INTRODUCTORY CONCEPTS

1-1 SCOPE OF MECHANICS

The subject of mechanics occupies a unique position in the physical sciences because it is fundamental to so many fields of study. In its broadest sense, mechanics may be defined as the science which describes and predicts the conditions of rest or motion of bodies under the action of forces. In a narrower sense, mechanics may be divided into areas which, with some overlap, are of separate interest to physicists and to engineers.

The topics of mechanics which interest physicists and astronomers (and, to some extent, electrical engineers) are particle mechanics, celestial mechanics, quantum mechanics, and relativistic mechanics. To engineers, the important areas of mechanics are the mechanics of solids and of fluids and their concomitant, continuum mechanics. A visual outline of how these topics are inter-related is shown in Fig. 1-1.1. In this book, we consider only the mechanics of rigid bodies which is divided into two parts: statics and dynamics. In statics, we consider the effects and distribution of forces on rigid bodies which are and remain at rest. In dynamics, we consider the various motions of rigid bodies and the correlation of these motions with the forces causing them.

1-2 PREVIEW OF STATICS

Throughout mechanics will recur the concepts of finding the components of a force and the moment of a force, plus the concept of a free-body diagram. In a very real sense, the components of a force and its moment effect are the

building blocks of statics. You will learn these concepts in your first week of study. Thereafter with the aid of a free-body diagram, (to be introduced shortly and discussed in detail in Chapter 3) you will learn how to apply these concepts to solve a broad variety of situations. In particular, the solution of many practical problems will teach you a method of reasoning, called engineering methodology, which is the hallmark of the engineer.
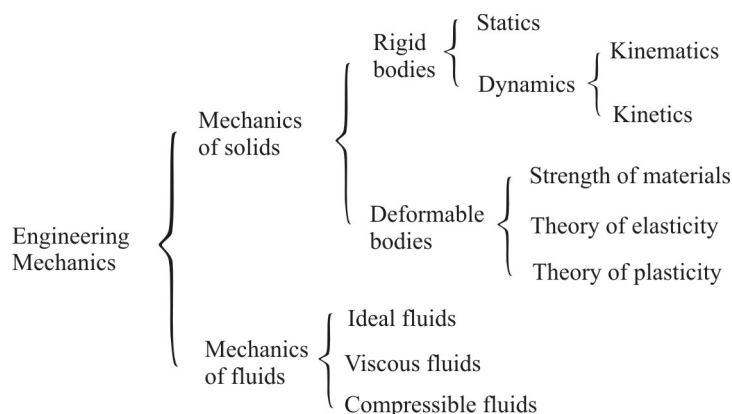

Figure 1-1.1 Abbreviated outline of engineering mechanics.

In Chapter 2 the concepts of the components of a force and its moment effect are applied to determine the resultant of any force system. In Chapter 3, we discuss the conditions which reduce the resultant of a force system to zero and thereby obtain the two basic equations of static equilibrium; namely, $\Sigma F = 0$ and $\Sigma M = 0$. In succeeding chapters, these two basic equations are applied to the analysis of structures and other situations.

1.3 FUNDAMENTAL CONCEPTS AND AXIOMS

RIGID BODY

A rigid body is defined as a definite amount of matter the parts of which are fixed in position relative to one another. Actually, solid bodies are never rigid; they deform under the action of applied forces. In many cases, this deformation is negligible compared to the size of the body and the body may be *assumed* rigid. Bodies made of steel or cast iron, for example, are of this type. The study of *strength of materials*, however, is based on the deformation (however small) of such bodies.

MASS

Mass is that invariant property of a body which measures its resistance to a change of motion.

FORCE

Force is the action exerted by one body upon another. Its *external* effect upon a body is manifested by a change in, or a tendency to change, the state of motion of the body upon which it acts. The *internal* effect of a force is to produce stress and deformation in the body. The *characteristics* of a force are:

1. its magnitude,
2. the position of its line of action, and
3. the direction (or sense) in which the force acts along its line of action.

The *unit* of force commonly used in India is Newton (N) or multiples of the Newton such as the kN (i.e., 1 kN = 1000 N).

The *principle of transmissibility* of a force states that the *external* effect of a force on a rigid body is the same for all points of application along its line of action; i.e., it is independent of the point of application. Its *internal* effect, however, definitely is associated with the point of application of the force. Thus, in Fig. l-3,l(a), the motion of the block will be the same whether it is pushed at *A* or pulled at *B*. The local internal effects at *A* and *B*, however, will be quite different. Note that the principle of transmissibility applies only to the external effect of a force on the *same* rigid body. Observe that the system of two bars in Fig. 1-3.l(b), hinged together at *D*, is not inherently rigid; it is the hinge supports at *C* and *E* that restrict relative movement of the bars. In this case,
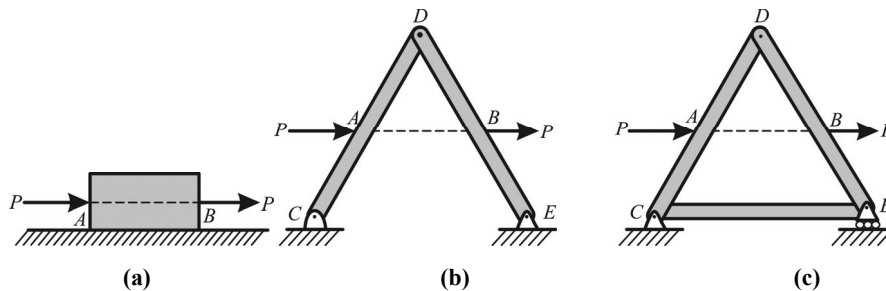

Figure 1-3.1 Principle of transmissibility is valid for (a) and (c), but not for (b).

therefore, since we do not have a single rigid body, both the external reactions and the internal effects will be different if *P* is applied first at *A* and then at *B*. However, if the hinge support at *E* is replaced by a roller support and another bar *CE* added as in Fig. 1-3.l(c), we shall have a triangular truss equivalent to a

single rigid body. Here the external reactions will be the same whether *P* is applied at *A* or at *B*, although the internal effects will differ since *P* applied at *A* tends to bend bar *CD* while *P* acting at *B* tends to bend bar *DE*.

AXIOMS OF MECHANICS

The principles of mechanics are postulated upon several more or less self-evident facts which cannot be proved mathematically but can only be demonstrated to be true. We shall call these facts the axioms of mechanics. They will be discussed at length in subsequent sections as they are used. At this time, we merely collate them for reference and state them in the following form:

1. The parallelogram law: The resultant of two forces is the diagonal formed on the vectors of these forces.
2. Two forces are in equilibrium only when equal in magnitude, opposite in direction, and collinear in action.
3. A set of forces in equilibrium may be added to any system of forces without changing the effect of the original system.
4. Action and reaction forces are equal but oppositely directed.

UNITS AND DIMENSIONS

We define *unit* as the numerical standard used to measure the qualitative *dimension* of a physical quantity. The three primary dimensions basic to mechanics are length, time, and either force or mass[1]. Combinations of these determine the secondary dimensions in which all other quantities can be expressed. Engineers everywhere prefer force instead of mass as the basic dimension because most experimental data used in the design of structural components involve direct measurement of force. To physicists, however, mass is used as the fundamental dimension. Because force and mass are not independent but are related by Newton's laws of motion, this results in two sets of primary dimensions. Those based on force are known as gravitational systems since force is measured in terms of the gravitational attraction upon a standard body at a standard location. On the other hand, the mass of a body is independent of its position so that systems using mass as a primary dimension are known as absolute systems. A detailed comparison between gravitational and absolute systems is discussed later in Section 10-4 when it will have more significance. For the present, Table 1-3.1 summarizes various dimensional systems and the units in which these dimensions are measured. Table 1-3.2 lists various equivalents between these systems of units.

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[1]In addition, there are two other primary dimensions, namely temperature and electric charge, but these are not used in this book.

Table 1-3.1 Systems of Units.

System	Length	Time	Force	Mass	Used by
Gravitational English (FPS)	foot (ft)	sec	pound (lb)	slug[a] (1b-sec^2/ft)	Engineers in English-speaking countries
Gravitational Metric (MKS)	meter (m)	sec	kilogram (kg)	metric slug[a] (kg.sec^2/m)	Engineers elsewhere
Absolute Metric (CGS)	centimeter (cm)	sec	dyne[a] (g-cm/sec^2)	gram (g)	Physicists Rarely
Absolute Metric (MKS)	meter (m)	sec	newton[a] (kg-m/sec^2)	kilogram (kg)	Physcists generally

[a]Derived unit

Table 1-3.2 Conversion Equivalents.

Lengths:	1 m = 3.281 ft = 39.37 inches
	1 km = 0.6214 mile
Force:	1 N = 0.2248 lb
Velocity:	1 km/hr = 0.9115 ft/sec
Acceleration:	g = 9.81 m/sec^2 = 32.2 fps^2
Power:	1 kw = 1.341 hp
	1 hp = 746 N-m/sec
	1 w = 1 N-m/sec
Pressure:	1 atm = 760 mm Hg
	1 mm Hg = 2.7852 psi
Volume:	1 litre = 0.03531 ft^3
	= 0.2642 gal

1-4 SCALAR AND VECTOR QUANTITIES

The quantities encountered in mechanics are of three types scalars, vectors and tensors. Quantities which possess magnitude only and can be combined arithmetically are defined a scalars. Common examples in mechanics are area, energy, mass, and time.

Vectors are defined as quantities having *both* magnitude and direction which combine *geometrically* according to the parallelogram law (Section 2-2). The result must be commutative; i.e., independent of the order of geometric addition. A vector of a quantity is represented geometrically (i.e., graphically) by drawing a line acting in the direction of the quantity, the length of this line

representing to some scale the magnitude of the quantity. An arrowhead is placed on the line, usually at the end, to denote the sense of the direction of the quantity. Also in this book, vectors are generally distinguished from scalars by a notation which is described in Section 2-6. Common examples of vectors are force, moment, displacement, velocity, acceleration, and momentum.

Quantities which possess magnitude but require two or more directional aspects to describe them completely are known as tensors. Other examples are the stress tensor and the strain tensor used in deformable body mechanics. The number of directions needed to fully describe a tensor is called the order of the tensor. Accordingly, observe that scalars and vectors are special cases of tensors, being respectively of zero order and first order, but we shall not treat them in this context. Incidentally, the study of mechanics rarely, if ever, involves higher than second-order tensors.

1-5   Introduction to Free-Body Diagrams

One of the most important concepts in mechanics is that of the free-body diagram. This concept is discussed in detail in Chapter 3 where we first really use it. It is introduced here to help you distinguish between action and reaction forces. To do so, it is necessary to *isolate* the body being considered by removing all elements that act upon the isolated body. A sketch of the isolated body which shows only the forces acting *upon it* by the removed elements is defined as a *free-body diagram*. The forces acting on the isolated or free body are the action forces, also called the applied forces. The reaction forces are those exerted by the free body upon other bodies. One exception to this concept concerns ground supports whose actions on the free body are commonly called reactions.

The free body may consist of an entire assembled structure or an isolated part of it, depending on which quantities are to be determined. For example, consider the derrick shown in Fig. 1-5.1(a). To determine the force in the boom *AC*, we would isolate pin *C* whose free-body diagram (Fig. l-5.l(b)) exposes this force as one of those acting upon it. These forces consist of the gravitational attraction of the weight *W*, the pull *T* exerted on the pin by the cable, and the force *P* exerted by the boom. If the effect of the supports at *D* and *E* were desired, a free-body diagram of the entire derrick, showing it isolated from these supports as in Fig l-5.l(c), would expose their action upon the derrick. Observe that the force *P* in the boom is not shown here since the boom is part of the entire derrick. Remember that a free-body diagram shows only forces exerted by the removed elements that are necessary to isolate the body. Therefore, since *P* acts in an element that was not removed, its action is internal and is not exposed.
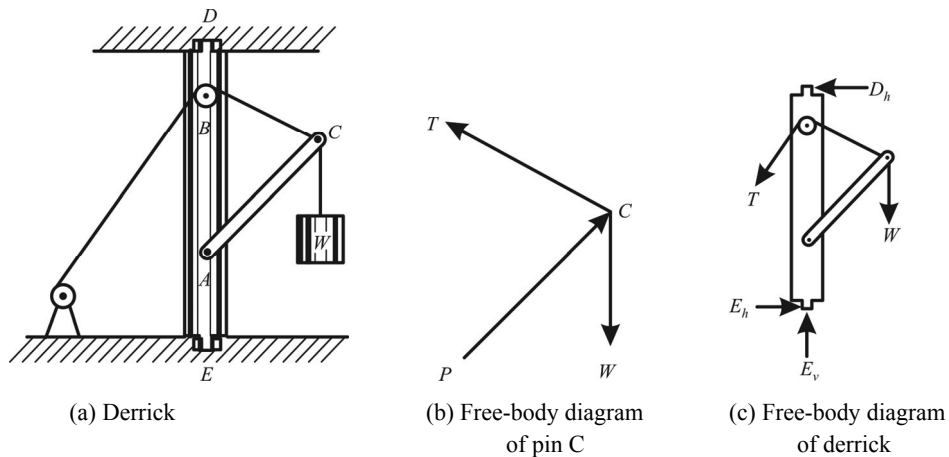
(a) Derrick   (b) Free-body diagram   (c) Free-body diagram
                    of pin C   of derrick

Figure 1-5.1 Free-body diagrams.

1-6   GUIDE TO SOLVING PROBLEMS

One of the first things a student should acquire is the ability to organize his work in a neat and orderly fashion. Properly arranged work not only helps to eliminate personal errors but also permits easy checking by another person – a frequent occurrence in engineering offices. To aid the student to achieve orderly work habits the following suggestions are offered:

1. After identifying the problem, start by constructing a neat diagram of the quantities involved. This diagram should be of sufficient size so that pertinent data and dimensions may be added without affecting its legibility. A freehand sketch is usually acceptable, although the use of a straight edge is preferred and will take little additional time. Some students use different-colored pencils to distinguish between known and unknown quantities.

2. State as concisely as possible what data are given and what information is required. Students who fail to realize what is required, often find themselves in the unenviable position of obtaining the right answer to the wrong question.

3. Errors are frequently caused by mental substitution in equations and subsequent failure to include the term in the equation. For this reason, write out the equation you intend to use before substituting in it. This will also make the process apparent to any person who may check it. If an equation is not used, write a short note indicating the principle used or the operation performed. This short statement of theory – be it equation,

principle, or operation – may be put at the left side of the sheet and the numerical work placed in line with it at the right of the page. In this book, whenever an equation or principle is used in the solution of a problem, it is stated at the left of the page in brackets and is followed by the solution in the same line. You cannot be urged too strongly to use this "theory-solution" technique when solving problems.

Experience has also shown that many students have difficulty in obtaining accurate numerical results eventhough they have correctly applied the principles. To indicate the way to more accurate computation, the illustrative problems discuss the technique of solution as well as the application of principles. The following sections offer several additional items for the student's guidance.

1-7 DIMENSIONAL CHECKS

The equations used in engineering computations must be dimensionally homogeneous; that is, the units on each side must be of the same dimensional form. An easy way to check the dimensions in an equation is to substitute the dimensional equivalents of each term and then multiply or divide these equivalents as though they were algebraic quantities. This process determines the dimensional unit of each term.

For example, consider the equation $v^2 = v_0^2 + 2as$ where v and v_0 are in meter per second, a in meter per second squared, and s in meter. The numeral 2 is a dimensionless number.[2] Substituting dimensionally in the equation, we get

$$\frac{m^2}{sec^2} = \frac{m^2}{sec^2} + \frac{m}{sec^2} \times m$$

which checks the equation since each term is in the same dimensional units.

A similar process may be used to determine the unit of an expression. For example, determine the dimensional unit of kinetic energy if it is expressed by the relation KE = $\dfrac{W}{2g}v^2$, where W is in Newtons, v in meter per second, and g in meter per second squared. Substituting dimensionally, we have

$$KE = \frac{N}{m \, per \, sec^2} \times \frac{m^2}{sec^2} = N\text{-}m \qquad\qquad \textbf{\textit{Ans.}}$$

Even the definition of acceleration, a = $\dfrac{d^2s}{dt^2}$ (Section 9-3), can be checked dimensionally. Here d^2s is a second differential of length, whereas dt^2 is a

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[2] In section 9-3, the numeral 2 is shown to be a factor of integration.

square of a differential of time. With units of length in meter and time in seconds, dimensional substitution in the definition for *a* yields,

$$a = \frac{m}{sec^2} \qquad \qquad \textit{Ans.}$$

1-8 CONVERSION OF UNITS

Occasionally it is necessary to convert a term from one system of units to another to make an equation dimensionally correct. The conversion is accomplished by multiplying the given term by unity where unity is a ratio of units (of the same kind but different in size) containing the required units and those given.

For example, convert a velocity of 60 miles per hour to units of meters per second. Begin by writing

$$v = 60 \frac{miles}{hour}$$

To express *v* in feet per second, multiply the right side by the following ratios, each of which has the value of unity.

$$v = 60 \frac{miles}{hour} \times \frac{1609\,m}{mile} \times \frac{hour}{3600\,sec}$$

Canceling out like units, we obtain

$$v = 60 \times \frac{1609}{3600} = 26.816\,m/sec$$

The ratio $\dfrac{1609}{3600}$ is the conversion factor by which miles per hour must be multiplied to yield meter per second. It is not necessary to remember conversion factors once the method is mastered.

As another example, consider the equation $\theta = \omega_o t + \dfrac{1}{2}\alpha t^2$ where ω_o is in revolutions per minute, *t* in seconds, and α in revolutions per minute per second. It is required to express θ in radians. Substituting dimensionally, we have

$$\theta(rad) = \omega_o \left(\frac{rev}{min} \right) \times t(sec) + \frac{1}{2} \times \alpha \left(\frac{rev}{min \times sec} \right) \times t^2 \left(sec^2 \right)$$

Converting by multiplying by ratios having the value unity gives

$$\theta(\text{rad}) = \omega_o \left(\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{sec}} \right) \times t(\text{sec})$$

$$+ \frac{1}{2} \times \alpha \left(\frac{\text{rev}}{\text{min} \times \text{sec}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{sec}} \right) \times t^2(\text{sec}^2)$$

$$\theta(rad) \;=\; \omega_o t \times \frac{2\pi}{60}(rad) \;+\; \frac{1}{2}\alpha t^2 \times \frac{2\pi}{60}(rad) \qquad\qquad \textbf{\textit{Ans.}}$$

Each term is now expressed in units of radians. In this example, θ was obtained in radians by multiplying each right-hand term by the factor $\frac{2\pi}{60}$.

1-9   NUMERICAL COMPUTATIONS
SLIDE RULES AND CALCULATORS

In engineering computations, the numerical data are rarely accurate to more than three significant figures. No increase in accuracy is attained by carrying more figures through a computation than those in the least accurate data. As in this book, slide rule computations will generally be sufficiently accurate since the known data are usually less precise than the percentage error in misreading the last three figures on a slide rule. Recently, however, pocket calculators at reasonable cost have become available to provide the engineer with an alternate means of computation. Indeed, pocket calculators have begun to replace the slide rule, but a suitable choice may be confusing since they are available in such a variety of types and advertising claims. We therefore present a short discussion of the major differences between calculators currently available. It is also pertinent to present a short comparison between the slide rule and calculator together with their relative advantages and disadvantages.

The advantages of the calculator over the slide rule, aside from its greater precision, are its ability to add and subtract, to automatically locate the decimal point, and the comparative ease of learning to use it. The following analogies should explain the major differences between calculators. In terms of the slide rule scales, most calculators possess only the equivalent of C and D scales and some have a "memory" which is a desirable feature. A slightly more advanced type of calculator has the equivalent of A, B, C, D, and CI scales. These are advertised as "slide rule" calculators since they automatically can square numbers, extract square roots, and take reciprocals. Finally, some calculators

are available that have, in addition, the logarithmic and trigonometric scales common to all log–log slide rules and can truly be called "electronic slide rules."

With the foregoing explanation, the choice of a calculator suitable for the engineer should be simplified. However, calculators vary widely in price according to their capability and numerical capacity so some additional information may be useful. At a minimum, the calculator should have square root, square, and reciprocal keys plus a separate memory and be able to perform mixed and chain numerical computations. Frequently, supplementary logarithmic and trigonometric tables must be available to augment their use. The necessity for supplemental tables, however, is offset by their relative low cost. Of this type, the Melchor 400 seems to offer the best value.

The few pocket calculators which have the functional capability of the log–log slide rule are two or three times as expensive as the Melchor 400. In a class by themselves are the Hewlett Packard models HP 35 and HP 45 which have direct entry keys for logarithmic and trigonometric functions. Although the HP 35 has fewer memory registers than the companion HP 45, it is completely satisfactory for practically all engineering computations. Its capability and versatility are truly remarkable because it uses a four-register operational stack and reverse "Polish" (Lukasiewicz) notation which enables it to solve equations directly without the need for writing down intermediate steps. Presently, the HP 35 is so much superior to its competitors that its higher cost is justified.

Other pocket calculators[3] achieve logarithmic and trigonometric capability by using double function entry keys. They usually require more key board entries than the HP 35 and have a smaller numerical range. However, their lower cost tends to offset these and several other minor deficiencies.

The use of a slide rule instead of a pocket calculator is still a viable choice for the student because of its low cost, versatility, and long life. As indicated before, the fact that a slide rule is not as accurate as a pocket calculator is not a

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[3]Typical of this group are the Rockwell International Unicorn 202/SR, the Sharp PC 1801, and the Bowmar MX-100. While all of these have essentially the same features, the Bowmar MX-100 appears to be best. Generally, the more direct entry keys a calculator has, the more useful it will be. Also available, but much more expensive, is the extremely versatile Hewlett Packard HP 65 which, in addition to being a pocket calculator of the same size and features of the HP 45, is also a programmable mini-computer especially useful for repetitive computations. It accepts programs up to 100 steps long which may be fed in either by direct keyboard entry or by using tiny magnetic program cards. Similar in many respects to the HP 35 is Texas Instrument's model SR 50.

serious deficiency in engineering calculations. Indeed, almost all the answers to problems in this book are computed only to slide rule accuracy. In some instances, moreover, such as computing proportions or applying the sine law, the slide rule will be more convenient and faster than a calculator although admittedly not as precise. These and other useful slide rule techniques are explained at pertinent places in the text. Also note that with a slide rule, the several parts of a computation are visually evident whereas a calculator usually shows only the last entry.

1-10 USEFUL TECHNIQUES AND INFORMATION

SIGNIFICANT FIGURES

It frequently happens that numerical work is computed to a greater degree of accuracy than is warranted by the given data. The accuracy of the final result depends on the least accurate figure used in the computation. For example, if the value of the gravitational constant g is taken as 9.81, the answer should be computed to three significant figures only. No increase in accuracy is attained by carrying more figures through a computation than those in the least accurate data. If for some reason four figures are carried through a computation where the data in the example are accurate to only three figures, the answer should be rounded off to three figures.

If a result is found to be 24.2, for example, the number indicates that the result is greater than 24.15 but less than 24.25; the last 2 of the result is doubtful. On the other hand, 24.20 means that the result is greater than 24.195 but less than 24.205; obviously, the last zero should not be added unless it is a significant figure.

LOCATION OF THE DECIMAL POINT

In slide rule work difficulty is sometimes experienced in locating the decimal point. It is not advisable to rely upon systems that locate this point by counting the number of times the slide extends to the left or right, or by counting the significant figures, etc. A better method is to compute an approximate answer for the problem by rounding off the figures and canceling as far as possible. This method not only will locate the decimal point but will check the slide rule work.

For example, compute the value of

$$\frac{2.196 \times 48.2 \times 289}{3420 \times 68.2}$$

Rounding off figures gives approximately

$$\frac{2\times 50\times 300}{3500\times 70}=\frac{6}{49}\approx\frac{6}{50}=0.12$$

By the slide rule, the numerals in the result are 131; hence, to resemble the approximate value; the answer must be 0.131.

SOLUTION OF QUADRATIC EQUATIONS

Quadratic equations may be solved by formula, slide rule, completing the square, etc. Completing the square is probably as fast and accurate as any other method. It is obtained as follows:

Consider a quadratic equation of the form $x^2 + ax - b = 0$. Rearranging this, we get

$$x^2 + ax + \left(\frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + b$$

hence

$$\left(x+\frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + b$$

$$x+\frac{a}{2} = \pm\sqrt{\left(\frac{a}{2}\right)^2 + b}$$

from which the value of x is easily found.

For example, solve for the roots of

$$x^2 - 22.8x - 31.3 = 0$$

From the above discussion, we obtain

$$x^2 - 22.8x + (11.4)^2 = (11.4)^2 + 31.3$$

$$(x - 11.4)^2 = 130 + 31.3 = 161.3$$

$$x - 11.4 = \pm\, 12.7$$

$$x = 24.1 \ \ \text{and} \ \ x = -\,1.3 \qquad\qquad \textbf{\textit{Ans.}}$$

TRIGONOMETRIC FUNCTIONS

The slide rule gives the functions of angles ranging from slightly below 0.6° to 84.3°. It is occasionally useful to know how to use the slide rule to find the functions of angles outside these limits.

For example, to find the value of sin 86°, proceed as follows:

Since

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

we have

$$\sin 86^\circ = 2 \sin 43^\circ \cos 43^\circ$$
$$= 2 \times 0.682 \times 0.731 = 0.998$$

For angles less than 0.6° a similar technique is available. Since $\sin \theta \approx \theta$ for small angles (where θ is in radians), the value of $\sin \dfrac{1}{2}^\circ$, for example, is given by the equation

$$\frac{1}{2}^\circ = \frac{1}{2} \deg \times \frac{2\pi \text{ rad}}{360 \text{ deg}} = \frac{\pi}{360} \text{ rad}$$

Therefore

$$\sin \frac{1}{2}^\circ = \frac{\pi}{360} = 0.00873 \qquad\qquad \textit{Ans.}$$

Likewise for cosine functions of small angles, we note that $\cos \theta \approx 1 - \dfrac{\theta^2}{2}$.

Hence, in computing the value of $\cos \dfrac{1}{2}^\circ$ we get

$$\cos \frac{1}{2}^\circ = 1 - \frac{1}{2} \left(\frac{\pi}{360} \right)^2 = 1 - 0.0000381 = 0.9999619$$

Trigonometric Functions

Angle	15°	30°	45°	60°	75°
sin	0.259	$\frac{1}{2} = 0.500$	$\frac{1}{2}\sqrt{2} = 0.707$	$\frac{1}{2}\sqrt{3} = 0.866$	0.966
cos	0.966	$\frac{1}{2}\sqrt{3} = 0.866$	$\frac{1}{2}\sqrt{2} = 0.707$	$\frac{1}{2} = 0.500$	0.259
tan	0.268	$\frac{1}{3}\sqrt{3} = 0.577$	1.000	$\sqrt{3} = 1.732$	3.732

$$\sin(-\theta) = -\sin\theta;\ \cos(-\theta) = \cos\theta;\ \tan(-\theta) = \tan\theta$$
$$\sin^2\theta + \cos^2\theta = 1;\ 1 + \tan^2\theta = \sec^2\theta;\ 1 + \text{ctn}^2\theta = \csc^2\theta$$

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos^2\theta; \ \cos^2\theta = \frac{1}{2} - \frac{1}{2}\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + ...$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{5!} - \frac{\theta^6}{6!} + ...$$

Differentials and Integrals

$$\frac{dx^n}{dx} = nx^{n-1}; \qquad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}; \qquad \frac{d\left(\dfrac{u}{v}\right)}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad\qquad \int \frac{dx}{x} = \log_e x + C$$

$$\int \sin x\, dx = -\cos x + C \qquad\qquad \int \cos x\, dx = \sin x + C$$

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C \qquad\qquad \int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int u\, dv = uv - \int v\, du$$