\frac{\rho^*}{\rho_\infty}\right)^{1-n}\left(\frac{\mu^*}{\mu_\infty}\right)^n\left(\frac{V_e}{V_\infty}\right)^{1-n}\left(\frac{L}{x}\right)^n Re^{*-n} Pr^{*-0.667}$$

The above equation is valid along a streamline as the local Reynolds and Stanton number is based on streamline running length 'x'. It is assumed that streamlines are parallel to the axis of the vehicle which holds quite well for small angle of attacks and removes the requirement of streamline computation. The averaged Stanton number over the surface is calculated by integrating the equation along the vehicle length over entire surface area.

Similarly, the stagnation point at the nose is the point experiencing the maximum heat flux which can be approximately predicted using the Fay Reidall equation described below. [8]

$$q_{nose} = 0.763\, Pr^{-0.667}(\rho_e\mu_e)^{0.5}\sqrt{\frac{du_e}{dx}}\,(H_r - H_w)$$

For the computation of St Number using empirical correlations, the vehicle geometry is broken down into simplified geometries as shown in Fig 4.
The overall heat flux is calculated from the respective geometry as follows:

$$\begin{aligned} q"_{conv}.Area_v = {} & q"_{nose}.A_{nose} \\ & + q"_{cone}.A_{cone} \\ & + q"_{plate}.A_{plate} \end{aligned}$$

The vehicle surface also radiates heat energy through radiation which becomes significant on increase of surface temperature. The radiation heat transfer is modelled simply using Stephan Boltzmann law as follows: -

$$q_{rad}" = \sigma\,\epsilon\,(T_w^4 - T_\infty^4)$$

Using the convective and radiative heat flux the 1-D heat transfer differential equation is solved along the trajectory.

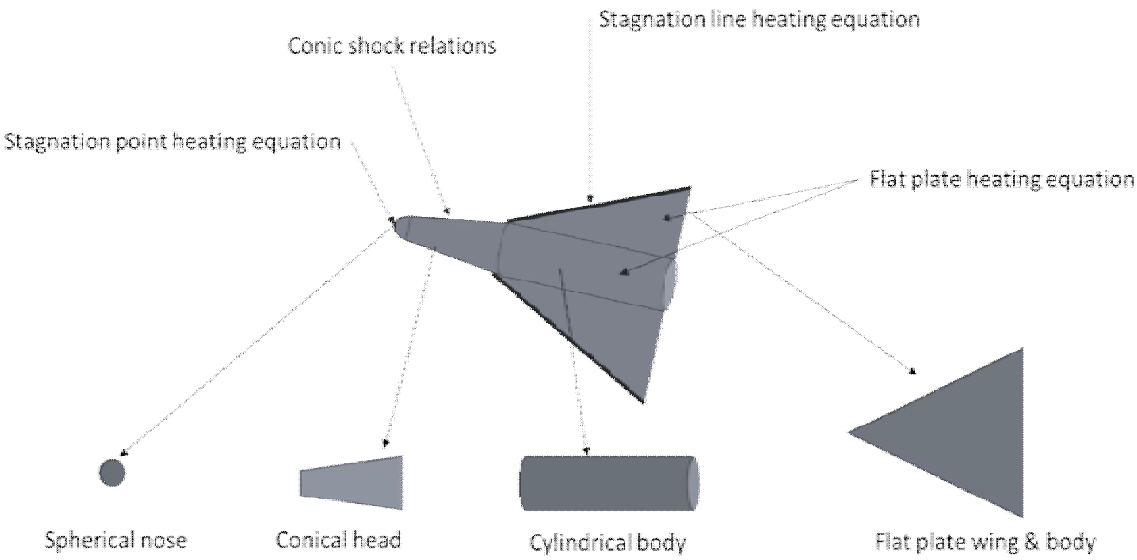

Fig 4 Simplified geometry

Model Validation

The Aero-heating model is tested to compute the heat flux and wall temperature of Re-entry Vehicle having conical shape with a blunt nose. The inputs to the model were trajectory data consisting of atmospheric pressure, temperature, density and Mach number with geometric and material properties of Re-Entry vehicle as summarized in following table.

The predicted heat flux profile and Temperature profile variation matches with the measured CFD simulations but the peak predicted heat flux value is over estimated by the heat transfer model and the response of the predicted temperature is slow leading to lower peak temperature estimates.

The predicted results are hence in close agreement with the CFD Simulated heat flux and temperature values. The inaccuracy can be attributed to the assumed material properties and limited flight path data available for the above configuration. The normalized results are compared in the following Figure 5-8: -

Table 2 Model Input Parameters

S. No.	Material/GeometricProperties	Trajectory parameters
1	Heat capacity C_{pv}	Altitude
2	Nose radius (R)	Velocity, Mach no.
3	Characteristic length (L_{ref})	Ambient Pressure
4	Emissivity (ϵ)	Ambient Temperature, Density
5	Density of TPS-CC (ρ_v)	Angle of attack

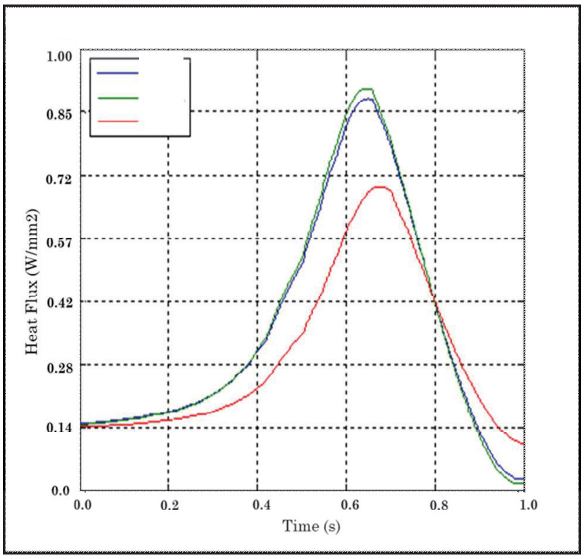

Fig 5 Estimated Heat flux from CFD

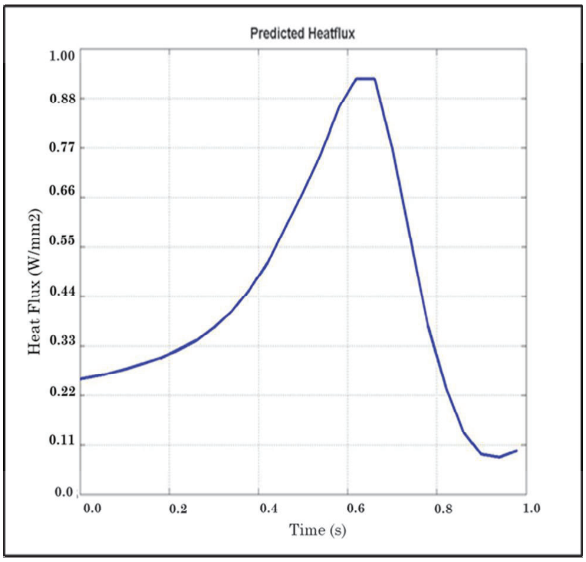

Fig 6 Estimated heat flux from Model

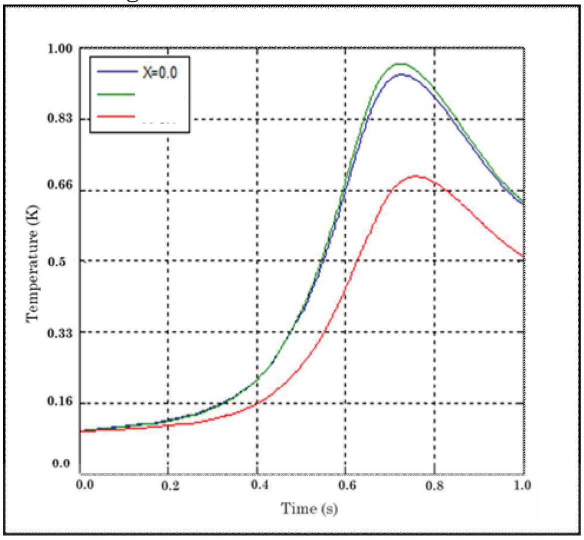

Fig 7 Estimated Temperature from CFD

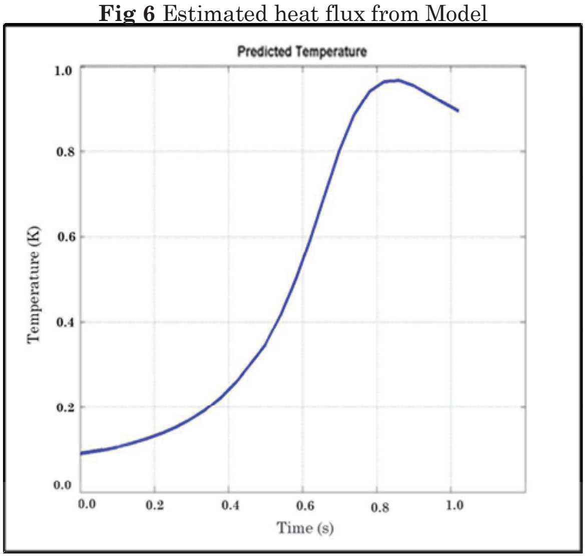

Fig 8 Estimated Temperature from Model

Conclusion

The proposed model is sufficient to capture the physics involved in the heat transfer process during re-entry for simplified geometries moving at hypersonic speeds. The model can be successfully utilized for design optimization, trajectory optimization and approximate aero-heating calculation tasks before utilizing complex numerical solvers for designing hypersonic vehicles. Hence this model can help to speed up the design and optimization process of hypersonic vehicles. The easy implementation logic with less computational load also makes this model suitable for onboard implementation, increasing missile intelligence.

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Abbreviations

ρ_∞	Free stream density
V_∞	Free stream velocity
P_∞	Free stream pressure
T_∞	Free stream temperature
H_∞	Free stream enthalpy
C_p	Coefficient of pressure
T_w	Wall temperature
$(mC_p)_v$	Heat capacity of vehicle
$q"_{conv}$	Convective heat flux
$q"_{rad}$	Radiative heat flux
St	Stanton number
H_r	Recovery enthalpy of boundary layer
H_w	Wall enthalpy
μ^*	Coefficient of viscosity
Re	Reynolds number
Pr	Prandtl number
L	Characteristic length
$\rho_e \mu_e$	Boundary layer edge properties (density, viscosity)