Heading Angle Estimation and Impact Angle Guidance

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***Abstract*—The impact angle is very important constraint for missile's warhead efficiency and kill capability. The impact angle guidance is used to increase the effectiveness of warheads especially for anti-ship applications where impact angle is define w.r.to ship hull surface. This paper presents some of the methods of estimating desired impact angle (γ_d) to increase the warhead penetration and overall effectiveness. Also presents an optimal guidance formulation for achieving desired impact angle to inflict maximum damage on ship targets. Estimating impact angle & using it in guidance to achieve the desired impact conditions will cause maximum damage to the ship targets. Besides it reduces the overall miss distance. Desired impact angle is estimated by finding approximate orientation of the ship in guidance reference frame. This paper also presents some of the methods to find the orientation of ship. At the end, this paper discusses limitations of the proposed approach & additional lateral acceleration demand.**

***Index Terms*—Impact Angle, Estimation, Guidance, Reference frame, Damage, Ship orientation.**

I. Introduction

The impact angle of an anti-ship missile is a very crucial factor in its effectiveness, as it directly influences warhead performance, efficiency in penetration, kill capability and overall probability of a successful kill. For a given warhead and target armour, there is an optimal impact angle for penetration. if the angle is too shallow, the missile may ricochet. This will cause failing to penetrate or detonating externally with limited damage. Ideally, a more perpendicular impact angle increases the probability of penetration and warhead detonation inside the ship hull. Different parts of the ship have different structural components and internal systems. Aiming specific areas with a precise impact angle can increase the chance of hitting critical systems like the propulsion, command and control, or weapon systems. A steep, near-vertical impact (top-attack) is generally preferred for anti-ship missiles because it targets the less armoured deck, maximizing the chance of penetrating vital internal compartments, leading to internal damage, fires, and flooding.

While the top-attack is often highlighted for its advantages against the less-armoured deck, a side-attack (horizontal impact) for an anti-ship missile also offers several tactical and damage related advantages. The primary advantage is reduced detection range by flying below the radio horizon at sea-skimming altitude. This low altitude flying makes the missile harder for Close-In Weapon Systems (CIWS) and surface-to-air missile (SAM) to acquire and intercept. In general, the sides of a warship houses ammunition, a direct hit on it can cause catastrophic secondary explosions and un-survivable damage. maintaining a sea-skimming with horizontal impact is generally simpler than top-attack, but achieving near perpendicular with respect to ship hull on side (port-side or starboard-side) is a challenging task discussed in this paper.

The development of sophisticated guidance laws to precisely control the terminal impact angle is a major on-going research work in the world wide. Researchers are continuously refining these guidance laws using various advanced control theories. Zarchan [1] outlines the significance of terminal angle constraints in tactical missile engagements and Shneydor [2] emphasizes guidance kinematics and presents proportional navigation (PN) modifications for angle control. Guidance field has progressed from basic Proportional Navigation (PN) to more advanced non-linear guidance laws specifically designed to meet complex terminal constraints. A suboptimal guidance for re-entry vehicle with constraint on body attitude angle at impact is designed [3]. The problem was formulated as a linear quadratic problem and Riccati equation control is derived to provide time-varying feedback gains.

An Impact Angle Control Guidance (IACG) law considering practical implementation is proposed by KS Kim and et al [4]. By selecting parameters properly, this guidance law can be applied to various engagement situations such as the rear-chase case and the face-to-face case. Polynomial trajectory shaping methods offer analytical solutions to control terminal conditions (e.g., impact angle and time-to-go). Yoon and Tahk [5] proposed a polynomial guidance law, shaping the trajectory to satisfy both impact angle and terminal position. The impact-angle-constrained guidance problem is solved as a non-linear regulator problem using the state-dependent Riccati equation (SDRE) technique by Ratnoo and et al. [6] which shows vey small miss distance.

A Modified Angle Constrained Biased Proportional Navigation Guidance (MACBPNG) designed where the required bias term is derived in a closed form considering non-linear equations of motion [7], [8]. In modern engagements, 3D angle control is essential due to evasive ship manoeuvres and the need for flexible terminal trajectories. Kim and Tahk [9] proposed a three-dimensional guidance law that estimates time-to-go to meet both pitch and yaw constraints. Yan and Lin [10] developed a terminal guidance law that optimizes trajectory considering physical limitations of missile dynamics.

Ohlmeyer et al. [11] proposed a Generalized Vector Explicit Guidance (GENEX) law which can achieve design specifications on miss distance and final missile-target relative orientation simultaneously.

An integrated guidance and control design to intercept a stationary target at a pre-specified impact angle presented by Abhinav [12]. This design incorporated both canard and tail aerodynamic surfaces to achieve the required lateral acceleration for a given angle-constrained interception.

Guidance law of Pure Pursuit and Proportional Navigation (PN) was developed by Akilandeswari et al. [13] for terminal phase of an anti-ship Bank to Turn (BTT) cruise missile in terms of course angle is implemented for stationary as well as moving targets in the lateral plane. It is a challenging task for Anti-Ship Missile (ASM) to detect/identify intended target ship from group of ship formation acquired by seeker [14]. Modified Iterative Closed Point (M-ICP) approach is proposed to identify the targeted ship and its motion.

Many studies leverage optimal control theory to derive guidance laws that minimize miss distance while simultaneously satisfying precise impact angle constraints. These methods often involve solving complex optimization problems. However, it's a complex trade-off between:

1) **Penetration vs. Surface Damage**: A near-perpendicular impact ensures maximum penetration, while a shallower angle might cause mere surface damage or ricochets.
2) **Stealth vs. Attack Angle**: Maintaining a low altitude for as long as possible (for stealth) might conflict with the need to climb for a high impact angle in case of top-attack.
3) **Missile Capabilities**: The missile's aerodynamic design, propulsion, and control systems limit the achievable impact angles and manoeuvrability.

The impact angle guidance is used to increase the effectiveness of warheads especially for anti-ship applications where impact angle is define w.r.to ship hull surface.

The definition of impact angle is schematically shown in figure 1 and 2. Ideal impact angle for top attack is 90^o which is called as T90 as shown in figure 1. Similarly in case of side attack scenario S90 is 90^o impact on side (either port side or starboard side) as shown in figure 2. This paper aims to achieve the ideal impact angle for side attack case. The first task is to do so is to estimate the orientation of ship.

This paper presents various methods of estimation of orientation of ship targets and us in guidance to attack the ship on broadside with in 90± 20 deg w.r.to ship's longitudinal axis. This also improves Seeker Signal to Noise Ratio (SNR) compared to narrow side engagement. Attacking broad side also reduces miss probability as the length of ship is 7 to 10 times higher compared to width of the ship.

II. Problem formulation

Above problem has been divided in two segments as mentioned below.

1) Estimating the accurate heading of the ship
2) Achieving the desired impact angle for various engagement scenarios.

Some of the methods for each problem mentioned above has been discussed below

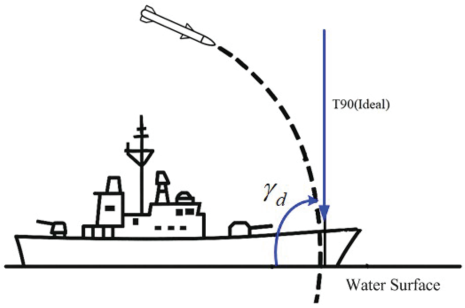

Fig. 1: Top attack (T90) engagement scenario

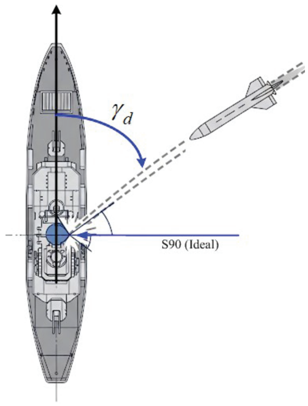

Fig. 2: Side attack (S90) engagement scenario

III. Heading Estimation

Estimating ship heading or orientation from seeker data is a critical task in various maritime applications, including navigation, surveillance, and target tracking for missiles. Ship heading can be estimated considering practical aspects. Some of the methods are as follows :

1) **Based on Known Target Heading** : In some cases, target velocity and heading information is already provided by the surveillance system. In such cases, target heading can be used directly to compute the impact angle (γ_d).
2) **Based on Seeker Track** : Mono-pulse seeker treats the target as a point of reflection and tracks it in the terminal phase of the trajectory. During track mode a Kalman filter can be used to estimate the target velocity and heading. More-ever, heading of the target can be estimated after first acquisition of target also by calculating the position vector direction between initial position which was received form surveillance system. This will give fair heading information and velocity if time stamping is also on common clock. This also helps in quick convergence of Kalman filter by initialising the states appropriately.
3) **Based on Seeker SNR and RCS** : Estimating target heading from seeker track is possible only if targeting is in motion. For stationary targets, radiation from the targets can be used to find the orientation of ship. Radar Cross Section (RCS) of ship is different in different azimuthal directions as shown in figure 5. Researchers are working on using SNR information pattern to estimate

the ship orientation using AI techniques which requires lot of training to estimate the required information.

Other methods like **wake analysis** will analyse the wake left by a moving ship are effective for optical imagery where the wake is discernible.

A. Heading Estimation Based on Seeker Track

Considering practical aspects, ship moves mainly along its longitudinal axis. Therefore, by taking this into consideration, orientation of ship can be computed by estimating the ship velocities in reference frame. Having known the velocities in reference frame, desired γ = (χ_T) is given as follows using simple trigonometric relation (Eq. 1). χ_T sign convention is shown in figure 3 for the sake of clarity. χ_T is intermediate variable used only for assignment. Subsequently, Desired impact angle (γ_d) can be found from right angle triangle geometry.

$$\begin{aligned} \chi_T &= \tan^{-1}\left[\frac{-V_{zT}}{V_{yT}}\right] \\ \gamma_d &= \chi_T \end{aligned} \tag{1}$$

An RF seeker, typically a radar provides measurements that are inherently noisy. For ship tracking and heading estimation, the primary measurements are **gimbal angles**, **range**, **range rate**, **line-of-Sight (LOS)**, **Signal-to-Noise (SNR)** and **Radar Cross Section (RCS)**. There are various filter techniques to estimate the heading and velocity from these noisy measurements. In this work, **two state** Kalman filter is used to estimate the velocity vector from seeker measurements. Main focus of this paper is not on estimating the velocities, but for completeness of the work, formulation of two state Kalman filter is given in figure 4 , similarly other component of the velocity (V_{zT}) can be estimated by passing appropriate arguments.

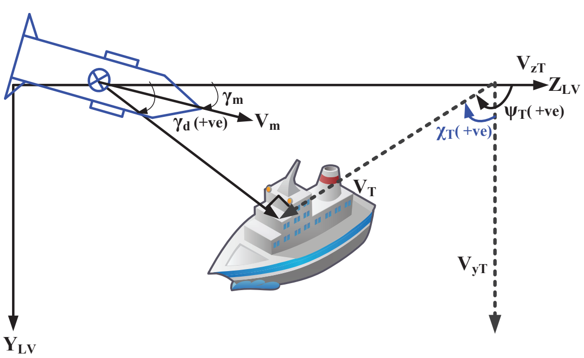

Fig. 3: Missile - Target impact geometry 2D view

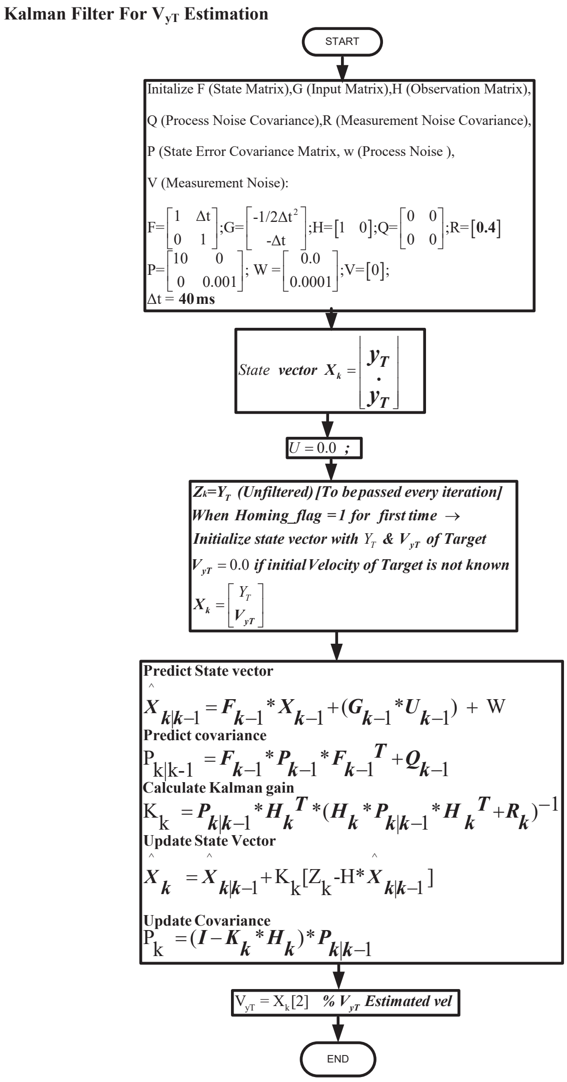

Fig. 4: Estimating the target velocity

B. Challenges involved in Orientation from Ship RCS

As discussed previously, orientation of the ship, can be found out from RCS mapping, But this requires extensive interpolation & co-relation, as same RCS can be present at multiple orientations. This scenario can be seen in figure 5. Besides that Signal to Noise Ratio (SNR) measured by seeker varies w.r.t relative distance (Rs) of the ship. This additional corelation is also required on-board which requires extensive computation & processing time involved. Considering these problems, this method is not persued in current work.

IV. Impact Angle [γ_d] Guidance

Now we will move to the next step of problem statement, that is how to achieve the desired impact angle (γ_d) with PN guidance. We know that, traditional PN guidance will persue LOS rates $[N^{'}V_c(\dot{\lambda})]$. This law cannot meet additional impact angle constraint. Therefore, an additional term (β) is derived to meet desired impact angle (γ_d) as follows. Consider the 2D geometry of missile & target geometry as shown in figure 6.

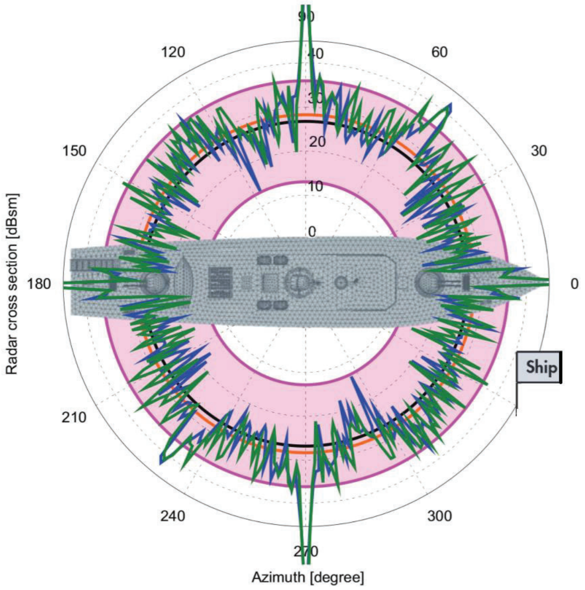

Fig. 5: Ship orientation & RCS mapping

$$n_M = N^{'} V_c(\dot{\lambda} - \dot{\lambda}_b) \tag{2}$$

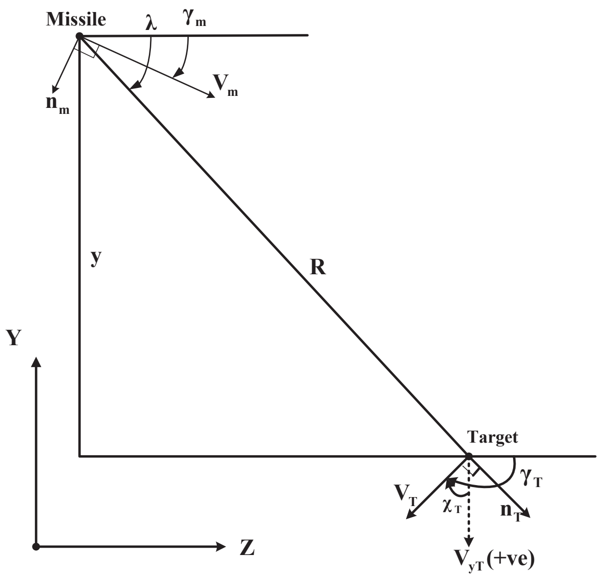

Fig. 6: Missile - Target geometry for Impact angle term derivation

Throughout the homing phase, $\dot{\lambda}_b$ term would induce the change of closing profile. Therefore in the end, it will end up with certain incidence angle w.r.to reference line known as impact angle. Multiplying and separating terms in Eq. 2, we get

$$\begin{aligned} n_M &= N^{'} V_c \dot{\lambda} - \beta \\ \beta &= N^{'} V_c \dot{\lambda}_b \end{aligned} \tag{3}$$

In anti-ship applications, target is moving with ≈ constant speed and not highly maneuvering. Hence target acceleration can be taken as zero. Similarly, compared to missile velocity, target velocity is small in the order of 15 m/s. Therefore, to simplify the problem further target velocity can be taken as ≈ 0

$$\begin{aligned} n_T &\approx 0 \\ V_T &\approx 0 \end{aligned} \tag{4}$$

With small LOS angle approximation, the equations of motion are,

$$\begin{aligned} \Delta\ddot{Y} &\approx -n_m \\ V_c &\approx V_m \\ \lambda &\approx \frac{\Delta Y}{\Delta Z} \\ t_f &\approx \frac{\Delta Z(0)}{V_c} \end{aligned} \tag{5}$$

Where $\Delta Z(0)$ is the range to go when homing phase starts. From Eqs. 3 and 5, we get

$$\Delta\ddot{Y} \approx -n_m \approx -N^{'} V_C \dot{\lambda} + \beta \tag{6}$$

Integrating above equation and substituting approximate expressions of λ, V_c from Eq. 5, we get

$$\Delta\dot{Y} = -N^{'}\Big(\frac{\Delta Z}{t_{go}}\Big)\Big(\frac{\Delta Y}{\Delta Z}\Big) + \beta t + C_1 \tag{7}$$

Cancelling ΔZ on numerator and denominator, and rearranging Eq. 7, we get

$$\Delta\dot{Y} + \Big(\frac{N^{'}}{t_{go}}\Big)\Delta Y = \beta t + C_1 \tag{8}$$

We have $t_{go} = t_f - t$, substituting in above Eq. and solving for ΔY using Bernoulli's differential eq. method, we get

$$\Delta Y = \frac{\beta \times (t_f - t)^2}{2 - N^{'}} - \frac{(\beta \times t_f + C_1)}{(1 - N^{'})}(t_f - t) + C_2 \times (t_f - t)^{N^{'}} \tag{9}$$

Substituting initial and final conditions in Eqs. 8 and 9 and solving for C_1 and C_2, we get

$$\begin{aligned} C_1 &= \Delta\dot{Y}(0) + \frac{N^{'}}{t_f} \times \Delta Y(0) \\ C_2 &= \Big[\Delta Y(0) - \Big(\frac{\beta \times t_f^2}{2-N^{'}}\Big) + \Big(\frac{\beta \times t_f + C_1}{1-N^{'}}\Big)t_f\Big] t^{-N^{'}} \end{aligned} \tag{10}$$

Substituting t = t_f in Eq. 9, We get that final value of ΔY is zero which is true for stationary and anti ship applications.

$$\Delta Y(t_f) = 0 \tag{11}$$

Flight path angle at $t = t_f$ given by

$$\gamma(t_f) = \tan^{-1}\left[\frac{-\Delta\dot{Y}(t_f)}{V_c \times \cos\lambda}\right] \tag{12}$$

Assuming $\Delta\dot{Y}(0) = 0$ for which is true at homing phase start and substituting t = t_f by differentiating Eq. 9, we get

$$\Delta\dot{Y}(t_f) = \frac{\Big(\beta t_f + C_1\Big)}{(1 - N')} \tag{13}$$

Substituting Eq. 13 in Eq. 12 and simplification, we get

$$\gamma(t_f) = \tan^{-1}\left[\frac{\beta\Delta Z(0)^2 + N^{'}\Delta Y(0)V_c^2}{\Delta Z(0)V_c^2(N' - 1)\cos\lambda)}\right] \tag{14}$$

Rearranging Eq. 14 and solving for β, we get

$$\beta = \frac{V_c^2}{\Delta Z(0)^2}\Big[\tan(\gamma(t_f)) \times \Delta Z(0)(N^{'} - 1)\cos\lambda) - N^{'}\Delta Y(0)\Big] \tag{15}$$

V. Results & Challenges

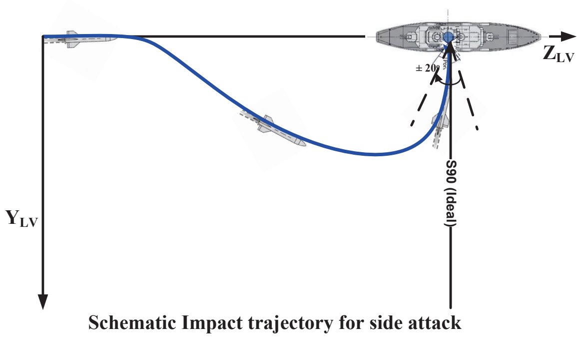

Fig. 7: Missile - Target schematic impact geometry

Figure 7 shows schematic impact trajectory for a receding target. If we use traditional PN guidance, missile persues LOS angle, hits along the narrow side following black line. The problems associated with this has been discussed above. With impact angle constraint, guidance modifies the interceptor path and impacts port/starboard side with in $\pm 20^o$ w.r.t to S90 reference as shown in figure.

A. Estimation results

Estimation has been done considering worst case scenario where target velocity & heading info is not known. Therefore filter initialisation starts with zero. Figure 8 shows true target velocity & estimation for two cases. From figure 8, heading of the target is computed in reference frame & shown in figure 9. From figure, it is infered that, the method is able to estimate the orientation of a moving ship with reasonable accuracy.

If $\approx$ target velocity (V_T) & heading (ψ_T) is known, then it can be resolved into components & filter can be initialised appropriatley. This helps in quick convergence of filter & same can be seen in figure 10. Here $\approx Vz_T$ of 16 m/s is initialized against true value of 20 m/s. Filter quickly converged within 5 seconds against 12 seconds without initialisation.

We will discuss the next challenge of the problem. As impact angle constraint requires additional lateral acceleration(n_m) demand, it leads to higher banking angles in case of cruise missiles, higher α_T in case of skid to turn missiles. This work emphasises more on cruise missiles. Considering missle which cruises at 0.7 to 0.8 Mach, results are worked out using real plant & 6DOF model.

Considering various engagement scenarios, Figure 11 shows impact geometry of interceptor & target which is moving with 20 m/s velocity. From figure, it can be seen that Objective of this work has been met to engage the target on broad side with in $\pm 20^o$. Desired & achieved impact angle ($\gamma \& \gamma_d$) is shown in figure 12. Maximum γ_d is limited to $\pm 75^o$ considering the lateral acceleration limit & banking angle limits of the interceptor. Maximum banking angle of the interceptor taken is $\pm 60^o$ & it is touching the limit for approaching target, which is critical as maximum

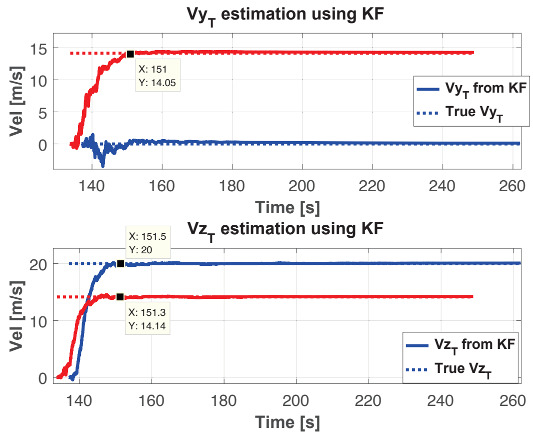

Fig. 8: Velocity estimation using 2 state Kalman filter

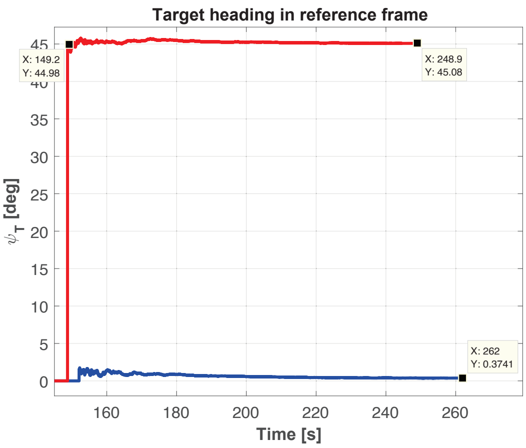

Fig. 9: Heading in refrence frame from estimation

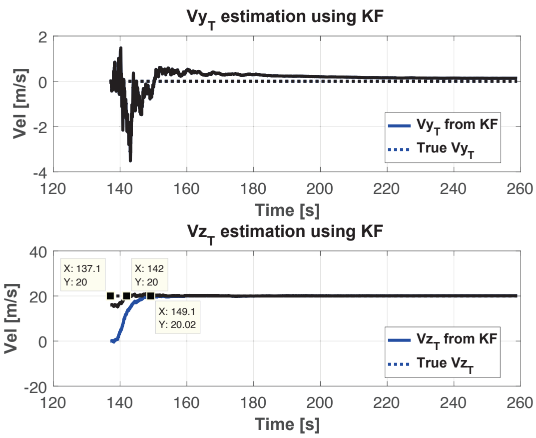

Fig. 10: Velocity estimation with Kalman filter initialisation

lateral acceleration demand (n_m) of $\approx$ 2.2*g is expected to meet impact γ_d constraint as shown in figure 14. This is one of the constraints from structural point of view. Considering this, maximum impact angle for critical trajectories is limited to $\pm75^o$.

B. *Figures and Tables*

TABLE I: Trajectory cases

Case No.	Trajectory parameters			
	Color	**Target heading $\psi_T[^o]$**	**$\gamma_d[^o]$**	**Imp $\gamma_f[^o]$**
Case 1	blue	0.0	-75.0	-71.4
Case 2	red	45.0	-45.5	-44.5
Case 3	black	90.0	-0.3	5.4
Case 4	magenta	135.0	46.5	48.2
Case 5	cyan	180.0	-75.0	-71.1
Case 6	green	225.0	-46.0	-51.3
Case 7	yellow	315.0	45.2	39.5

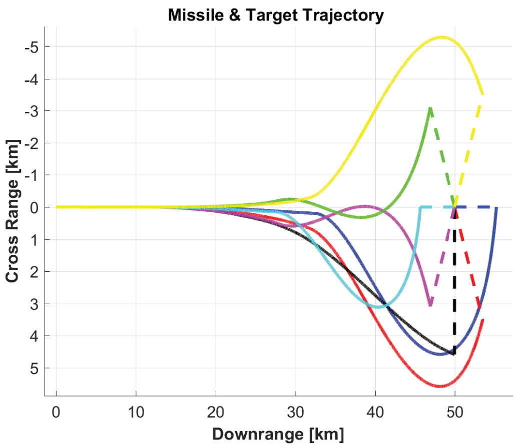

Fig. 11: Missile - Target 360^0 impact geometry considering various scenarios

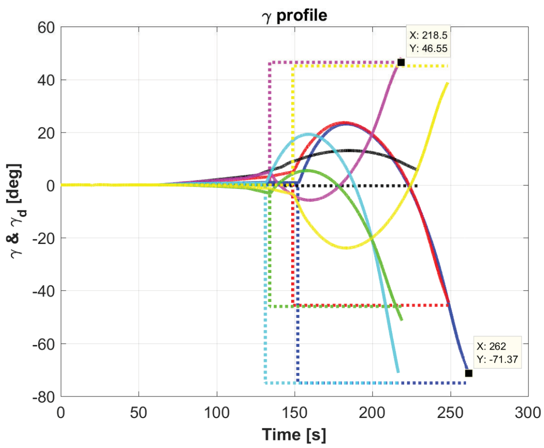

Fig. 12: Impact angles desired & achieved (γ&γ_d) for various scenarios

VI. Conclusions

1) The proposed approach is effective for moving targets($\geq 2.0m/s$) to estimate orientation of the ship.

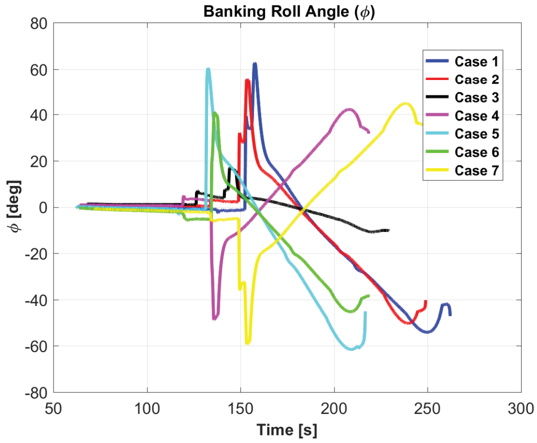

Fig. 13: Interceptor banking angle for various engagement scenarios

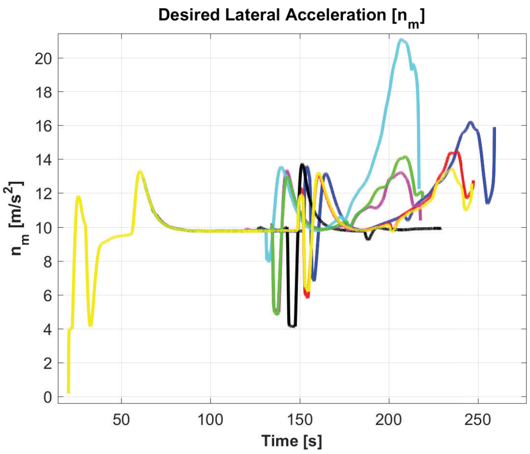

Fig. 14: Interceptor lateral acceleration demand

2) Proposed scheme for estimating orientation will have limitation for stationary ship targets. In these cases, alternative schemes for estimating orientation can be explored.
3) Impact Angle Constrained Biased PN (IACBPN) guidance scheme is proved in achieving the desired impact angle (γ_d). This can be seen from results given above.
4) This approach requires additional lateral acceleration demand(n_m). Hence, maximum lateral acceleration (n_m) constraint to be considered during design.
5) This paper presents 6DOF results for side attack (azimuth plane) impact angle. Similar guidance approach can be extended for elevation plane (top attack) impact angle.

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