Altitude Bias Estimation Using Kalman Filter

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***Abstract*—An Inertial Navigation System (INS) vertical accuracy degrades with time and absence of Satellite Navigation (Sat. Nav / GPS) leads to unacceptable uncertainty, especially in the vertical channel (it varies typically from 50 m to 2000 m). Moreover, the hybrid navigation solution is always w.r.to WGS-84 Ellipsoidal Earth model which is a smooth curve fit, but real altitude measurement is required above local terrain which is rough and uneven. Radio Altimeters (RA) measures the altitude above the ground/sea surface by sending the radio waves and measuring the time it takes to return after reflecting-off the surface. RA is recommended for all low altitude applications due to its accuracy and faster update, but relying only on RA readings and discarding inertial altitude information may not give accurate results as RA prone to noisy altitude measurement and random spurious outages in measurement. INS and RA have different benefits and drawbacks. The reason for integrating these two navigators is mainly to combine the best features and eliminate the shortcomings.**

A two state Kalman Filter (KF) is devised to estimate altitude information. A bias computation scheme is worked out to compute altitude bias between RA and INS altitudes. This paper describes the design, implementation, and real-time validation of Kalman filter-based altitude bias estimation methodology. This work aims to provide high accuracy and integrity estimates of vertical parameters (altitude above ground/sea and vertical velocity). The results from simulation, aircraft sorties and flight test, demonstrate that the vertical parameters estimated by the this approach satisfy vertical performance requirements across different scenarios.

***Index Terms*—INS, Radio-Altimeter, Kalman Filter, Error Covariance Matrix, Measurement Noise, Process Noise, State Matrix, State Vector**

I. Introduction

An **Inertial Navigation System (INS)** vertical accuracy degrades with time and absence of Satellite Navigation (Sat. Nav / GPS) leads to unacceptable uncertainty, especially in the vertical channel (it varies typically from 50 m to 2000 m). Navigation solution is always w.r.to **WGS-84** Earth model, but real altitude measurement is required above local terrain or sea surface. **Radio Altimeters (RA)** measures the altitude above the ground/sea surface by sending the radio waves and measuring the time it takes to return after reflecting-off the surface. RA is recommended for all low altitude applications due to its accuracy and faster update, but relying only on RA readings and discarding inertial altitude information may not give accurate results as RA provides noisy altitude measurement and randomly provides spurious outages in measurement.

There are a few comprehensive investigations about sea skimming guidance of these flight vehicle using RA as a main sensor in the literature. An altitude control system design with Kalman filter for sea wave adaptation is proposed in [1] to estimate the accurate altitude above sea surface. A Kalman filter for the integration of a radar altimeter into a terrain database-dependent guidance system presented and performance analyzed using low-altitude helicopter flight test data acquired over moderately rugged terrain in [2]. In [3], Kalman filter based radar-altimeter inertial vertical loop (RIVL) system enabling a highly accurate estimation of vertical parameters like altitude above ground and vertical velocity presented. In the paper [4] an integrated navigation system, consisting of radio and INS altimeters, is presented. The integration is achieved by using an indirect Kalman filter. Hereby, the error models of the navigators are used by the Kalman filter to estimate vertical channel parameters of the navigation system.

Current problem is a landing application with high vertical velocity. In order to address this problem a two state Kalman filter is devised to estimate altitude information which is better than two individual sources INS and RA. **Kalman Filter (KF)** is an algorithm that estimates the state (in this case, altitude) of a system by using noisy measurements and a **model of the systems's dynamics**. The KF uses vertical acceleration (A_X) information from INS as control input and altitude measurement from RA.

This integration of INS and RA helps to reduce errors caused by noise, measurement errors and other factors, resulting in a more reliable and accurate altitude estimate, especially in challenging environments of rough terrain and during high attitude manoeuvres. The sequential measurement processing Kalman filter performs well in the presence of measurement anomalies and data drop-outs. An input data rejection test has been implemented which successfully identifies and disregards erroneous data based on statistical criteria.

II. Brief about Kalman Filter

A Kalman filter takes information which is known to have some error, uncertainty or noise [5], [6]. The goal of the filter is to operate on this imperfect information, sort out the useful parts of interest, and reduce the uncertainty or noise. The functional diagram of Kalman filter is as depicted in figure 1. The Kalman filter uses a prediction followed by a correction in order to determine the state of the filter. This is sometimes called predictor-corrector, or prediction-update. The main idea is that using information about the dynamics of states, the filter will propagate state equation and estimate what will be the next (future) states (altitude).

Basically KF consist of two steps: **prediction step** estimate the future (predicted) states based on present states and control inputs. **Updation step** corrects the predicted states by using measurements from sensor (RA) to produce more accu-

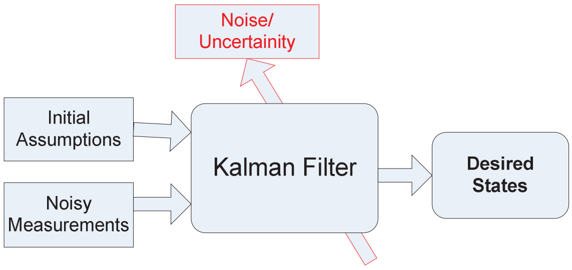

Fig. 1: Kalman Filter functional diagram

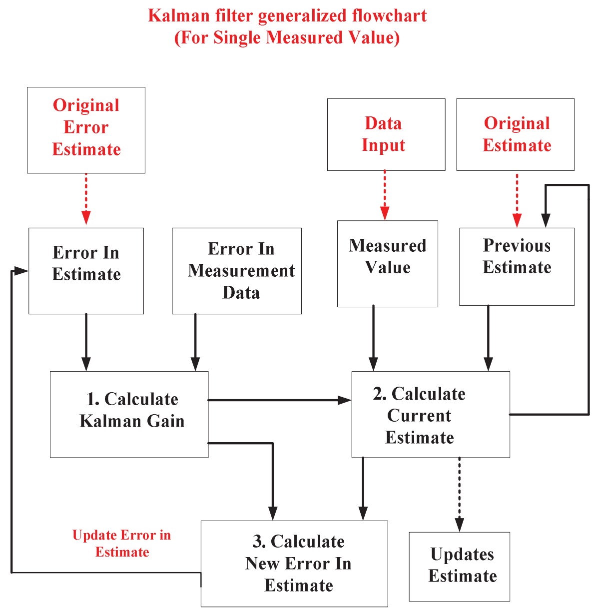

Fig. 2: Kalman Filter generic flowchart for single measured value

rate posterior states. This is a recursive process, current states will be used to predict the future states and measurements are used to correct the predicted states. The correction or update step then involves comparing a measurement with prediction that measurement should be based on our predicted states. The generic flow of Kalman filter functioning as shown in the figure 2.

The Kalman filter is designed to operate on systems in linear state space format

$$\mathbf{X_k} = F_{k-1}X_{k-1} + G_{k-1}U_{k-1} + W_{k-1} \quad (1)$$
$$\mathbf{Y_k} = H_k X_k + V_k \quad (2)$$

Where variable details and dimensions are given in table I.

1) State Vector (X) : The state vector X are the values that will be estimated by the filter.
2) Output Vector(Y): It is not the output of filter, but rather things able to measure. We need to express measurements in terms of the states so that we can compare them with the measurements. **For state estimation problems using Kalman filtering, the State is actually the desired result.**

TABLE I: Variable details and dimensions

Variable	Description	Dimension
X	State Vector	$n_x \times 1$
Y	Output Vector	$n_y \times 1$
U	Input Vector	$n_u \times 1$
F	State Matrix	$n_x \times n_x$
G	Input Matrix	$n_x \times n_u$
H	Observation Matrix	$n_y \times n_x$
W	Process Noise Vector	$n_x \times 1$
V	Measurement Noise Vector	$n_y \times 1$
P	Error Covariance Matrix	$n_x \times n_x$
Q	Process Noise Covariance Matrix	$n_x \times n_x$
R	Measurement Noise Covariance Matrix	$n_y \times 1$

3) Input Vector (U) : This vector contains information that is necessary to define system dynamics.
4) Process and Measurement Noise (W & V): These terms are used to represent the uncertainty (or noise) in the state equations and measurements. The ‘W’ represents modelling error called process noise which is uncertainty in the state equations. ‘V’ account for measurement error.
5) F, G and H Matrices : These matrices will depend on the considered problem, and are used in order to represent the equations as a linear system of states and inputs. F will contain coefficients of the state terms in the state dynamics, while H serves a similar function in the output equations. The G matrix contains coefficients of the input terms in state dynamics. These matrices in general vary with time, but cannot change with respect to the states or inputs.
6) Process Noise Covariance Matrix (Q) : This matrix represents the uncertainty in the process model. It captures the inherent randomness or variability in the system dynamics that can not be accounted by the deterministic process model. It is often referred as the system noise and it is typically assumed to be zero mean Gaussian noise. This matrix is used to update the error covariance matrix (P) during the prediction step of the Kalman filter.
7) Error Covariance Matrix (P) : This matrix represents the uncertainty in the estimate of the system state. It quantifies error between the predicted state and the true state of the system. The Kalman filter updates this matrix at each iteration based on measurements and process model.
8) Measurement Noise Covariance Matrix (R) : It represents the uncertainty or noise associated with the measurements of the system and is used in the Kalman filter to appropriately weigh the importance of each measurement during the update step.

III. Altitude Mathematical Model

Using particle kinematics [7], we expect that the acceleration of the object will be equal to the acceleration due to gravity defining height (h) of the object in meters,

$$\ddot{h}(t) = -g \quad (3)$$

where g is is the acceleration due to gravity (g = 9.80665 m/s^2).

This relation can be rewritten as below

$$\ddot{h}(t) = \frac{\dot{h}(t) - \dot{h}(t - \Delta t)}{\Delta t} = -g \tag{4}$$

which is a backward difference equation, which is useful for Kalman filtering applications due to the recursive structure of the filter, i.e. each time step in the Kalman filter always references the previous time step. Simplifying this expression gives

$$\dot{h}(t) = \dot{h}(t - \Delta t) - g\Delta t \tag{5}$$

Integrating above equation yields a commonly used kinematic equation relating successive positions of a particle with respect to time for constant acceleration

$$h(t) = h(t - \Delta t) + \dot{h}(t - \Delta t)\Delta t - (\frac{1}{2}g(\Delta t)^2) \tag{6}$$

The equations in continuous time, can be represented in discrete time as follows :

$$h(t) = h(k\Delta t) = h_k \tag{7}$$
$$h(t - \Delta t) = h(k\Delta t - \Delta t) = h(\Delta t(k - 1)) \quad = h_{k-1} \tag{8}$$

Rewriting the kinematic relationships for the above problem in terms of the discrete time variable, k, gives

$$\dot{h}_k = \dot{h}_{k-1} - g\Delta t \tag{9}$$
$$h_k = h_{k-1} + \dot{h}_{k-1}\Delta t - (\frac{1}{2}g(\Delta t)^2) \tag{10}$$

From these kinematic expressions, we have two equations describing the motion of the object: one for velocity and one for position. The state vector for the Kalman filter can be defined as

$$X_k = \begin{bmatrix} h_k \\ \dot{h}_k \end{bmatrix} \tag{11}$$

$$X_k = \begin{bmatrix} h_{k-1} + \dot{h}_{k-1}\Delta t - (\frac{1}{2}g(\Delta t)^2) \\ \dot{h}_{k-1} - g\Delta t \end{bmatrix} \tag{12}$$

we can rewrite these equations in terms of the state vector X, as

$$X_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} -\frac{1}{2}(\Delta t)^2 \\ -\Delta t \end{bmatrix} g \tag{13}$$

Now, we have the problem in necessary format for the Kalman filter

$$\mathbf{X_k = F_{k-1}X_{k-1} + G_{k-1}U_{k-1}} \tag{14}$$

where the system matrices F and G are given by

$$F_{k-1} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \tag{15}$$

$$G_{k-1} = \begin{bmatrix} -\frac{1}{2}(\Delta t)^2 \\ -\Delta t \end{bmatrix} \tag{16}$$

and the input vector can be defined as U_{k-1} = g

Prediction :

$$\mathbf{\hat{X}_{k|k-1} = F_{k-1}\hat{X}_{k-1} + G_{k-1}U_{k-1}} \tag{17}$$
$$\mathbf{P_{k|k-1} = F_{k-1}P_{k-1}F_{k-1}^T + Q_{k-1}} \tag{18}$$
$$\mathbf{K_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R_k)^{-1}} \tag{19}$$

Correction :

$$\mathbf{\hat{X}_k = \hat{X}_{k|k-1} + K_k(Z_k - H_k\hat{X}_{k|k-1})} \tag{20}$$
$$\mathbf{P_k = (I - K_kH_k)P_{k|k-1}} \tag{21}$$

The Kalman filter sequence of operations as shown in the figure 3.

IV. Low Altitude Application

Typically for all flight vehicle applications navigation solution is available in two forms, **pure** (Only based on inertial sensors - INS) and **hybrid** (Inertial + Sat. Nav) solutions. The following table II summarizes the altitude accuracy of the navigation solution in different scenarios.

TABLE II: Altitude error in different scenarios

S. No.	Condition	Altitude error
1	Hybrid Navigation	50 m (1σ)
2	Pure Navigation	2000 m (1σ)

The altitude information provided by hybrid navigation itself is not accurate enough to meet low altitude requirements due to following reasons.

1) On Ground : Hybrid navigation solution is always w.r.to **WGS-84** Earth model, but real altitude measurement is required above local terrain surface in case of flying over ground.
2) On Sea Surface : Sea skimming guidance, by default, is achieved with altitude feedback from radar altimeter measurement. However, radar altimeters measure the height from instantaneous sea surface. The **Geoid** is an equipotential surface that approximates the mean sea level accounting for Earth's gravity field and rotation. It is an irregular surface that undulates across the Earth's surface, influenced by variations in mass distribution. The geoid serves as a reference surface for measuring elevations and is used in geodesy. Real altitude may differ slight even with geoid

Based on the above for all low altitude applications it is recommended to use Radio Altimeter (RA). The accuracy of RA depends on altitude of operation, SNR and platform attitude (roll and pitch angle). In-spite of several advantages, RA altitude can not be used directly for decision making as it may corrupted due to flight orientation or invalid data sample or spurious noisy pulses (outliers).

The algorithm given in the figure 6 is used to estimate the altitude above the sea surface using raw measurements from RA. Flight acceleration in vertical channel is computed after necessary transformation from sensor frame to vertical frame and used in KF to propagate the state equation. The flowchart in the figure 7 depicts necessary pre-processing check carried out on raw data.

The fault-tolerant robust algorithm is as follows :

1) Radio altimeter starts its transmitter (Tx)-ON at the altitude of h_0 (based on the inertial/hybrid altitude)

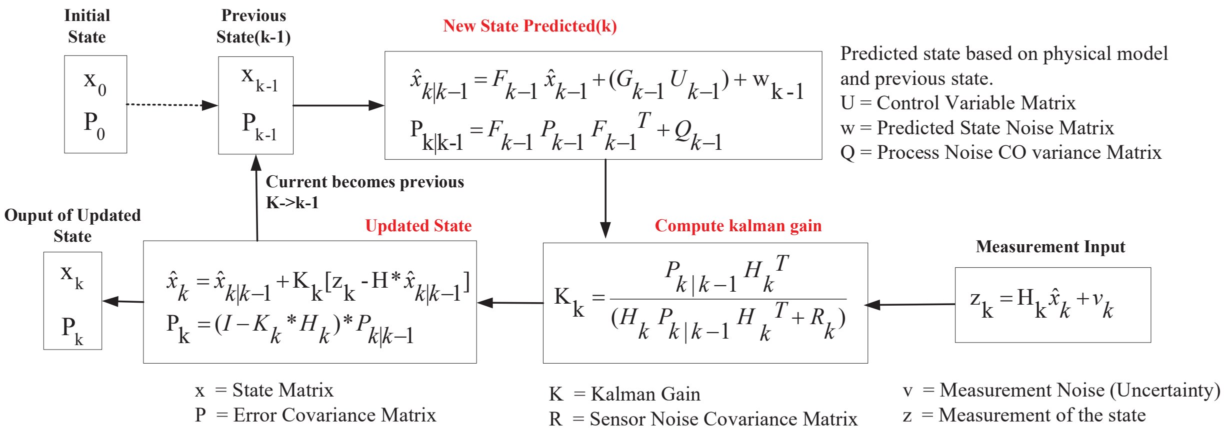

Fig. 3: Kalman filter sequence of operation

and starts getting data within one second. $h_0 = 6$ km considered for current application.

2) Validity of measured data is ensured by checking the validity bit which is available along with the altitude data. This will ensure the health of the sensor as well as validity of the data under perturbation of SNR and orientation of the platform. This check indicated as a condition C1 in figure 7
3) The data received from RA should be within the expected extreme limits between h_{low} and h_{high}. This check indicated as a condition C2 in figure 7.
4) ΔV is defined as the velocity difference between V_{INS} and V_{RA}. Where, V_{INS} is the vertical velocity of navigational unit and V_{RA} the estimated velocity from Kalman filter. Condition C3 is mean to check that ΔV should not exceed specified limit of 0.5 m/s.
5) If any of the above conditions fails, then the data from RA is not trust worthy and con't be used for decision making or feedback of guidance/control. Under such situations, h_{INS} corrected with bias (past estimation) is used as a valid altitude information.
6) If all conditions are fulfilled then the altitude data from RA and h_{INS} is used in filter to estimate the best altitude (h_{RA}) information and also the valid altitude bias (h_b) which is the difference between h_{INS} and h_{RA} (state estimate). This bias indicates the h_{INS} offset from valid RA measurements. This valid bias will be updated as long as all the three conditions fulfils and the Kalman filter gives the height and velocity estimates.
7) After computing the valid bias, if RA due to some reason not giving proper data then INS data corrected with valid altitude bias (h_b) be used as a valid altitude information.
8) Initially before valid RA data is available, Filter is initialized with h_{INS} and V_{INS} and the altitude bias h_b with pre-loaded geoid height at flying Latitude / Longitude. This value is replaced with computed bias upon availability of valid bias.
9) Measurement noise covariance matrix (R) represents the apportionment of uncertainty involved in measurement. The value of R is generated based the exhaustive aircraft sorties as indicated in figures 4.
10) The Radio Altimeter data was analyzed to determine the noise characteristics [8] of the device in the general operating range bracket of 10 m to 4500 m. The captive flight sortie data discussed in the section VI at different phases were analyzed. Near constant altitude cruise section has been identified and used for extracting the measurement noise (R) which is around 0.2 m (1σ) as shown in figure 5. The obtained R is used for filter tuning as per the standard deviation.
Higher noisy pattern seen during take-off and landing (see figure 13), this is mainly due to the terrain elevation changes from runway to sea coast and the same ignored.
11) For this problem, process noise covariance matrix (Q) roughly determined based on the disturbances from air resistance or other sources. However, assuming that these errors are small enough to ignore, the process noise covariance matrix, Q, can be set to zero.
The conditions of C1 and C2 eliminates all possible scenarios of data unavailability from RA due to failure or data freeze on bus and also eliminates spurious values in altitude data. The condition of C3 is obtained filter states with various bias and offset cases and in most of the cases the differences obtained between INS and estimates are with in satisfactorily limits.

V. Simulation Results

The Kalman filter implemented and integrated with 6DOF model to evaluate the performance under different scenarios. INS measurements are available at every 20 ms and RA updates at every 8 ms. The state propagation of KF is carried out at every 10 ms and measurement updated at 40 ms which ensure the synchronization of INS and RA updates.

Decent phase of trajectory considered here to show the efficacy of proposed algorithm. Here the vertical velocity is reducing very fast compared to typical aircraft. Further large

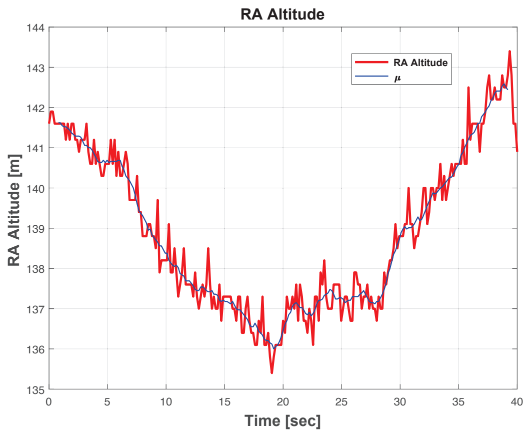

Fig. 4: RA Altitude data along with moving average mean (μ)

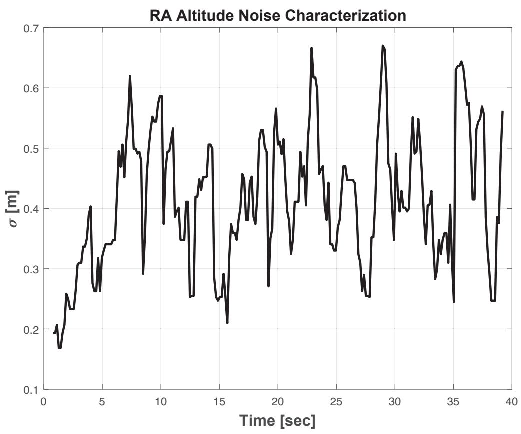

Fig. 5: RA Altitude standard deviation (σ)

variation in altitude also gives good test case to demonstrate the performance of Kalman estimator.

The filter performance is tested under various noise levels with zero mean (μ) Gaussian noise with different variance (σ^2) from 10 to 100 m^2. Initial error in h_{INS} is considered up to 1000 m. Kalman filter convergence is quite fast with 150 ms in-spite of large initial error. The figure 8 depicts altitude when RA data is having variance from 10 to 100 m^2 with initial offset varies from 50 m to 1000 m. The figure 9 depicts comparison of V_{INS} and V_{RA} and figure 10 shows the difference in various cases.

If the ΔV (velocity difference) exceed certain value, which means estimation is not correct. Under these situations, it is better to us h_{INS} with altitude bias h_b correction.

VI. Field Test

Field test carried out in two stages. First an aircraft sortie carried out to characterize the RA noise levels and generated measurement noise covariance matrix (R). Secondly, flight vehicle test for varying altitude and vertical velocity is carried out.

A. Aircraft Sorties

L-39C Albastros aircraft from Zypher Aerospace was used to evaluate the performance in real field. This aircraft can fly

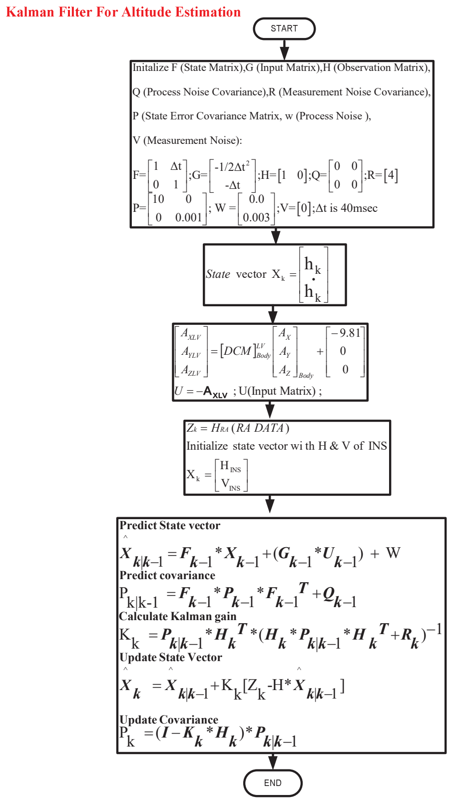

Fig. 6: **Flowchart of Kalman Filter implementation**

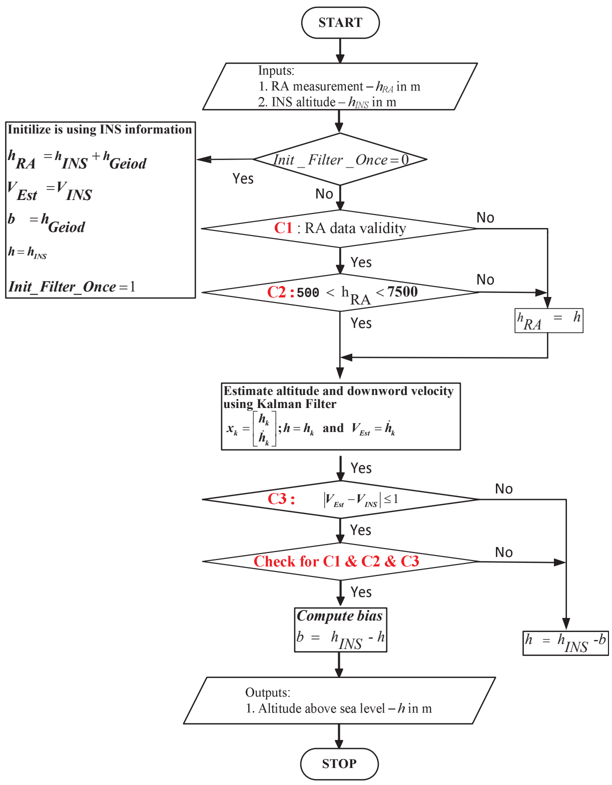

Fig. 7: **RA Data validity checks**

at very low altitudes of the order of 10 m above sea level at a speed around 150 m/s. Aircraft carries both INS(hybrid mode) and Radio altitude sensors in a specially designed **pod** as a payload under its wings. The aircraft used for this sorties along with integrated pod is as shown in figure 11. The figure 12 depicts the flight path flown by aircraft.

The flight recorded data of INS and RA is used to estimate the altitude using designed two state Kalman filter in off-line mode. The sortie was dedicated to extensive attitude manoeuvres at low altitudes to test the robustness of the algorithm and validity of attitude thresholds used to exclude radar-altimeter measurements at excessive vehicle attitudes as shown in figure 13. Figure 13 also indicates the altitude as measured by both the sensor. It is clear from figure that, RA is noisy and deviated from INS measurement by 100 m at some points (zoomed figure during take-off and landing).

Figure 13 shows h_{INS} below zero which is unrealistic (zoomed figure between 1900 to 2100 s). This is due to that h_{INS} on WGS-84 Earth model and in some locations it is below Geiod. This section of flight path also got one spurious point which has been eliminated by the algorithm.

B. Flight Vehicle Test

The developed KF based altitude and bias estimation was implemented in the mission software and altitude bias computation was successfully demonstrated. This test case is unique as the altitude of the flight vehicle is reducing from 5 km to ground level simulation landing application at very high velocity. Kalman filter worked as expected and the altitude estimation was in close match with true altitude. The figure 14 depicts velocity estimate from filter and the difference between true and filter estimate. The figure 15 depicts the altitude estimate from the filter and bias which is difference of true and filter estimated altitude.

VII. Conclusions

A two state Kalman filter is devised in to estimate altitude information which is better than two individual altitude data sources (INS and Radio Altimeter). The Kalman filter uses vertical acceleration (A_X) information from INS as control input and measurements from RA. In prediction step of the filter based on the present state estimate and control input the estimates at next time step are computed. In updation step, measurement from RA is used to correct the predicted states and produce more accurate posterior states. The filter is tested under different update cycles like 20, 40 and 160 msec. The developed filter is tested under different scenarios like varying RA variance from 10 m to 1000 m, initial INS bias from 100 m to 1000 m and also with RA outages. The performance of designed filter compared with NRT and HILS results. The Kalman filter is implemented in OBC, performance was evaluated using INS and RA data obtained from flight trials and results were also presented.

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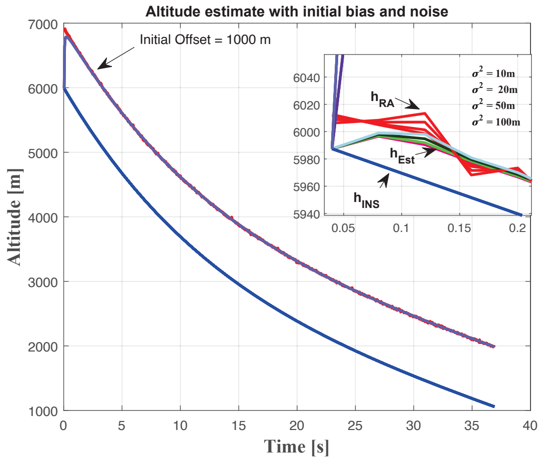

Fig. 8: Altitude estimate with initial bias and variance in RA measurement

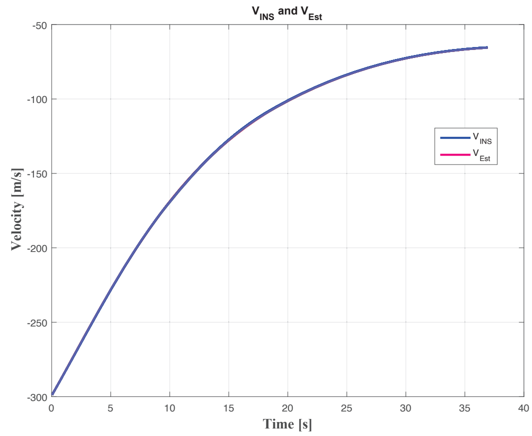

Fig. 9: Velocity estimate

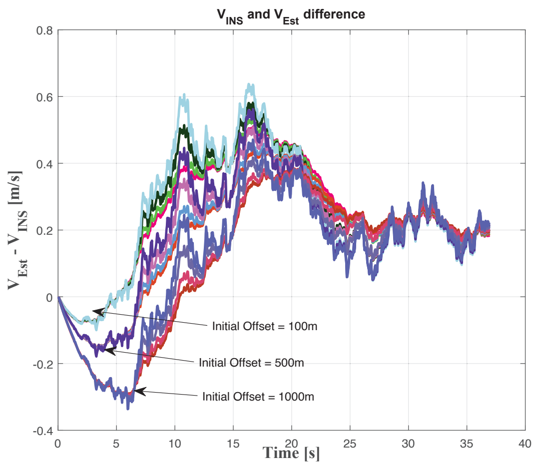

Fig. 10: Velocity difference between INS and estimated state

Fig. 11: Aircraft L39C Albastros Aircraft along with Integrated Pod

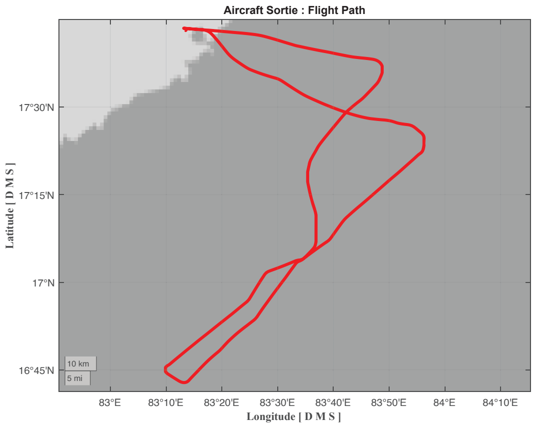

Fig. 12: Flight path flown by Aircraft

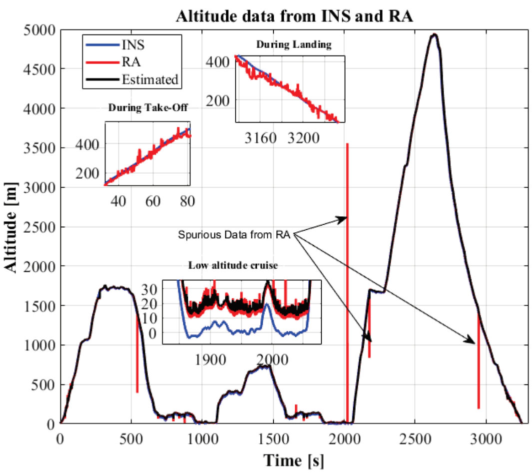

Fig. 13: Altitude data from INS and RA

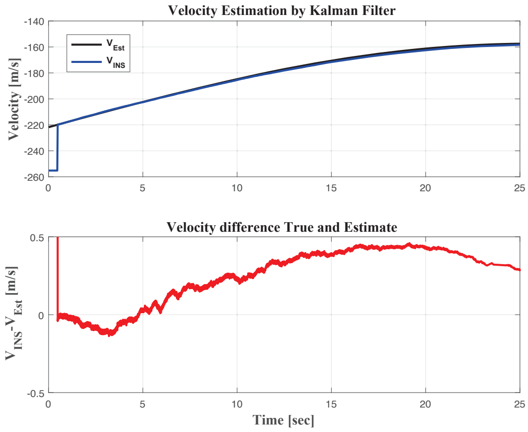

Fig. 14: Velocity estimation and difference from true

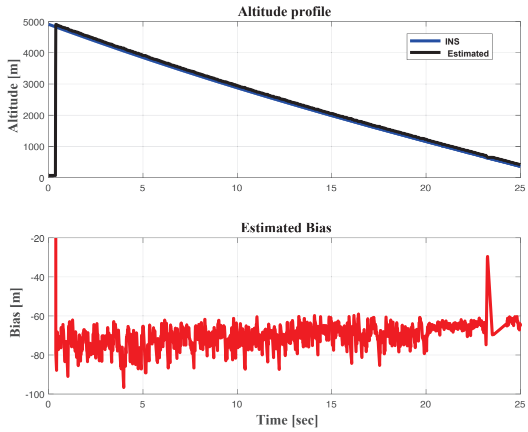

Fig. 15: Altitude estimation and computed bias

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