Reentry Maneuvering Vehicle Ascent Phase Trajectory Design

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***Abstract*—Maneuvering Reentry vehicles are designed to alter the trajectory during the atmospheric reentry phase unlike the predictable ballistic path. These vehicles are engineered to evade the missile defense systems, achieve glide capability and increase accuracy by controlled reentry maneuvers. For the vehicle to successfully execute its reentry maneuver an optimal reentry angle is to be maintained. Shallow reentry flight path angle is very crucial so as to enable maximum latax pull with less control effort. A typical Long Range missile system for a very wide Range Bracket is modelled as a two stage solid propelled vehicle and Reentry vehicle capable of maneuvering by means of aerodynamic surfaces. In order to have maneuvering reentry for the defined Range bracket, optimum reentry angle to be maintained for all range and azimuth conditions. For a solid propelled vehicle where engine cut-off is not possible, the reentry constraints are achieved by ascent phase trajectory shaping which ensures required conditions to enable maneuvering. This paper describes the Re-entry condition based ascent phase trajectory design approach for a maneuvering reentry vehicle to cater for the wide Range bracket and multiple terminal constraints are considered.**

***Index Terms*—Reentry Maneuver, Ascent Trajectory Design,Trajectory Optimization**

I. Introduction

In present warfare scenario, Guided Weapon systems with precision and deception are of greater importance that can be achieved by vehicle maneuvering capability. In order to get the desirable re-entry conditions most of the System engineers traditionally design vehicle coupled with constraints such as dynamic pressure, load factors, and thrust. These constraints significantly limit the vehicles flight envelope. To ensure the vehicle operates within permissible constraint boundaries, it is crucial to plan a flight corridor that satisfies multiple constraint conditions. The emphasis on design is to have better control margins and Flight load bearing capability during atmospheric phase. The trajectory design options of a ballistic missile guidance are lofted trajectory and Shallow Trajectory solutions as shown in Fig.1

A. *Lofted Trajectory Design:*

Lofted Trajectories follows a high arcing flight path where primary aim is to achieve higher altitudes where path control is very limited.

- **Advantages:** Attitude stability, less heating, reduced drag and atmospheric disturbances resulting in limited control requirements.
- **Disadvantages:** Increased vulnerability to detection and interception, complexities in maneuverability. It requires more sophisticated guidance systems to manage the longer flight time and potential for navigation errors.

B. *Shallow Trajectory Design:*

Shallow trajectory follows low altitude flight path.

- **Advantages:** Potential for Stealth, Terrain following capabilities, Maneuverability and Precise Targetting
- **Disadvantages:** Increased Drag, Lower Throw weight, Precise Control Requirements, Reduced Accuracy, increased Heating and Deceleration

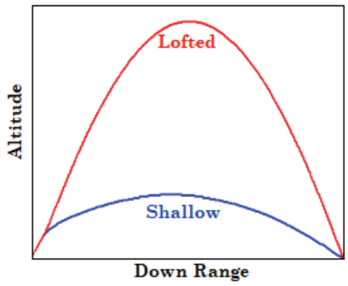

Fig. 1: *Trajectory solutions*

The typical Mission profile of a ballistic missile has ascent powered phase, followed by ballistic free flight and atmospheric re-entry. During the ascent phase the missile follows a pre-determined sequence of events where the missile is guided to specific injection point to reach the intended target. The ascent phase guidance decides whether the trajectory is a shallow trajectory or a lofted trajectory based on the total energy of the vehicle and the constraints of controllability and flight loads. A typical Mission profile of a Ballistic missile for solid propelled vehicle during atmospheric phase [1] is shown in Fig.2

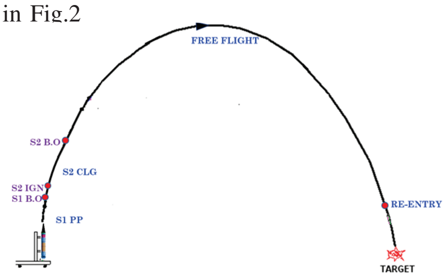

Fig. 2: *Typical Events of Flight Profile*

Typical Ballistic missiles have no maneuverability. In order to increase the terminal precision and deception capability maneuverability is employed. However, to have a maneuvering re-entry desired re-entry conditions to be achieved by appropriate trajectory planning [2]- to meet the re-entry requirements. Trajectory optimization is one of the critical aspect of mission design.Indirect and direct Trajectory optimization methods are available for the solving optimization problem [3]. Specific energy-based trajectory design [4] is essential to cater for propulsion variation in solid rocket motors. Any optimization technique can be adapted to the energy framework. Indirect methods use Pontryagins Minimum Principle to derive optimality conditions in the energy domain to solve the resulting boundary value problem (BVP) for adjoint variables associated with specific energy. It has high accuracy and is ideal for ballistic missile range maximization like optimizing a ballistic missiles lofted trajectory to maximize specific energy at apogee. Direct Methods use Direct Transcription based on Non-Linear programming (NLP) [5]. Trajectory Optimisation of Long Range and Tactical Systems [3]-Direct Collocation, Direct Shooting, Pseudo spectral Methods, Model Predictive Control (MPC) algorithms etc. In Direct collocation method, the trajectory is discretized in the energy domain, representing states and controls with polynomials. It handles complex constraints well and best suited for Cruise missiles maintaining constant specific energy for low-altitude flight. Direct Shooting method parameterizes controls as functions of specific energy, integrating energy-state dynamics. It is Simpler for real-time guidance and suitable for Air-to-air missile intercept trajectory. Pseudo spectral Methods use orthogonal polynomials to approximate energy-state trajectories and are efficient for smooth, long-range trajectories suitable for hypersonic missile boost-glide trajectory. Model Predictive Control (MPC) solves a finite-horizon optimization problem in real-time, updating the energy-based trajectory based on target tracking and Adapts to dynamic targets or disturbances and suitable for Missile intercepting a maneuvering aircraft.

Guidance schemes for launch vehicles and ballistic missiles can be described in three categories Explicit, Implicit and Perturbation Guidance Schemes.In implicit guidance scheme, the predefined nominal trajectory is stored and corrects the deviation on-board which is not suitable for long range ballistic flight where parametric variations are influenced by various factors such as propulsion variation, atmospheric disturbances etc. In explicit guidance, the optimal trajectory path is computed on-board based on the present vehicle state. Perturbation Guidance scheme is used to guide a vehicle along the desired trajectory by making small perturbations to its control inputs based on deviations from nominal path. These schemes have relatively lower tolerances to parameter variations compared to explicit guidance schemes but were preferred because of ease of implementation [1].

Various guidance schemes for re-entry maneuvering have been explored such as, the cross-product, Bank to Turn, Proportional and Tangent Cubic Guidance Laws. Here, Proportional and tangent cubic law is implemented in phased manner as per the desirable terminal conditions given in Guidance and Control of Maneuvering Re-entry Vehicles [6].

In order to enhance the maneuvering and deception capability of any existing vehicle, which is not designed for shallow trajectory, a novel approach of Ascent phase trajectory shaping through engineering design method is designed. This approach not only minimizes ascent phase flight load but gives better desirable re-entry conditions essential for re-entry maneuvering. This technique can be applicable for any Re-entry glide vehicle trajectory design. This sub-optimal trajectory encompasses a lofted and then a shallow trajectory where the vehicle initially has almost vertical flight path then transitions to a flatter and more horizontal trajectory. The quintessential suggested design utilizes features of Lofted and Shallow trajectory to achieve desirable impact conditions.

II. PROBLEM DEFINITION

Ballistic missile has three phases of flight i.e. powered phase, free flight phase and the re-entry phase. Guidance can be carried out only during the powered phase. Powered phase is again divided into two parts, the pre-programmed open loop guidance scheme for the atmospheric phase and the closed loop guidance (CLG) scheme for the upper atmospheric phase.

For a guided missile system, to ensure maneuvering trajectory during re-entry phase, it has to re-enter the atmosphere at a shallow flight path angle so that the control effort and the flight loads are minimum to attain a higher latax pull up capability. This desired flight path angle at the start of re-entry guidance is attained by judicial ascent phase trajectory design.

For a two-stage vehicle, a novel guidance scheme is designed. The vehicle first follows a Lofted trajectory path in the Stage-1 atmospheric phase followed by a steeper turn in Stage-2 for achieving a Shallow trajectory path. This scheme though is sub-optimal for a given Range and energy conditions but achieves desirable conditions suitable for re-entry maneuvering trajectory path.

The Mission profile for the proposed ascent phase trajectory design is shown in Fig.3

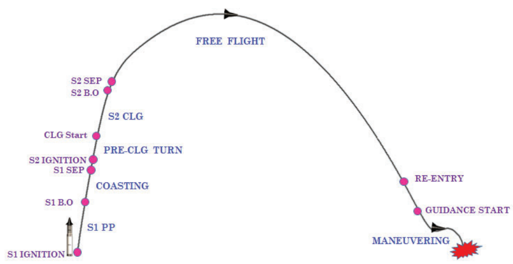

Fig. 3: *Modified Mission Profile*

III. DESIGN PHILOSOPHY

A typical two stage vehicle follows a pre-defined pitch turn during Stage-1. On-board guidance is not preferred in Stage-1 due to structural limitations and flight performance requirements as vehicle is in dense atmosphere, and limited pitch turns are employed. During Stage-2, inertial navigation and guidance scheme is followed which guides the missile to the pre-defined target even in presence of external disturbances based on on-board computations. The guidance steers the vehicle to a burnout point, uniquely related to the velocity and flight path angle condition based on the specified range of the target. During the free-flight phase the vehicle follows an elliptical path based on the injection conditions till the re-entry point. For a two-stage vehicle, the re-entry point conditions are achieved by adapting the terminal constraint based design philosophy. This ascent phase design scheme ensures desired Re-entry conditions for all Range brackets.

For a Re-entry maneuvering vehicle the ascent phase trajectory design is slightly different from the traditional design.

Design philosophy is divided into four phases i.e.

- Stage-1 Pitch Program Phase: Optimized Energy Based Pitch Program during Stage-1
- Coasting Phase: To get desirable Stage-1 Separation conditions after S1 Burnout.
- Pre-CLG Turn Phase: Range based Pitch turn prior to Closed Loop Guidance
- Stage-2 Closed Loop Guidance: Explicit Energy Management Guidance Scheme

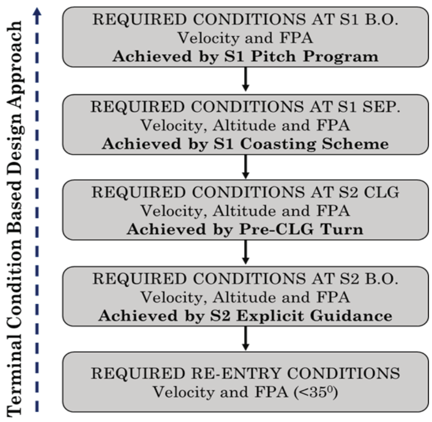

Fig. 4: *Design Philosophy*

To achieve desired stage burnout altitude and flight path angle with flight load constraints during the atmospheric phase Energy based Pitch Program is implemented. Coasting scheme is implemented with on-board computation of coasting time in order to achieve the desired altitude for Stage-1 Separation such that the disturbances on the vehicle during separation will be minimum. During Stage-2, required pitch turn will be computed on-board to achieve the desired input conditions required for Stage-2 explicit energy management guidance solution and the pitch turn is executed prior to the Closed loop guidance phase. This scheme achieves similar Re-entry conditions for all Range and azimuth conditions with the variation in Thrust profiles, thus ensuring desirable Maneuvering Re-entry under all conditions.

The typical theta profile and flight path angle profile with the new design philosophy are shown in Fig.5 and Fig.6 below.

IV. IMPLEMENTATION METHODOLOGY

A Typical Ballistic missile orbit [7] as shown in Fig.7. consists of initial powered phase, free flight and atmospheric re-entry phase. The free flight path of the missile is governed by the ascent phase guidance during the powered phase. The design philosophy discussed gives various schemes adopted

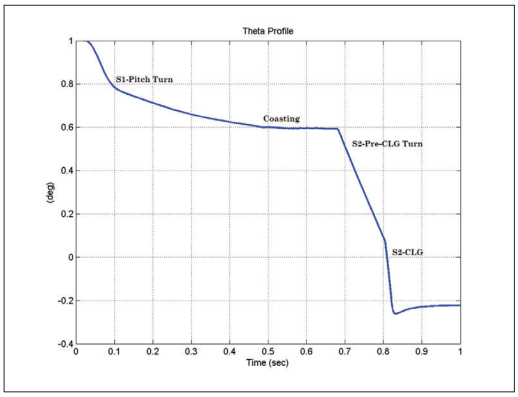

Fig. 5: *Ascent Phase Theta (Normalised)*

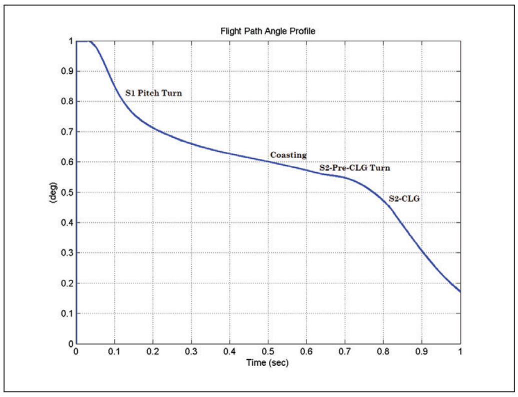

Fig. 6: *Ascent Phase Flight Path Angle(Normalised)*

to achieve a shallow ballistic trajectory to facilitate for an enhanced re-entry maneuvering capability. The implementation of the design philosophy is carried out as discussed below.

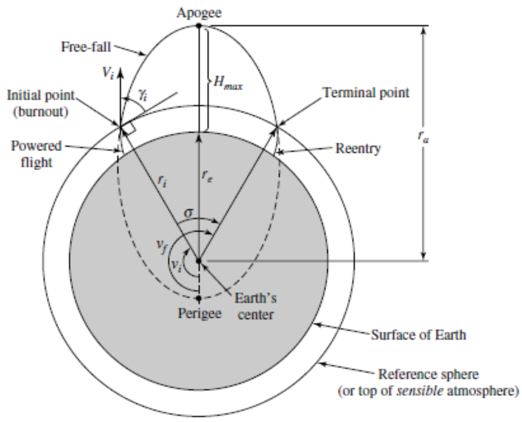

Fig. 7: *Typical Ballistic Missile Orbit*

A. Optimized Energy Based Pitch Program during Stage-1 (S1 PP)

This design technique basically includes nonlinear programming for design of energy based Pitch Program in Stage-1 [4] to ensure optimum Flight Path Angle at S1 burn out condition for all range brackets catering variations in thrust profile, mass and aero perturbations. The simplified or 3 degrees of freedom equation as given below are used for optimizing the trajectory. 3 DOF system dynamics is considered with states of the system being relative velocity magnitude (V), flight path angle (Y) and altitude (h).

$$\dot{V} = \frac{(T-D)\cos(\alpha)}{m} - g\sin(\gamma)$$

$$\dot{\gamma} = \frac{1}{mV}\{(T-D)\sin(\alpha) - (QSC_{N\alpha}\alpha)\} - \frac{g}{V}\cos(\gamma)$$

$$\dot{h} = V\sin(\gamma)$$

The outcome of energy based pitch program minimizes S1 Burn out altitude variation w.r.to the various propulsion profiles as compared to Height based and time based pitch programs. The results are shown in Fig.8, Fig. 9 and Fig.10.

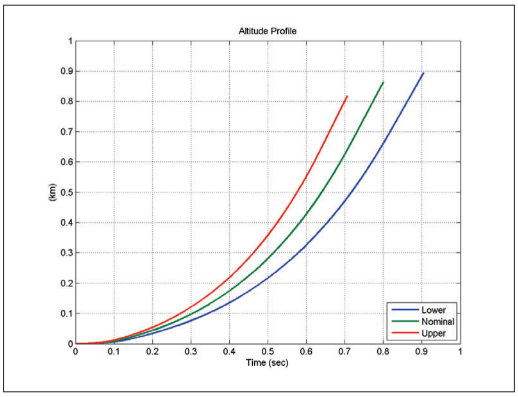

Fig. 8: *Ascent Phase Altitude Profile(Normalised)*

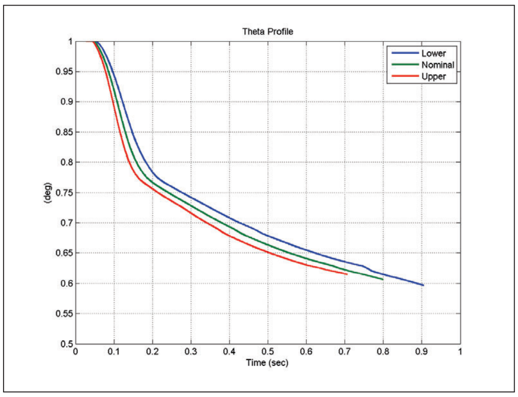

Fig. 9: *Ascent Phase Theta Profile (Normalised)*

B. Coasting after Burnout for desirable Stage-1 Separation conditions (Coasting)

In general ,for lofted trajectory design in ascent phase S1 burnout is expected to occur above 30 km altitudes for various range conditions. Coasting scheme is incorporated during Stage-1 prior to S1 separation to achieve the desirable Stage-1 Separation altitude for Stage-2 controllability, Velocity and Flight path angle conditions optimum for Target Range before

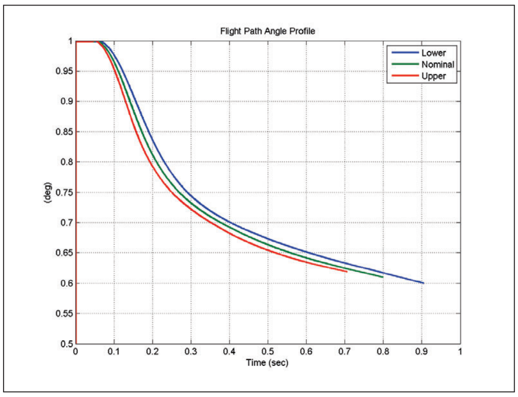

Fig. 10: *Ascent Phase Flight Path Angle (Normalised)*

CLG. The Duration of Coasting is to be computed on-board based on the calculation of velocity required for a vehicle to reach an intended Range. As per the Wheelon's equations [8], the relation between the Range given by Range angle(Φ), burnout velocity(V) from both the stages and the burnout angle(γ) and height(h) is given by

$$V_r^2 = GM/r\left[\frac{(1-\cos\phi)}{\left(\frac{r}{r_t}\right)}\sin^2\gamma + \sin(\phi-\gamma)\sin\gamma\right]$$

The maximum coasting time is decided based on the amount of velocity imparted from Stage-1 and assuming lower impulse from Stage-2 and gravity loss during the period of coasting. Also, the altitude at the end of the coasting will be favorable for the Stage-2 controllability.

C. Range based Pitch turn prior to Closed Loop Guidance (Pre-CLG Turn)

During Stage-2, Pre-closed loop guidance Pitch turn is essential to get the shallow trajectory solution at the start of CLG. As the Stage-1 follows a pre-programmed lofted trajectory, the explicit management guidance scheme will converge to a lofted solution with the initial conditions. So an algorithm is designed to calculate pitch turn desirable prior to Closed Loop guidance start to attain shallow guidance solution. The algorithm takes Target Range in terms of Range Angle((Φ)) and estimated burnout velocity of the vehicle(V) to obtain the required flight path angle(γ) for a shallow guidance solution based on the Spherical Hit equation [7].

$$\frac{r}{r_t} = \frac{1-\cos\phi}{\lambda\sin^2\gamma} + \frac{\sin(\gamma-\phi)}{\sin\gamma}$$

The amount of pitch turn required is computed from the Flight Path Angle requirement and executed prior to Closed Loop Guidance. This computation is carried out on-board based on the present state vector of the vehicle and assumption of Stage-2 Lower impulse. The additional flight path angle requirement due to assumptions considered will be catered by the Stage-2 Closed Loop Guidance.

D. Explicit Energy Management Guidance Scheme during Stage-2 (S2 CLG)

The thrust termination of solid rocket motors as and when required is not possible before Motor burn-out. For any given Range the total energy has to be utilized as per injection condition requirements. As per the maximum energy capability of the missile, burnout state vectors are estimated on-board and the present state vectors are obtained from the inertial navigation system. For the estimated burnout vector, range angle is computed for the specified target. Based on the Range Angle, flight path angle for the specified target is estimated. The oblateness effects are considered and flight path angle is solved iteratively by Newton-Raphson technique and the time of flight is computed. The Target is updated based on the Time of flight computation. The required velocity vector is computed based on the present state vector. Steering law (Vg X aT) generates the required commands to drive the velocity to be gained (Vg) to zero.

The closed loop guidance flow chart is given in reference [1] and as shown in Fig.11.

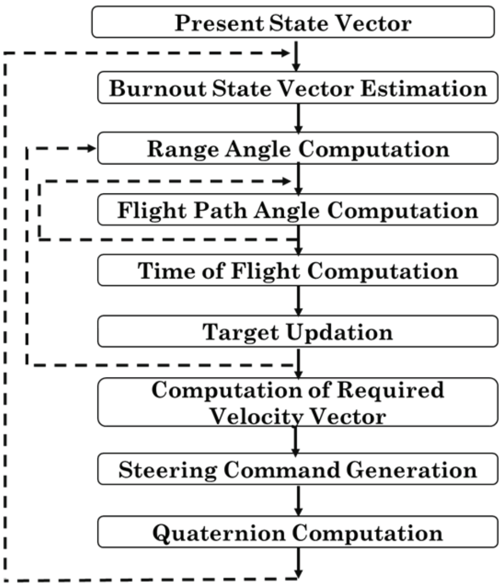

Fig. 11: *Closed Loop Guidance*

V. Modeling and Simulation

A. Vehicle Model

For the novel guidance design, a two stage solid propelled vehicle configuration is chosen. The vehicle is assumed to have a payload with lift to drag ratio greater than 2.0 and equipped with Fins. During ascent phase Pitch and Yaw is controlled by Flex nozzle control system and Roll by Aerodynamic Control (ADC) fins during atmospheric phase and by Reaction control system (RCS) outside atmosphere. RCS is also used for Roll, Pitch and Yaw control during free flight phase. ADC fins are used for Roll, Pitch and Yaw control during Re-entry. A typical Vehicle configuration is as shown in Fig.12.

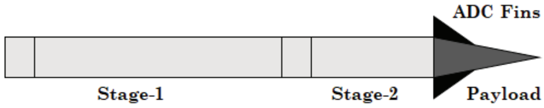

Fig. 12: *Vehicle Configuration*

B. System Dynamics

After vehicle configuration, the trajectory is shaped to achieve the required re-entry conditions. The performance of the system with the trajectory shaping is validated by a 6-Degree of Freedom (6-DoF) simulation model. In 6 DoF model, three rotational and three translational equations of motion are computed to simulate the dynamics of the system. The simulation studies are carried out to predict the vehicle parameters, to validate the design philosophy and ensure control and propulsion margins. In a non-real time simulation, the vehicle and the system external environment are modeling by mathematical models. The systems are divided into sub-systems and the mathematical models of these sub-systems are modeled. The six DoF simulation model is given in Fig.13.

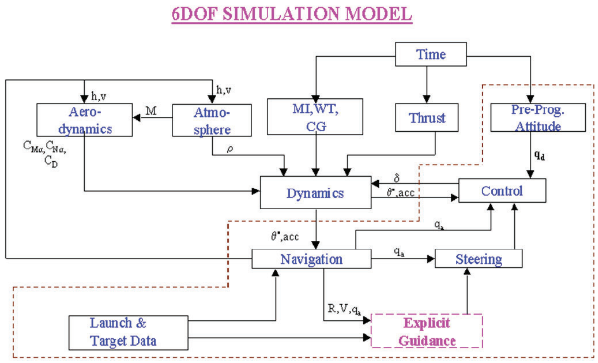

Fig. 13: *Six DoF Block Diagram*

1) Thrust Model: Thrust Model consists of pre-stored thrust and mass consumption profile bounds with variation in temperature, burn rate and mass for both stages. Typically, a nominal profile along with upper and lower performance bounds is used for simulation. This data is obtained based on the ballistic performance evaluation of the motor design which is validated by means of motor static tests. This data is used to calculate the thrust force and moments acting on the vehicle based on the control inputs. A very accurate thrust modelling leads to accurate simulation and also control system design and performance analysis. This forces and moments calculated along with other forces and moments are used for computation of accelerations and body rates of the vehicle.

2) Aerodynamics Model: The aerodynamic model consists of aerodynamic force and moment coefficients stored as a function of Mach number, altitude and angle of attack for complete vehicle, second stage and payload and payload alone configurations. The lift force, side force, pitching moment and yawing moments acting upon the vehicle are computed based on the vehicle present state. Also the drag force acting on the body is computed. These computed aero forces are used in addition with the thrust force and moments for computation of accelerations and body rates.

3) Atmospheric Model: For the computation of atmospheric effects on the vehicle, Indian Standard Atmospheric Model proposed by Shri MR Ananthasayanam and R Narasimha called ISTA 8 is being used. This model gives the sea level temperature and various lapse rates w.r.t altitude for computation of temperature, pressure, density and velocity of sound. This model assumes that the atmosphere above 80 Km has negligible effect. The distribution of pressure, temperature, density and velocity of sound is shown in Fig.14.

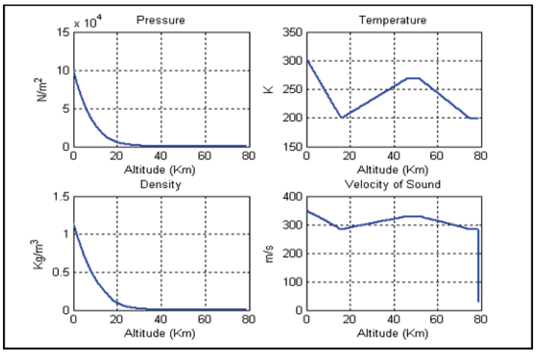

Fig. 14: *ISTA-8 Atmos Model*

4) Earth's Gravity Model: The gravitational potential on the surface of the earth at various points is computed based on the WGS-84 Model. The WGS 84 defines a reference ellipsoid, which gives the mathematical model of earth's shape, and a set of parameters that define its shape and orientation in space.

5) Wind Model: Winds are the local disturbances of the free stream. Wind model consists of Wind magnitude in Eastward and Southward components up to a height of 60km for various seasons with gust at different key altitudes. This data is obtained from the mean wind profile observed over various months, Wind disturbances effect the angle of attack and side slip angle which cause additional aero loads on the vehicle. Typical wind pattern till 60km is shown in Fig.15.

6) Navigation Model: An Inertial Navigation System uses accelerometers and gyroscopes to measure and estimate vehicle's position and orientation relative to a known reference. In the navigation model, acceleration and body rates computed from system dynamics are used for the calculations of vehicle velocity, position and attitude at defined update rates. All the calculations are done in the form of matrix of quaternions and DCMs. The present state vector from navigation, forms the input for guidance and control system.

7) Guidance and Steering Model: Guidance ensures a vehicle reaches its intended path by generating guidance commands based on present and estimated vehicle state.

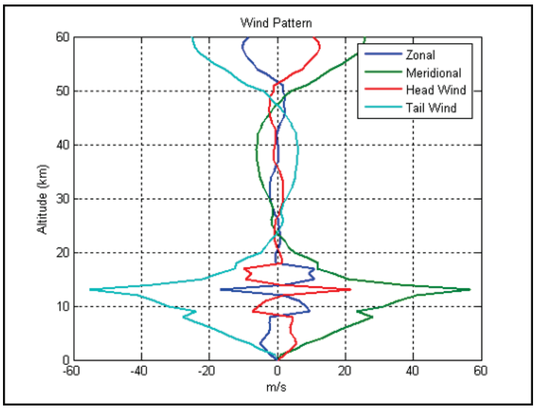

Fig. 15: *Wind Model*

The explicit guidance scheme is a two-point boundary value problem. The guidance model estimates the burnout state vector on-board and computes the required velocity vector and velocity to be gained to reach the target. The steering commands are generated and fed to control. This scheme though has high computational load, provides robust solution by the advent of FPGA based faster processors.

8) Control Model: The flight Control system uses attitude control for all three planes using a digital autopilot controller. Control system stabilizes the movements of the vehicle and executes the guidance demands to maneuver the vehicle to follow a trajectory to reach the destination. Type of control used in different phases are:

- First Stage: This stage is controlled by a Flex Nozzle Control (FNC) system for Pitch, Yaw and Roll Control by Control Fins located in the payload.
- Second Stage: During this stage, Pitch and Yaw control is performed by Flex Nozzle Control system and Roll Control through fins and Reaction Control System for Roll control after meaningful atmosphere.
- Free Flight Phase: Reaction Control System (RCS) is used to control the vehicle in the exo-atmospheric free flight. During this phase the vehicle is oriented to achieve zero angle of attack during re-entry. Initial body rates generated due to stage separation are counter balanced by RCS. Control forces are generated by the reaction motors operating in Bang- Bang mode.
- Re-entry Guidance Phase: Pitch, Yaw and Roll Control by Control Fins located in the payload. This will be used for steering of Re-entry Vehicle during Re-entry guidance phase also.

The simulation is carried out after implementation of all the models and the critical parameters are analyzed for satisfactory performance of the vehicle under all possible flight conditions. The performance of the vehicle from simulation is shown by various parameters such as theta, flight path angle, vehicle velocity, attitude and trajectory profiles are shown below.

The Theta Profile is given in Fig. 16 for various ranges depicting amount of pitch turn required for each ranges to achieve Shallow Trajectory guidance solution.

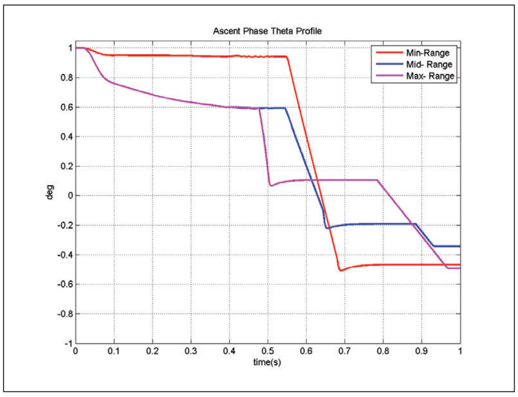

Fig. 16: *Theta Profile(Normalised)*

The Flight Path Angle Profile is given in Fig.17 which is essential to achive the required re-entry conditions.

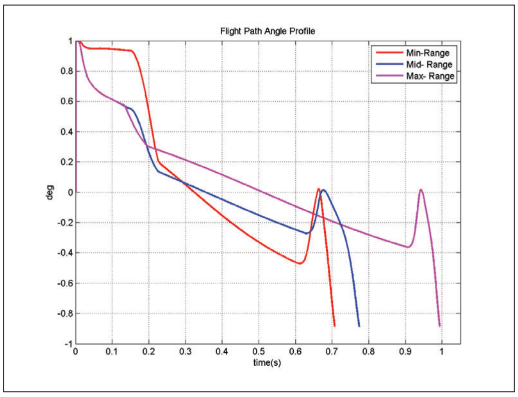

Fig. 17: *Flight Path Angle Profile (Normalised)*

The Relative Velocity is given in Fig.18, which shows the velocity variation based on Pitch turn and Range .

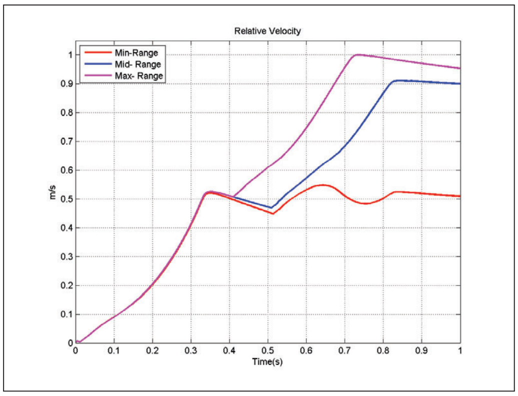

Fig. 18: *Relative Velocity Profile (Normalised)*

The attitude of the vehicle in terms of Quaternions is given in Fig.19 to Fig.22.

The Trajectory Profile for various Ranges is given in Fig.23. In all the Ranges the shallow re-entry during descent is ensured by proposed ascent phase trajectory design methodology.

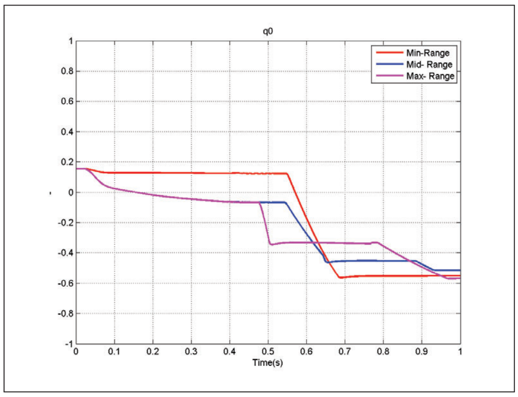

Fig. 19: *Quaternion q0 (Normalised)*

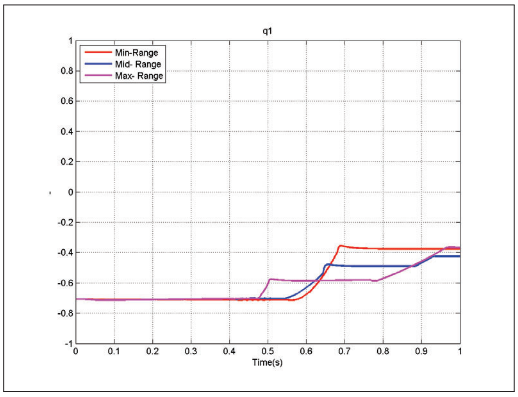

Fig. 20: *Quaternion q1 (Normalised)*

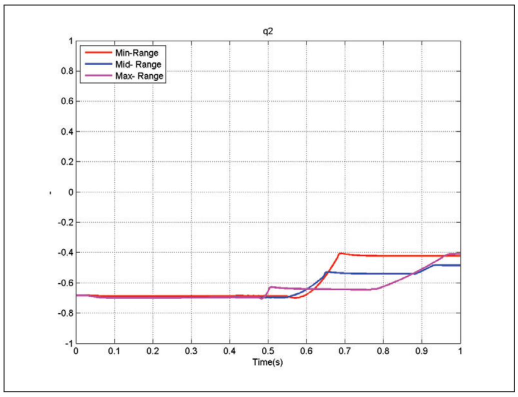

Fig. 21: *Quaternion q2 (Normalised)*

VI. Summary and Conclusion

This new trajectory design method is a novel approach which gives an effective solution for impact and velocity constraint based Payload applications. This technique can be applied for achieving required re-entry conditions suitable

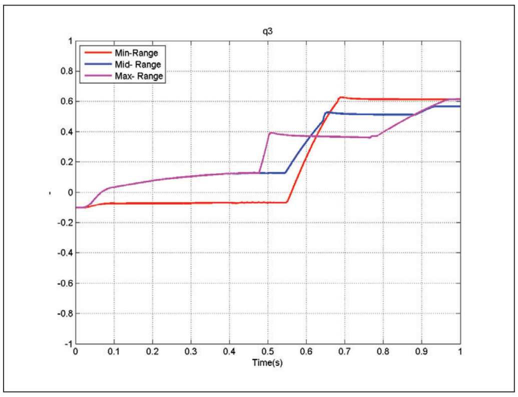

Fig. 22: *Quaternion q3 (Normalised)*

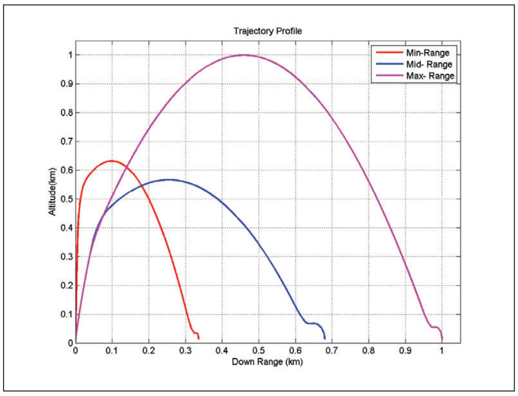

Fig. 23: *Trajectory Profile (Normalised)*

for maneuvering the re-entry vehicle. This guidance scheme computes the required velocity and flight path angle essential for achieving desirable re-entry state vector for Maneuvering. Simulation results confirmed the convergence of the guidance equations and high impact accuracy. The operational complexities of the thrust termination requirements are eliminated for the Solid Rocket Motor systems by use of coasting after Stage-1 burnout and Pre computed Theta turn before Closed Loop Guidance Start. This technique can also be used to dodge enemy RADAR and interceptor installations due to near horizontal re-entry angle.The proposed Ascent Phase Trajectory design is an effective scheme for Maneuvering Reentry (MaRV) class of vehicles.

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