I : Units, Dimensions and Standards

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UNITS, DIMENSIONS AND STANDARDS

RECALL

Units have been associated with physical quantities from the ancient times, inadvertently or with full awareness by way of trade or exchange. They have had special significance in engineering, increasingly so with the continual advances and developments of technology. Thus, whilst in the beginning units generally pertained to simple entities such as mass, distance and assorted articles, as well as time, there are now more important physical quantities like force, pressure, current, potential and so on which involve a variety of units associated with them.

The process of measurement of a quantity – electrical or non-electrical – in particular cannot be considered complete, however elaborately and accurately the measurement might have been carried out, unless associated with the *proper* unit. Accordingly, the result of measurement of a physical quantity must be expressed *both* in terms of quantum and a unit.

For example:

10 metres
15 kilograms
25 seconds
230 volts
5 amperes
100 ohms

etc.

Here, the second part of each expression refers to a particular unit, *in a given system*, the first being the magnitude, pertaining to a physical quantity; that is, distance, mass, time, potential difference, current and resistance, respectively.

FUNDAMENTAL AND DERIVED UNITS

Fundamental or absolute units may be defined as those in which the various other units may be expressed, either in whole or small number or fraction of the fundamental units. The word "absolute" in this sense does not necessarily imply supremacy and extreme accuracy; rather it is used as opposed to "relative."

The Committee of the British Association of Electrical Units and Standards which met in 1863 came with the decision that even electrical units should be defined by some *natural* law that expresses the relation between the quantity concerned and the fundamental quantities of length, mass and time.

The units that relate to these fundamental quantities are known as fundamental units[1].

Each one of these units itself may be indentified in terms of mile, kilometre, metre; gramme, pound, tonne, ton; and second, minute, hour, respectively, depending on the *system* of units used or the 'ease' of expression.

Units in Electrical Engineering

In electrical engineering, and measurements related to electricity and magnetism, two more aspects related to units in use, in addition to the fundamental units (of length, mass and time) must be considered depending on the properties of the media in which the electrical and/or magnetic actions take place. These are known as *specific electric constant*, also called specific inductive capacitance or "permittivity of free space" [ε_0], and *magnetic space constant* or "permeability of free space" [μ_0].

DERIVED UNITS

CGS esu and emu Systems of Unit

The earliest system of units used to express nearly all quantities of electricity and magnetism was based on the use of centimetre[c], gramme[g] and second[s] as the fundamental units *and* either one or both of the above constants. Owing to the process involved in *deriving* the required units in terms of length, mass, time and/or ε_0 and μ_0, these units are called derived units and are generally related mathematically, usually in the form of ratios, to the fundamental units.

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[1]As seen, these (fundamental) quantities represent the basic existence and motion of beings in the universe.

The CGS esu system

This system involves only the permittivity ε of the medium[1] as well as units of length, mass and time. In the basic system, evolved in the very beginning, the permittivity *as a whole* is taken as unity, or $\varepsilon = 1$. The acronym esu stands for "electrostatic units".

The CGS emu system

This system is based on the use of permeability, μ, as well as units of length, mass and time and is known as CGS "electromagnetic unit" system. The emu system is found to be more convenient from the point of view of most electrical measurements and hence has been more generally used than the esu system.

The MKS (or Giorgi) System of Units

The CGS systems of unit are reduced to only historic interest, being of not much practical use in the modern engineering practices due to the units being too "small" or too "large", or otherwise. For example, the magnetic flux used to be expressed as maxwells (or the number of lines of force) and flux density by gauss (or number of lines of force per square centimetre)[2]. Over the decades, therefore, systems of practical applications of units have been evolved, consistent with engineering practices.

The system of practical units in vogue is called the MKS system, or rather the "rationalised MKS system" of units.

The system was originally suggested by Prof. G. Giorgi in 1901. The characteristic feature of the system is that the units of length, mass and time are expressed in terms of metre [M], kilogramme [K] and second [S], respectively, and hence the acronym MKS. The system was finally adopted by the International Electrotechnical Commission (the IEC) at its meeting in 1938 at Torquay.

The MKS system is an "absolute" system of units and has the advantage that a single set of units covers all electrical and magnetic quantities, and is applicable both to electromagnetic and electrostatic effects. It differs from the historic CGS system in the expression of values of the "permeability and permittivity of free space". Thus, μ_0 the absolute permeability is assigned the value 10^{-7} and, using the relation

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[1] ε being the product of ε_0 and ε_r, the *relative* permittivity, depending on the medium, or $\varepsilon = \varepsilon_0 \, \varepsilon_r$.

[2] The modern, practical units of these quantities being weber (= 10^8 maxwell) and tesla (= 10^4 gauss), respectively.

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3 \times 10^8 \text{ m/s (the velocity of light)}$$

the absolute permittivity ε_0 is obtained to be $\varepsilon_0 = 1.113 \times 10^{-10}$.

As an example, some of the common practical mechanical and electrical units in the MKS system and their symbols are

Area, A	:	square metre, m^2 [*note that the unit for area may best be written sq.m.*]
Volume, V	:	cubic metre, m^3
Velocity, v	:	metre per second, m/s
Acceleration, a	:	metre per second per second, m/s^2
Force, F	:	newton, N
Work, W	:	newton-metre, Nm
Energy, E	:	joule, J
Current, I	:	ampere, A
Potential difference, PD	:	volt, V
Electric power, P	:	watt, W

Rationalised MKS System

Further considerations of the MKS system of units to be more applicable, esp. to *electrical* engineering practice, gave rise to "rationalised" MKS system of units, also known as SI system as the abbreviation for "System International d' Units" in French, and is now being followed universally. The main difference between the MKS and the rationalised system of units is in the values assigned to the permeability and permittivity (of free space) as in the case of the difference between CGS and MKS systems. This is because the basis of the rationalised system is the conception of unit magnetic flux issuing from a unit magnetic pole, or unit electric flux issuing from a unit charge, instead of a flux of 4π as was assumed earlier. Thus, in the rationalised system of units, the permeability of free space, μ_0, is assigned the value $4\pi \times 10^{-7}$ (instead of simply equal to 10^{-7} in the un-rationalised MKS system). Correspondingly, by still using the relation

$$c = 1 \big/ \sqrt{\mu_0 \varepsilon_0}$$

the permittivity of free space is found to be

$$\varepsilon_0 = 8.854 \times 10^{-12}$$

(as against 1.113×10^{-10} in the un-rationalised system).

Definitions of Some Units in Rationalised MKS System

In the definitions given below, the standard symbol for the quantity under steady-state (DC or rms AC) and the name of each unit is followed by the symbol of the unit as indicated in parenthesis.

Unit *force* (F, *newton*, N) is the force which produces an acceleration of one metre per second per second to a mass of one kilogram.

Unit *work* or *energy* (W, *newton-metre* or *joule*, Nm or J) is the work or energy associated with a force of one newton when it acts through a distance of one metre.

Unit *power* (P, *watt*, W) is the rate of work done at one joule per second. (accordingly, the unit of energy, joule, can be specified as

1 joule = 1 watt-second).

Unit *current* (I, *ampere*, A) is the current which, flowing in a long, straight conductor at one metre distance (in vacuum) from a similar conductor carrying an equal current, experiences a force of 2×10^{-7} N per metre length.

Unit *charge* (Q, *coulomb*, C) is the charge, or quantity of electricity, which passes in one second through any cross-section of a conductor through which a current of one ampere is flowing.

Unit *potential difference* (PD or V, *volt*, V) exists between two points if one joule of work is done in transferring one coulomb from one point to the other.

[The unit of *electromotive force* (E) is also the volt].

Unit *resistance* (R, *ohm*, Ω) is a resistance such that a potential difference of one volt exists across it when a current of one ampere is flowing.

Unit *inductance* (L or M, *henry*, H) is an inductance such that a rate of change of one ampere per second induces an EMF of one volt[1].

Unit *capacitance* (C, *farad*, F) is a capacitance such that a charge of one coulomb results in a potential difference of one volt.

Unit *magnetic flux* (Ø, w*eber*, Wb) is the flux such that when its linkage with a single turn is removed in one second, an EMF of one volt is induced in the turn.

Unit *magnetic flux density* (B, *tesla*, T) at a point exists when the flux per unit area over a small surface perpendicular to the direction of flux, surrounding the point, is one weber per square metre.

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[1]The definition applies to *self inductance* (L) if the changing current and induced EMF are in the *same* circuit, or to *mutual inductance* (M) if the current is in one circuit and the induced EMF is in the other.

Unit *magneto motive force* ($\mathcal{F}$, *ampere*, I) is that associated with a current of one ampere flowing in a single turn.

Unit *magnetising force* (H, *ampere per metre*, A/m) is an MMF gradient of one ampere per metre of (magnetic) flux-path length.

Unit *electric flux* (ψ, *coulomb*, C) is the flux emanating from a charge of one coulomb.

Unit *electric flux density* (D, c*oulomb per square metre*, C/m^2) at a point exists when the electric flux per unit area over a small surface perpendicular to the direction of flux surrounding the point is unity.

Unit *electric field strength* (Є, *volt per metre*, V/m) is the electric potential gradient of one volt per metre of (electric) flux-path length.

Multiple and sub-multiple units

For convenience in practice, multiples and sub-multiples of the above units are frequently used. The multiplying factor in such cases is usually a *power of ten* with the corresponding standard prefixes being milli, Mega (or simply Meg) etc. In addition, for certain purposes, other multiples may be convenient; for example, watt-hour, kilowatt-hour, ampere-hour for units of energy and quantity of electricity, respectively.

The various MKS units in common use are summarised in Table 1.1.

Table 1.1 : Some practical units and symbols : Rationalized MKS system[1]

Quantity	Symbol	Unit	Remarks
Length	L (or l, *l*)	metre, m	Fractional units: millimetre(mm); centimetre(cm) etc. Multiples : kilometre(km) etc.
Mass	M (or m)	kilogram, kg	Fractional units: milligram(mg); gram(gm) or simply (g) etc.
Time	s	second, s	Fractional units: micro-second(μs); millisecond(ms) Multiples : minute, hour etc.
Force	F	newton, N	Also, $m \times g$ when m is mass in kg and g acceleration due to gravity in m/s/s (= 9.81)
Distance	d (or D)	metre, m	See “length”
Acceleration	α (or a)	metre/s/s, m/s^2	
Work	w (or W)	newton-metre, Nm	
Torque	T	newton-metre, Nm	Same unit as work as both quantities are dimensionally identical

[1]See Appendix I for more details about various units and their conversions.

Table 1.1 *Contd…..*

Quantity	Symbol	Unit	Remarks
Power	W	watt, W	Fractional units : milliwatt (mW) etc. Multiples: kilowatt (kW); Megawatt (MW) etc. Can also be expressed in terms of (the old unit) hp where 1 hp = 746 W
Energy	E	joule, J	Also, W-s
Current	I	ampere, A	Fractional units : milliamp(mA) etc. Multiples : kiloamp(kA) etc.
Charge	Q	coulomb, C	Also, electric flux (symbol : ψ)
Charge surface density	σ	coulomb per metre^2, C/m^2	
Potential difference	V	joule per coulomb, J/C; volt, V	Also, electromotive force (EMF)
Electric field strength	Є	volt per metre, V/m	
Electric flux density	D	coulomb per metre^2, C/m^2	
Capacitance	C	coulomb per volt, C/V or farad, F	A very large unit; common practical unit : micro-farad (μF) Very small fraction: pico-farad (pF)
Permittivity	ε	farad per metre, F/m	Actually refers to unit of permittivity of free space ε_o . Relative permittivity of media, ε_r is a constant.
Resistance	R	volt per ampere, V/A or ohm (Ω)	Fractional units : milliohm (mΩ), micro ohm $(\mu\Omega)$ etc. Multiples : kilo ohm (kΩ), Megaohm (MΩ) etc.
Magnetic flux	Ø	weber, Wb or volt-sec	Equal to 10^8 maxwell
Magnetic flux density or magnetic induction	B	tesla, T	Also, weber per metre^2; equal to 10^4 gauss Fractional unit millitesla (mT) etc.
Magneto-motive force	$\mathcal{F}$	ampere, A	In an N turns coil, equal to NI, I the current in each turn
Magnetic field strength	H	ampere per metre, A/m	Also called magnetising force
Inductance	L	weber per ampere, Wb/A or henry, H	Fractional units : millihenry (mH); microhenry (μH) etc.
Permeability	μ	henry per metre, H/m	Actually refers to unit of permeability of free space μ_o. Relative permeability of media, μ_r , is a constant

International Units

With the universal adoption of the rationalised MKS system of units, the following four units were defined as international or absolute units.

The international ohm

The ohm was chosen as the "primary" reference quantity. This is the resistance offered to the passage of an unvarying electric current (or DC) by a column of mercury at the temperature of melting ice, of mass equal to 14.4521 gm of uniform cross sectional area and of length equal to 106.30 cm.

The cross sectional area would work out to be very nearly 1 mm^2.

The international ampere

This is defined as the steady (or direct) electric current which when made to pass through a solution of silver nitrate in water would deposit the quantity of silver at the rate of 0.0011180 gm per second[1].

The international volt

This is the steady electric potential difference which, when applied across a conductor of resistance of one international ohm, would produce a current of one international ampere. (As would be expected, this is straightforward, derived from the definitions of the international resistance and current).

The international watt

Once again, following the definitions of the resistance and current, the international watt, as the basic reference of power, is defined as the electrical energy per second expanded when an unvarying (direct) current of one international ampere flows under a potential difference of one international volt.

In accordance with the above, the international units of charge (one coulomb), capacitance (one farad) and inductance (one henry) were later obtained as "derived" units.

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[1]It is interesting that definitions of both, the international ohm and ampere, are based on natural phenomena : the former on physical properties of mercury and the latter on the process of electrolysis involving silver.

> ***Comment***
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> *Note that the names of all units are written in small-case letters: for example, metre, kilogram, second, ampere, volt, watt, hertz, newton, weber, tesla etc.*
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> *The symbol of units are generally denoted by upper case letters in most cases, esp. when the name of the unit relates to a scientist: for example Hz (for hertz), N (for newton), A (for ampere) and so on. Symbols for others may be written in small-case: for example, m (for metre), s (for second), cd (for candela), lm (for lumen), lx (for lux) etc.*

DIMEMSIONS OF UNITS

Dimensions of units – of mechanical, electrical or other quantities – form an important aspect, especially when dealing with the derived units and expressions incorporating the same.

Units Pertaining to Mechanical Quantities

All mechanical quantities can be expressed in terms of the three (fundamental) quantities, viz., mass (denoted as [M]), length (denoted as [L]) and time (denoted as [T]) for "dimensional" representation – the basics to motion of bodies and in mechanics. Thus, dimensional "equations" are used to relate *any* chosen quantity to any two or all of the above three quantities.

For example, velocity of a body in motion is defined as the distance, or length, traversed per unit time. Or

$$\text{velocity} = \frac{\text{length}}{\text{time}}$$

This relation can be expressed in the dimensional notation as

$$[\mathrm{v}] = \frac{[\mathrm{L}]}{[\mathrm{T}]}$$

where [v], [L] and [T] represent the dimensions of velocity, length and time, respectively. The square brackets indicate that the equality is dimensional only and *does not refer to numerical values.*

Writing the above expression as

$$[\mathrm{v}] = [\mathrm{LT}^{-1}]$$

gives the dimensions of velocity in "standard" form. Thus, velocity has the "dimensions" $[\mathrm{LT}^{-1}]$, *independent of any system of units*.

Similarly,

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{\text{distance or length}}{\text{time} \times \text{time}}$$

and, therefore, dimensionally

$$[a] = \frac{[L]}{[T^2]} = [LT^{-2}]$$

Likewise

$$\text{force} = \text{mass} \times \text{acceleration}$$

thus, representing the dimensions of mass by [M]

$$[F] = [M] \times [LT^{-2}] = [MLT^{-2}]$$

Accordingly, the dimensions of other *derived* units in mechanics can be obtained by first writing the expression of the quantity in terms of combination of length, mass and time as applicable and working out the final dimensions as above.

Dimensions in Electrostatic and Electromagnetic Systems

In electrostatics

To obtain the dimensions of *charge*, the Coulomb's Inverse Square Law can be applied, that is,

$$F = \frac{Q_1 \times Q_2}{\varepsilon\, r^2}$$

where Q_1 and Q_2 are the two charges, distance r apart, ε is the permittivity of the medium, and F is the resulting force.

Dimensionally,

$$F = \frac{[Q]^2}{\varepsilon\, [L^2]}$$

in which, for the purpose of obtaining the dimensions of charge, the product $Q_1 \times Q_2$ is replaced by the term (quantity of charge)^2. Now the dimensions of force are $[MLT^{-2}]$ as derived above.

Therefore

$$[MLT^{-2}] = \frac{[Q]^2}{\varepsilon\, [L^2]}$$

from which

$$[Q] = [M^{1/2}\ L^{3/2}\ T^{-1}\ \varepsilon^{1/2}]$$ [1]

[1]Thus, in addition to the dimensions of the fundamental quantities, L, M, and T, another fundamental quantity, the permittivity of medium, ε, has to be introduced in the (CGS) electrostatic system, as an example of deriving dimensions of Q. Similarly, the constant μ will have to be introduced in the (CGS) electromagnetic system.

In electromagnetics

The corresponding inverse square law in electromagnetics involves the pole strengths m_1 and m_2 of a magnet to express the force between the two magnetic poles, given by

$$F = \frac{m_1 \times m_2}{\mu r^2}$$

where r is the distance between the poles and μ the permeability of the medium.

That is,

$$\text{force} = \frac{\text{pole strength} \times \text{pole strength}}{\mu \times (\text{length})^2}$$

Therefore, dimensionally

$$[MLT^{-2}] = \frac{[m]^2}{[\mu L^2]}$$

from which

$$[m] = [M^{1/2} L^{3/2} T^{-1} \mu^{1/2}]$$

Dimensions of ε and μ

If ε and μ are assumed to be "fundamental" quantities, their dimensions cannot be expressed in terms of length, mass and time. However, a relationship *between them* can still be deduced as follows:

The dimensions of the charge in electrostatic system is expressed as

$$[Q] = [\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-1}]$$

Now, in electromagnetism, the force exerted upon a magnetic pole of strength m units, placed at the centre of a circular wire of radians r due to a current i flowing in an arc of the circle of length l is given by

$$F = \frac{m\, i\, l}{r^2}$$

or

$$i = \frac{F r^2}{m\, l}$$

and the quantity of electricity flowing in time t, or the charge, is

$$Q = i \times t = \frac{F r^2 t}{m\, l}$$

Dimensionally,

$$[Q] = \frac{[MLT^{-2}][L^2][T]}{[\mu^{1/2}L^{3/2}M^{1/2}T^{-1}][L]}$$

by substituting the dimensions of F, r^2, t and *l*, and that of m derived previously.

Therefore

$$[Q] = [M^{1/2}L^{1/2}\mu^{-1/2}]$$

giving the dimensions of Q in the electromagnetic system, in an alternative form.

Since Q must have the same dimensions in *either* system,

$$[\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-1}] = [M^{1/2}L^{1/2}\mu^{-1/2}]$$

or $$[\varepsilon^{1/2} LT^{-1}] = [\mu^{-1/2}]$$

or $$[LT^{-1}] = [\mu^{-1/2}\varepsilon^{-1/2}]$$

Now $[LT^{-1}]$ are the dimensions of a velocity;

$$\therefore \qquad \frac{1}{\sqrt{\mu\varepsilon}} = \text{a velocity}$$

In any system of units, the "permeability of free space", μ_0, and the "permittivity of free space", ε_0, are related by the equation

$$\mu_0\varepsilon_0 = \frac{1}{c^2}$$

where c is the velocity of light *in the system of units considered.*

From this relationship, the dimensions of any electrical quantity can be converted from those of the electrostatic system to those of the electromagnetic system and *vice versa*.[1]

Dimensions of Electrical and Magnetic Quantities

The dimensions of the various electrical and magnetic quantities can be derived from their known relationships between them and using one of the fundamental quantities μ or ε in addition, as illustrated below.

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[1]Instead of having to use either μ or ε as the necessary fourth fundamental quantity, any of the electrical or magnetic quantity could be used; for example, the quantity of electricity Q. Thus, using the product QV which represents work done, dimensionally

$$[QV] = [\text{work}] = [\text{force} \times \text{distance}]$$
$$= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

from which $$[V] = [ML^2T^{-2}Q^{-1}]$$

being independent of either μ or ε.

Electric current

The electric current can be defined by

$$\text{current} = \frac{\text{quantity of electricity or charge}}{\text{time}}$$

$$\therefore \qquad [I] = \frac{[\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-1}]}{[T]} \qquad \text{using the dimensions of electric charge derived earlier}$$

$$= [\varepsilon^{1/2}\, L^{3/2}\, M^{1/2} T^{-2}] \quad \text{in the ES system.}$$

To convert these dimensions to the ones of the EM system, involving μ instead of ε, substitute $[\mu^{-1/2} L^{-1} T]$ for $\varepsilon^{1/2}$ as derived previously. Then, in the EM system,

$$[I] = [\mu^{-1/2}\, L^{-1}\, T\, L^{3/2} M^{1/2}\, T^{-2}]$$

$$= [\mu^{-1/2}\, M^{1/2}\, L^{1/2}\, T^{-1}]$$

Electric potential[1]

The electric potential is defined by

$$\text{potential} = \frac{\text{“work”}}{\text{quantity of electricity or charge}}$$

Then, expressing work and charge in their dimensional forms

$$[V] = \frac{[ML^2 T^{-2}]}{[\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-1}]}$$

$$= [\varepsilon^{-1/2}\, M^{1/2}\, L^{1/2}\, T^{-1}] \quad \text{in the ES system.}$$

And by the same reasoning as before, the potential in the EM system will be given by

$$[V] = [\mu^{1/2}\, L T^{-1} L^{1/2} M^{1/2}\, T^{-1}]$$

$$= [\mu^{1/2}\, L^{3/2}\, M^{1/2}\, T^{-2}]$$

Resistance

The dimensions of resistance can be best derived as the ratio of electric potential to electric current

$$\text{or} \qquad [R] = \frac{[\varepsilon^{-1/2} L^{1/2} M^{1/2} T^{-1}]}{[\varepsilon^{1/2} L^{3/2} M^{1/2} T^{-2}]}$$

$$= [\varepsilon^{-1}\, L^{-1}\, T] \qquad \text{in the ES system}$$

[1] Also, electromotive force or EMF.

Also, $$[R] = \frac{[\mu^{1/2}L^{3/2}M^{1/2}T^{-2}]}{[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]}$$

or $$[R] = [\mu LT^{-1}] \qquad \text{in the EM system}$$

Since the two dimensions must 'equal'; by doing so

$$\frac{1}{\sqrt{\mu\varepsilon}} = [LT^{-1}]$$

or (dimensionally) velocity as was obtained earlier.

Magnetic flux

To obtain the dimensions of flux, use can be made of Faraday's law which states that

EMF (induced in any circuit) = rate of change of flux

or $$\text{the EMF} = \frac{\text{flux}}{\text{time}}$$

or $$\text{flux} = \text{EMF} \times \text{time}$$

$\therefore$ dimensionally

$$[\text{Ø}] = [\varepsilon^{-1/2}L^{1/2}M^{1/2}T^{-1}] \times [T]$$

$$= [\varepsilon^{-1/2}L^{1/2}M^{1/2}], \qquad \text{in the ES system}$$

Likewise $$[\text{Ø}] = [\mu^{1/2}L^{3/2}M^{1/2}T^{-1}], \qquad \text{in the EM system}$$

Flux density

Since by definition flux density is flux per unit area

$$[B] = \frac{[\text{Ø}]}{[L^2]} = [\varepsilon^{-1/2}L^{-3/2}M^{1/2}], \qquad \text{in the ES system}$$

or $$[B] = [\mu^{1/2}\, L^{-1/2}\, M^{1/2}\, T^{-1}], \qquad \text{in the EM system}$$

and similarly for the other derived units.

The dimensions of some of the important electrical and magnetic quantities, other than the above, in both ES and EM systems, together with the relationships from which they are derived are given in Table 1.2.

Table 1.2 : Dimensions of electrical and magnetic quantities

Quantity	Symbol	Relevant equation	Dimensions: electrostatic system : ES	Dimensions: electromagnetic system : EM	Practical (MKS) unit
Quantity of electricity, charge	Q	$F = \frac{Q_1 Q_2}{\varepsilon\, r^2}$	$L^{3/2} M^{1/2} T^{-1} \varepsilon^{1/2}$	$L^{1/2} M^{1/2} \mu^{-1/2}$	coulomb/ ampere-hour
Electric power	P	$P = V\,I$	$L^2\, M\, T^{-2}$	$L^2\, M\, T^{-2}$	watt
Electric energy	E	$E = V\,I\,t$	$L^2\, M\, T^{-1}$	$L^2\, M\, T^{-1}$	watt-hour/ joule
Inductance	L	$e = L\,\frac{di}{dt}$	$L^{-1}\, T^2\, \varepsilon^{-1}$	$L\, \mu$	henry
Capacitance	C	$C = \frac{Q}{V}$	$L\, \varepsilon$	$L^{-1}\, T^2\, \mu^{-1}$	farad
Impedance	Z	$Z = \frac{V}{I}$	$L^{-1}\, T\, \varepsilon^{-1}$	$L^{3/2} M^{1/2} T^{-1} \mu^{1/2}$	ohm
Magnetic field intensity	H	$H = \frac{B}{\mu}$	$L^{1/2} M^{1/2} T^{-2} \varepsilon^{1/2}$	$L^{-1/2} M^{1/2} T^{-1} \mu^{-1/2}$	ampere per metre
Magneto-motive force	$\mathcal{F}$	$\mathcal{F} = H\,l$	$L^{3/2} M^{1/2} T^{-2} \varepsilon^{1/2}$	$L^{1/2} M^{1/2} T^{-1} \mu^{-1/2}$	ampere
Permeability	μ	$\mu = \frac{B}{H}$	$L^{-2} T^2 \varepsilon^{-1}$	μ	

Observe that in all the above cases, one of the two fundamental quantities, μ or ε, in addition to the other three, appears invariably in the dimensions. This is the basic characteristic of ES or EM system.

Dimensions of Electrical and Magnetic Quantities in Rationalised MKS System

The dimensions of the various quantities in the Rationalised System are based on FOUR fundamental quantities and their dimensions. These are: metre [M], kilogramme [K], second [S] (or time [T]), and *ampere* (or current) [I]. The use of ampere as the fourth fundamental unit was recommended by the IEC in July, 1950 and was subsequently adopted universally. The main feature of the system is that it does away with the use of either permittivity or permeability which do not appear in any of the dimensions.

The current itself being the rate of electric charge (charge per unit time), can be dimensionally expressed as

$$[I] = \frac{[Q]}{[T]} \qquad \text{or} \qquad [Q] = [I]\,[T]$$

It follows that when viewed in *electrical* context, therefore, the fourth fundamental quantity will appear to be charge instead of the ampere or current.

The dimensions of the various quantities can now be derived as illustrated below.

Electromotive force or potential difference

Since potential difference = work per unit charge

$$[PD] = [V] = \frac{[W]}{[Q]}$$

$$= \frac{[ML^2T^{-2}]}{[Q]}$$

$$= [M\,L^2T^{-3}\,I^{-1}]$$

substituting for the dimensions of [Q].

Electric flux

This is dimensionally equal to the charge producing it,

or $$[\psi] = [Q] = [I]\,[T] = [I\,T]$$

Similarly worked out, the dimensions of other quantities are listed in Table 1.3.

Table 1.3 : Dimensions of quantities in rationalised MKS system

Quantity	Symbol	Relevant equation	Dimensions
Electric flux density	D	$D = \frac{\psi}{\text{area}}$	$[I\,T\,L^{-2}]$
Electric field strength	ϵ	$\epsilon = \frac{V}{L}$	$[M\,L\,T^{-3}I^{-1}]$
Capacitance	C	$C = \frac{Q}{V}$	$[M^{-1}L^{-2}T^4I^2]$
Magnetomotive force	$\mathcal{F}$	$\mathcal{F} = (N)\,I$	[I]
Magnetising force	H	$H = \frac{\mathcal{F}}{L}$	$[I\,L^{-1}]$
Magnetic flux	Ø	$Ø = V \times t$	$[ML^2T^{-2}I^{-1}]$
Magnetic flux density	B	$B = \frac{Ø}{\text{area}}$	$[MT^{-2}I^{-1}]$
Inductance	L	$L = \frac{V \times t}{I}$	$[ML^2T^{-2}I^{-2}]$
Resistance	R	$R = \frac{V}{I}$	$[ML^2\,T^{-3}I^{-2}]$
Power	P	$P = V \times I$	$[ML^2T^{-3}]$
Energy	E	$E = P \times t$	$[ML^2T^{-2}]$

Then, the permeability and permittivity are associated with the field quantities as follows:

$$B = \mu H, \quad \text{where } \mu = \mu_0 \mu_r; \ \mu_0 = 4\pi \times 10^{-7}$$

and

$$D = \varepsilon \epsilon, \quad \text{where } \varepsilon = \varepsilon_0 \varepsilon_r; \ \varepsilon_0 = 8.854 \times 10^{-12}$$

Comment

What is the significance associated with units and dimensions of quantities in engineering practice?

In the course of measurement, quantities are generally 'measured' using a given supply as an input and a current or PD as a manifestation in parts of a circuit. For example, in the simple case of measurement of a "medium" resistance by the voltmeter-ammeter method, the input is a given DC voltage, resulting in a current that is measured by an ammeter; whilst the PD across the unknown resistance is in volts. Dividing the PD by the current gives the unknown resistance directly in ohms. However, if the resistance is of the order of a few kilo ohms, the current may be measured in mA. And the unit associated with the measured quantity is to be carefully recorded.

As to the importance of dimensions, consider the measurement of inductance of a coil using Hay's bridge (to be discussed in a later chapter), the expression for the inductance being obtained as

$$L = \frac{R_2 R_3 C}{1 + \omega^2 R_4^2 C^2}$$

where R_2, R_3, R_4 and C denote the various branch components of the bridge and ω the angular frequency of the supply.

Here, if R_2, R_3 and R_4 are in ohm, ω in rad/s, and C is in farad, the inductance must be obtained in henry if the derivation of the above expression is correct. However, whilst the *magnitude* of L will be obtained in terms of the values of resistances etc. in the expression, it is essential that the expression must be *dimensionally balanced* to yield the true value of the inductance. Thus, in the above expression, after some simplification,

$$[L] = \left[\frac{1}{[\omega^2][C]}\right]$$

or

$$= \left[\frac{1}{T^{-2}L^{-1}T^2\mu^{-1}}\right]$$

by substituting the dimensions of $[\omega^2]$ and $[C]$ in EM system.

or $\quad [L] = [L\,\mu] = [L]$, in the EM system.

Hence, the expression provides the correct value of the inductance.

STANDARDS

All measurements, whether of electrical or non-electrical quantities, are characterised by a numeric value or a number representing the magnitude of the quantity followed by the unit. For example, the *voltage* of the single-phase domestic supply may be expressed as 230 volt (written 230 V) ; or the supply *frequency* as 50 Hz. Here, the numbers 230 and 50 are the magnitudes of the supply voltage and frequency, respectively, whilst volt and hertz are the corresponding units. How accurately the voltage and frequency are known? Could the voltage be 235, 220 or 215 V? And the frequency 52 or 48 Hz?

When measuring these quantities, a simple voltmeter and frequency meter may be employed to measure the voltage and frequency, respectively. How accurate are these instruments?

Obviously, the accuracy of measurement of the quantities of interest – voltage and frequency in the present case – would depend on the *accuracy* (or the error, in other words) of the measuring instruments. How would one ensure the accuracy of these instruments? A logical answer would be: to compare the instruments with those having a better or higher accuracy; for example, to compare a ±2.0% accuracy instrument with the one having an accuracy of ±0.5%, and the latter to be compared with the instruments that are ±0.1% accurate and so on[1]. The ultimate thus would be to approach the condition of comparing instruments with the ones having zero error, at least theoretically.

If the physical quantity being measured is just 1 volt, the device having a value of 'exactly' one volt can be considered as a "standard" of the unit of voltage. The term standard is thus applied to a piece of equipment or instrument, having a known measure of a given physical quantity, which can be used, usually by a 'comparison-method' to obtain the values of the physical properties of other equipment.

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[1]This aspect acquires a particular significance when extremely low or high magnitudes of the quantity of interest are to be measured; for example, mV, μV, mA or μΩ or MV and MΩ. In such cases, a slight deviation in the accuracy of the instrument(s) based on a specified percentage (±1.0 or ±0.5% etc.) can lead to considerable deviation in the *actual* value of the quantity.

Thus, care must be exercised in the choice or selection of the measuring instruments for a given application, the accuracy of the instruments being compatible with the accuracy with which the measurement is being carried out. Note, however, that measuring instruments of superior accuracy are, in general, quite expensive and may not be justified for routine measurements.

Classification of Standards

The standards can be broadly classified as

(a) Absolute standards
(b) Primary standards
(c) Laboratory or practical standards.

In addition, there may also be "legal standards" which 'enforce' a legal binding on the users of a class of instruments, or their manufacturers.

Absolute Standards

An absolute standard is one whose value can be determined directly from physical dimensions of the device; for example, the self–inductance of a suitably shaped coil whose value can be deduced directly from its size and number of turns etc. Similarly, the mutual inductance between a long solenoid and a short co-axial search coil situated at mid-length can be calculated from physical dimensions of the two coils and their number of turns. In such cases, there is no reference that may be necessary to any other physical quantity or constant.

Primary Standards

Primary standards which can be devised by means not simply dependent on physical dimensions are standards of such high accuracy that they can be used as the *ultimate* reference standards, especially for all electrical instruments and equipment. It is imperative that in addition to highest possible accuracy, such standards must possess extremely good stability, that is, their values may vary only negligibly over long periods of time, at times stretching to many years, despite changes in atmospheric and other conditions (typically the ambient temperature). Thus, the construction of such standards must include consideration of

(i) long-term stability of materials;
(ii) very small temperature coefficient responsible for expansion, or change of resistance etc.;
(iii) avoidance of deterioration of materials caused by moisture or other atmospheric conditions;
(iv) precision of machining and fabrication;
(v) accuracy of measurement of dimensions;
(vi) rigidity of construction.

By refinements perfected over past many decades, the accuracy of such standards has been raised to extremely high levels. For example, the accuracy of a few of the quantities in use is now ensured as follows:

unit of inductance : 5-10 ppm
unit of current : 10-20 ppm
unit of resistance : 10-20 ppm

or better.

Standards of fundamental quantities

As previously specified, these pertain to the (fundamental) units of length, mass and time.

Length

Metre

The *working standard* of metre now universally adopted as unit of length has been defined for many decades as the distance between two precision blocks of steel having two parallel surfaces. With this arrangement, an accuracy of 1 ppm is claimed.

The International Standard (or unit) of Metre

This is specified as 1,650,763.73 times the wave-length in vacuum of the orange-red light radiation of Krypton-86 atom under controlled atmospheric conditions.

The latest (since 1983) definition of metre is defined as the distance travelled by light in 1/299, 792, 458 second; that is, the inverse of the speed of flight in m/s in 'vacuum'.

Mass

Kilogramme

The international prototype of a kilogramme since 1889 is the mass of a cylinder of platinum-iridium about 39 mm high and 39 mm in diameter maintained under controlled conditions[1]. The latest proposal is to define $(1/1000)^{th}$ of a kilogramme (that is, one gramme) as the mass of exactly 18×140744813 carbon-12 atoms. Also, it is being attempted to link the kilogramme to a fundamental unit of measurement based on quantum physics – the Planck's constant.

Time

Second

The earliest standard or unit of time emerged from the time the earth took to rotate on its axis and the duration of its revolutions around the Sun.

The modern and latest standard of time, and definition of a second, is related to frequency of oscillations of crystals and quartz under controlled environmental conditions; that is, on their atomic properties and thus form

[1]However, not being completely independent of time, the "standard" kilogramme is known to have 'lost' around 50 micro-gramme (almost the weight of a grain of sand!) over the past more than 100 years.

the basis of "atomic clocks" regarded to be accurate to within ±1 second in 300, 000 years![1]

Practical or Laboratory Standards

These standards are of direct relevance to the measurement practices in general and are dealt with in detail; here and in later chapters. The main purpose of these standards is either to check the accuracy of general laboratory instruments or to act as a "standard" variable in precision methods, such as in bridge measurements discussed later [see Chapter XI], or both. The design and construction of such standards may vary.

For example

(i) standards of resistance for general laboratory use take the form of specially fabricated resistance coils whose values are determined by comparison, the ultimate reference being a resistance derived by the absolute method;

(ii) laboratory standards of inductance may again be obtained by comparison methods or in terms of physical dimensions as in the case of primary standards, but with less emphasis on accuracy;

(iii) a standard of EMF may be in the form of a "standard cell", or the voltage drop across a known resistor when a known current is flowing through it.

The accuracy and stability of laboratory standards will range from values approaching those of the primary standards to relatively (somewhat) low accuracies which are adequate for most purposes. The term "standard" may be applied to fixed or variable resistors, inductors and capacitors, and to "high-quality" indicating instruments as well which are meant to accurately measure voltage or PD, current and power[2].

The precautions mentioned in connection with the design and construction of primary standards are also relevant to practical standards, even if to a lesser extent in many cases. Other stringent requirements may apply for some variable standards. For example, in continuously variable types, the bearings of the moving part must be of the highest quality whilst in switched types, the switches must be extremely reliable having low contact resistance and lasting for thousands of operations.

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[1]The first such atomic clock built decades ago is housed in the British Science Museum, London.

[2]There are a class of instruments which are inherently more accurate than others owing to their principle of operation, design and construction as dealt with in a later chapter. [See Chapter V : Measurement of Power].

Laboratory standards of resistance

Resistance standards range in values from $0.001\ \Omega$ to $10^6\ \Omega$ or more and usually are required for use at frequencies ranging from DC to very high values. No single design or method of fabrication may be suitable over such wide range, but the construction and resistance material must comply with the following properties :

- permanence
- an extremely low temperature coefficient
- very low thermoelectric EMF, usually with copper
- adequate heat dissipating capacity
- low values of residual inductance and capacitance
- robustness

The most suitable material for constructing coils that satisfies nearly all requirements generally is **manganin**, an alloy of manganese, copper and nickel having almost all desirable properties; its main disadvantage being its high cost.

In general, a resistor in practice, comprising 'large' number of turns, is bound to have some inductance due to the magnetic field of the current through it (which may be quite large in some measurements), and capacitance due to appreciable electrostatic field in high-voltage applications. Both these effects can be significant at higher frequencies.

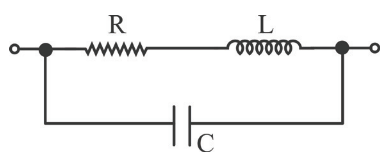


Fig.1.1 : Equivalent circuit of a practical resistor

The resistor may therefore be represented by the equivalent circuit of Fig.1.1 in which the resultant *impedance* of the two parallel branches can be given by

$$\dot{Z} = \frac{(R + j\omega L)\ (-j/\omega C)}{R + j\omega L - j/\omega C}$$

at the angular frequency, ω rad/s.

or
$$\dot{Z} = \frac{R(1 + j\omega L/R)}{1 + j\omega CR - \omega^2 LC}$$

which simplifies to

$$\dot{Z} \simeq R\ [1 + \omega^2\ LC + j\omega\ (L/R - CR)]$$

neglecting "squares" and higher powers of the terms containing L and C as both are too small. The resulting impedance will be non-reactive if

$$L/R = CR$$

(that is when the imaginary term is equal to zero) or $$L = CR^2$$

Hence, in the design of resistors for *AC measurements* care is taken to fulfill the above condition as much as possible. Then, the component behaves as a pure resistance of the value $R(1+\omega^2LC)$. Clearly, at DC when $\omega = 0$, there is no error due to inductance or capacitance. Over the commonly used range of frequency, too, the term ω^2LC may be quite small.

Construction of standard resistors

Standard resistors constitute the most frequently used component in electrical measurements. The form of a laboratory standard resistor depends greatly on its value, purpose and the frequency range over which it is to be used.

Low-resistance Standards

Typical values of these standards may lie between 0.001 Ω to 0.1 or 0.5 Ω. These are usually constructed using strips of manganin (typically about 15 cm long, 3 cm wide and 1 mm or less thick, depending on the value and design of the resistance), arranged in parallel leaving about 5 mm gap between adjacent strips to allow for cooling, and rigidly joined at the two ends to blocks of copper or brass. A typical feature of these resistors is the "4-terminal" construction, with two current terminals to carry the load current in and out and two parallel or potential terminals for external connections as shown in Fig.1.2. This is further discussed in later chapters.

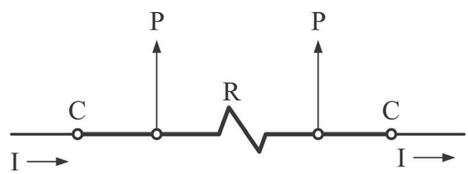
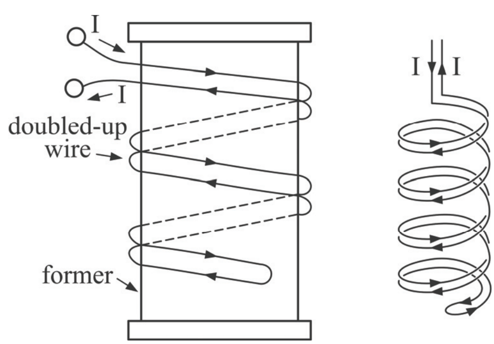


Fig.1.2 : A 4-terminal low, standard resistance

Medium-resistance Standards

These range in values from about 1 Ω or tens of ohm to several hundred or thousands of ohms. The typical construction of resistances in this class comprises coils that utilise "bifilar" loops as shown in Fig.1.3. The wire is first doubled back on itself and then wound helically on a non-metallic, non-magnetic cylindrical former. With the neighbouring portions



(a) wound on a former (b) alternative design

Fig.1.3 : Bifilar construction of resistance

of wire carrying currents in opposite direction at any instant, the magnetic field due to the current and hence the inductance effect is cancelled. However, the frequency range is somewhat limited because of capacitive effect.

The Ayrton-Perry Winding

This is often employed being better than the bifilar construction in terms of compensation for inductive and capacitive effects. Two wires of the winding are connected in parallel and are wound with opposite sense of winding on a flat insulator strip or card as shown in Fig.1.4.

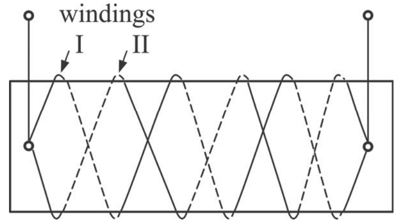


Fig.1.4 : Ayrton-Perry winding

Here, too, the magnetising effect of the two wires is in opposition and so the resulting inductance is extremely small. Since the portions of wire do not run in parallel the capacitance effect would also be very small.

For higher values of resistance (> 10,000 Ω), it is impossible to completely eliminate the capacitive effect. It is therefore usual to wind the coil allowing a small amount of inductance in order to compensate for the capacitance. A simple, single winding is used on a flat card for the purpose for resistance values up to about 100,000 Ω.

Resistance Boxes

These boxes, also known as decade resistance boxes, find extensive use in bridge methods; for example, the Wheatstone bridge discussed in detail later. Typically, the boxes comprise a number of sets of precision resistance coils (for example, the bifilar type) housed in a box, the appropriate coils being connected into circuit by means of rotary switches. Accordingly, a ten ohm decade box consists of ten 1 Ω coils in series and the switch can connect any number of these from 0 to 10 Ω into the measuring circuit. Thus, four decades, 10 Ω, 100 Ω, 1000 Ω and 10,000 Ω each, will provide an adjustable resistor having any value from 1 Ω to 11,110 Ω in steps. However, the accuracy of a resistance value at a given setting, especially at lower range may be affected by various factors from individual coil due to the contact error(s) of the switches; the switch contacts may at times be gold plated to minimise such errors.

Laboratory standards of inductance

Fixed standards of inductance are simple in principle and are constructed in the form of a coil or coils based on considerations of stability and high-frequency performance. A single coil can act as a self-inductance standard whilst two coils wound in close proximity on rigidly connected formers can act as a standard of mutual inductance. For low values, it is customary to adopt air-core construction; for higher values, use of high permeability soft iron core may be imperative, *operating in non-saturating state of iron*.

It is important to adopt suitable choice of the size of the former and number of turns of a particular wire size. Use of a relatively thin wire and a very large number of turns, to obtain high value of inductance, may result in appreciable resistance introduced in series with the inductance, with corresponding heating and temperature rise for long periods of testing. For improved accuracy, elaborate methods of compensation for the resistance and means for cooling and ventilation may be employed.

Variable Standards

Whilst the self-inductance of a coil may be varied by altering number of turns, the mutual inductance of two coils can be adjusted by altering the relative position of the coils. One of the commonly used device consists of a fixed coil inside which a second coil can be rotated about a diametre, along horizontal or vertical axis. Clearly, the maximum inductance is obtained when the two coils are co-axial and very nearly zero when the axes of the coils are perpendicular to each other.

Variable standards of inductance can also be used as standards of self-inductance by connecting the fixed and moving coils in series. The resulting net inductance L is then given by

$$L = L_1 + L_2 + 2M$$

where L_1 and L_2 are the self inductances of the two individual coils and M the mutual inductance between them. If the coupling between the coils is fairly close and the variation of M can be effected to be both positive and negative, a wide range of variation of L can be achieved.

Laboratory standards of capacitance

Fixed Standards

For the very low capacitances, in the range of pico-farad, parallel-plate or concentric-cylinder configuration, with *air* as dielectric, is usually employed which provides fairly good accuracy. For higher values, up to about 0.01 μF, multiple capacitors form very dependable standards. These may use some dielectric other than air and may entail some dielectric loss, especially at higher frequencies. By careful design and use of high-grade materials, the loss may be minimised, with the power factors (or values of tan δ) brought down to as low as 0.00001.

For high-voltage work, in special bridge methods, capacitors which are practically loss-free, yet having adequate dielectric strength, are constructed using compressed nitrogen or carbon-dioxide as the dielectric in the concentric-cylinders form.

Variable Standards

Variable standards of capacitance covering a range of about 0.001μF to 1.0 μF are available in the form of decade boxes. Capacitor units with mica as the dielectric are selected by rotary switches, the arrangements being similar to a decade resistance box. A usual 4-decade box may provide capacitance values accurate to 0.1% or better, with tan δ values being 0.0001 to 0.0005.

Laboratory standards of EMF

Standards of EMF, or what are known as "standard cells", find extensive use in nearly all potentiometer methods of measurement. In this context, the EMF of certain primary cells is found to be remarkably constant with respect to time and environmental conditions. A few of these have thus been developed into EMF standards of high accuracy and precision. The most satisfactory and commonly employed of these is the Weston cell developed and patented by Dr. Weston in 1892. It has a "standard" EMF of 1.01859 V (usually rounded off to 1.0186 V) at 20°C, decreasing by about 40 μV per 1°C temperature rise. The construction of a typical cell is shown schematically in Fig.1.5. A sealed H-shaped vessel contains the electrolyte E, being a saturated solution of cadmium sulphate. The two lower ends constitute the positive and negative poles, P and N; these being mercury and cadmium amalgam, respectively. A layer of mercurous sulphate D is the depolariser (to maintain the two polarities), and the solution is maintained saturated by crystals of cadmium sulphate, C[1].

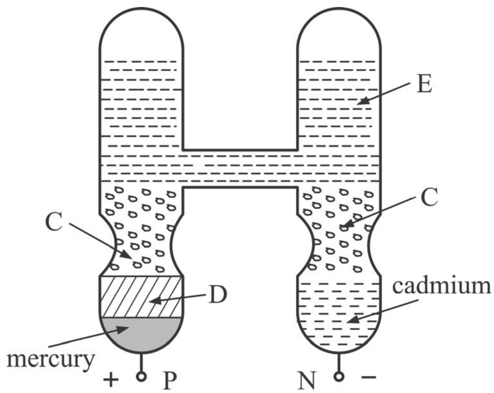


Fig.1.5 : Schematic of the Weston cell

[1]Characteristically, the value of the EMF is specified and accurate to *four* decimal places. This has special significance in the standardisation process of various potentiometers as discussed later in Chapter IX.

Laboratory standards of current

It is not possible, nor practicable, to set up a standard of current in the same manner as the standard of resistance, or indeed that of the EMF. In practice, therefore, recourse is made to the combination of a standard EMF and a resistance to achieve a “standard” of current. The voltage drop across a suitable standard resistance (for example a 4-terminal resistance of $1\,\Omega$) is measured using a precision potentiometer by reference to a standard cell in the first instance. The PD divided by the resistance then provides the (standard) current in the circuit, or otherwise.

Alternatively, as a simple, direct means, the current can be measured by using a high precision ammeter, the reading being assumed to be of sufficient accuracy.

WORKED EXAMPLES

On Units

1. A force of 500 N pulls a sledge of mass 100 kg and overcomes a constant frictional force to motion of 100 N. What would be the acceleration of the sledge in motion?

Resultant force on the sledge for acceleration

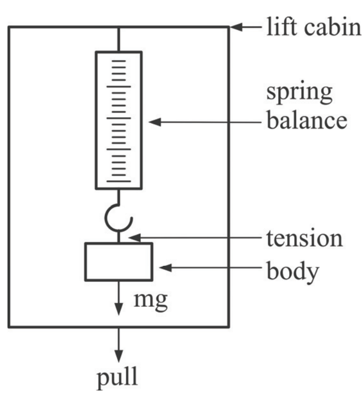
$$F = 500 - 100 = 400 \text{ N}$$

Using the expression : force = mass × acceleration,

the acceleration of the sledge

$$\alpha = \frac{400}{100} = 4 \text{ m/s/s} \qquad (4 \text{ m/s}^2)$$

2. A body of 10.0 kg mass is attached to the hook of a huge spring balance, the same being suspended vertically below the top of a lift cabin as shown in the adjoining figure.



What would be the reading on the spring balance when the lift is

(a) ascending with an acceleration of 0.5 m/s^2 ;

(b) descending with an 'acceleration' of 1.0 m/s^2 ?

Assume $g = 10 \text{ m/s}^2$.

As seen, the suspended body is acted upon by *two* forces :

(i) the *tension*, T, (in N) in the spring balance, acting *upwards*;

(ii) the *weight* of the body, resulting in a gravitational pull acting *downwards*.

The weight is given by m × g or 10.0 × 10 or 100 kg-force or 100 N.

(a) when the lift is ascending, the body is experiencing an acceleration which is upwards.

This means $T > 100$ N

The net force, F, acting on the body will then be

$$F = (T - 100) \text{ N}$$

Also $F = m\,\alpha$

where α is the acceleration by which the body is "moving" upward.

or $F = 10 \times 0.5$ or 5 N

$\therefore \quad T - 100 = 5$

whence $T = 105$ N

(b) when the lift is descending, the body weight would appear to be greater than the tension in the spring balance.

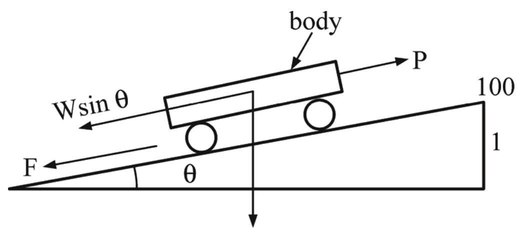
or $100 - T = F = m\,\alpha$

$= 10 \times 1.0$

$= 10$

$\therefore \quad T = 90$ N

3. A body of mass 800 kg is made to move upward on an inclined plane, the slope of the plane being given by "1 in 100". Down the slope, the frictional force acting on the body is 500 N. Calculate the force required to move the body when it is



body weight (W = m g)
F: frictional force

(a) to accelerate at 5 m/s^2;

(b) to move with a constant velocity of 10 m/s.

Assume g = 10 m/s^2.

The configuration of the body movement is shown in the above figure.

As seen, the body is acted upon by *three* forces :

(i) the upward (applied) force, P, to pull the body up;

(ii) the component of body's weight along the inclined plane, acting downward 'opposite' to P;

(iii) the frictional force, F, acting downward, again in opposition to P.

Now the weight of the body acting *vertically* downward is

$$W = m \times g$$

$$= 800 \times 10 \quad \text{or} \quad 8000 \text{ N}$$

and its component downhill is

$$W' = W \sin\theta$$

$$= 8000 \times (1/100) \quad \text{or} \quad 80 \text{ N}$$

With F = 500 N, the net force required *uphill* is given by

$$F' = P - 80 - 500 \text{ ; also, } F' = 800 \times 5 \quad \text{or } 4000 \text{ N}$$

$$= P - 580$$

whence $P = 4000 + 580$ or 4580 N

(b) when the body is supposed to be moving with a constant velocity, there is no acceleration.

Hence, $\alpha = 0$

and $F' = 0$

[the velocity does not figure in the calculation]

Then

$$0 = P - 580$$

or $P = 580$ N

[Note that if the incline was steeper, say 1 in 50, the pulling force in the second case would be 660 N].

4. A train weighing 300 tonne is hauled up an incline of 1 in 80 at a steady speed of 40 km per hour by means of an electric locomotive fed from an overhead traction system at 750 V. The overall efficiency of the locomotive is 75% in terms of the energy input. The rotational inertia of the train is 10% whilst the resistance to motion is 20 N per tonne.

Calculate

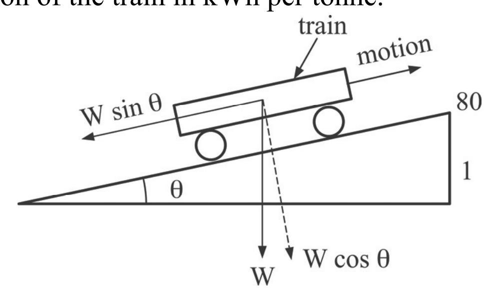
(a) the energy expended in the traction in kWh for pulling the train for a distance of 1 km;

(b) the current fed to the motors of the locomotive;

(c) specific energy consumption of the train in kWh per tonne.

When the train is being hauled up the incline, the component of its weight to oppose the motion will be

$$W \sin\theta$$



where W is the weight of the train and θ the inclination of the track as shown in the adjoining figure.

Here $W = 300$ tonne

$$\sin\theta = \frac{1}{80} \quad \text{[the incline being 1 in 80]}$$

'Effective' weight of the train

$$W_e = 1.1 \times 300 = 330 \text{ tonne,}$$

considering rotational inertia @ 10%

Tractive effort required to pull the train up the incline

$$T_1 = W_e \times g \times \sin\theta$$

$$= 330 \times 9.81 \times (1/80) \times 1000,$$

assuming $g = 9.81 \text{ m/s}^2$

[since the weight of the train is in tonne]

$$= 40466 \text{ N}$$

Tractive effort required to overcome the resistance to train motion

$$T_2 = W_e \times 20$$

$$= 330 \times 20$$

$$= 6600 \text{ N}$$

Total tractive effort required

$$T = T_1 + T_2 = 47066 \text{ N}$$

When the train is hauled for a distance of 1 km or 1000 m, the work done

$$W = 47066 \times 1000$$

$$= 47.066 \times 10^6 \text{ Nm or J or W-s}$$

$$= \frac{47.066 \times 10^6}{1000 \times 3600} \text{ kWh}$$

$$= 13 \text{ kWh}$$

And the energy *input* at the overall efficiency of 75%

(a) $$W_{in} = \frac{13}{0.75} = 17.34 \text{ kWh}$$

At a steady speed of 40 km per hour, time taken to travel 1 km

$$t = \frac{1}{40} \text{ hr}$$

Therefore power input

$$P_{in} = \frac{17.34 \times 1000}{(1/40)}$$

$$= 693.6 \times 10^3 \text{ W or } 693.6 \text{ kW}$$

(b) At the supply voltage of 750 V, the current input to the motors

$$I_{in} = \frac{693.6 \times 1000}{750}$$

$$= 925 \text{ A}$$

(c) Specific energy consumption

$$\text{SEC} = \frac{17.34}{300}$$

$$= 0.0578 \text{ kWh/tonne (or 57.8 Wh/tonne)}$$

On Dimensional Analysis

5. Obtain suitable units for μ_0 and ε_0.

$$[\mu_0] = \frac{[B]}{[H]}, \text{ by definition and with } \mu_r = 1$$

$$= \frac{[MT^{-1}Q^{-1}]}{[L^{-1}T^{-1}Q]}$$

in practical units, with $Q = I\,T$ as the fourth "fundamental" quantity.

or $$[\mu_0] = [M\,L\,Q^{-2}]$$

Comparison of this result and the list of dimensional expressions of various units [see Table 1.3] shows that $[\mu_0]$ may conveniently be expressed as

$$[\mu_0] = [M\,L\,Q^{-2}] = \frac{[ML^2Q^{-2}]}{[L]}$$

$$= \frac{[\text{inductance}]}{[L]}$$

Hence the unit of μ_0 may be called *henry per metre*.

Now $$[\varepsilon_0] = \frac{[D]}{[\epsilon]} = \frac{[L^{-2}Q]}{[MLT^{-2}Q^{-1}]}$$

$$= [M^{-1}L^{-3}T^{2}Q^{2}]$$

Again, by comparison, it is seen that this may conveniently be written

$$[\varepsilon_0] = \frac{[M^{-1}L^{-2}T^{2}Q^{2}]}{[L]}$$

$$= \frac{[\text{farad}]}{[L]}$$

Therefore, the unit of ε_0 may be called *farad per metre*.

6. In a derivation, the resistance portion of a capacitive component is given by

$$R' = \frac{1+\omega^2C^2R^2}{\omega^2CR}$$

where R and R' are resistances, C a capacitance and ω the angular frequency.

Check whether the expression is dimensionally correct.

It is necessary to first check the dimensional consistency of the numerator of the right-hand side of the expression, remembering that the dimensions of $[\omega]$ is simply $[T^{-1}]$.

Then

$$[\omega^2\,C^2\,R^2] = [T^{-1}]^2\,[M^{-1}L^{-2}T^{2}Q^{2}]^2\,[ML^{2}T^{-1}Q^{-2}]^2$$

$$= [M^{0}L^{0}T^{0}Q^{0}]$$

that is, $[\omega^2\,C^2\,R^2]$ is dimensionless which is consistent with the fact that the numeric 1 is added to it in the numerator of the given expression.

Thus, the dimensions of the whole of right-hand side are

$$\left[\frac{1}{\omega^2CR}\right] = [\omega^{-2}C^{-1}R^{-1}]$$

$$= [T^{-1}]^{-2}\,[M^{-1}L^{-2}T^{2}Q^{2}]^{-1}[ML^{2}T^{-1}Q^{-2}]^{-1}$$

$$= [T], \text{ after simplification}$$

But the dimensions of the left-hand side, that is of the resistance R', are

$$[R'] = [ML^{2}T^{-1}Q^{-2}]$$

which can be balanced only if a “C” is added to the denominator of the expression, making it

$$\frac{1}{\omega^2 C^2 R}$$

and R' then given by

$$R' = \frac{1 + \omega^2 C^2 R^2}{\omega^2 C^2 R}$$

[Clearly, 'disregarding' 1 from the numerator, R' simplifies to

$$R' = \frac{\omega^2 C^2 R^2}{\omega^2 C^2 R} = \text{“R”} \qquad \text{- a resistance]}$$

7. In an electric circuit fed from a supply voltage V and comprising a 'load' resistance, the power 'consumed' in the load is known to be a function of applied voltage and the resistance, R. Determine the expression of proportionality in terms of exponents of V and R.

Assume that the electric power, P, can be expressed as

$$P = K\, V^x\, R^y$$

where K is a constant. It is required to deduce the values of x and y.

For this, substitute the dimensions of P, V and R, respectively, in the above expression, say in the EM system.

Then

$$[L^2 M T^{-2}] = K\{[L^{3/2} M^{1/2} T^{-2} \mu^{1/2}]^x \times [L T^{-1} \mu]^y\}$$

To balance this equation *dimensionally*, equate the indices of L, M, T and μ on both sides.

Thus, for L

$$2 = (3/2)\, x + y$$

and for M

$$1 = (1/2)\, x$$

whence $\qquad x = 2 \qquad$ and $\qquad y = -1$

Also, for T

$$-3 = -2x - y$$

which itself is satisfied when $x = 2$ and $y = -1$.

Further, for μ

$$0 = (1/2)\, x + y$$

which is also satisfied for $x = 2$ and $y = -1$.

Therefore, substituting the results in the assumed expression for power,

$$P = K\,V^2\,R^{-1}$$

or

$$P \propto \frac{V^2}{R}$$

8. The expression for the mean torque T of an electrodynamic wattmeter may be written

$$T \propto M^p E^q Z^t$$

where M = mutual inductance between fixed and moving coils

E = applied voltage

Z = impedance of the load circuit.

Determine the values of p, q, and t from the dimensions of the quantities involved.

Assuming a dimensions-less constant, the torque expression can be written

$$T = K\ M^p E^q Z^t$$

Now the dimensions of the various quantities in the EM system are

$$[M] \rightarrow [L\mu]$$

$$[E] \rightarrow [L^{3/2} M^{1/2} T^{-2} \mu^{1/2}]$$

$$[Z] \rightarrow [LT^{-1}\mu]$$

and

$$[T] \rightarrow [ML^2T^{-2}]$$

Substituting these dimensions in the above expression,

$$[ML^2T^{-2}] = K\{[L\mu]^p\ [L^{3/2}M^{1/2}T^{-2}\mu^{1/2}]^q\ [LT^{-1}\mu]^t\}$$

$$= K\{[L^{(p+3q/2+t)}]\ [M^{q/2}]\ [T^{(-2q-t)}]\ [\mu^{(p+q/2+t)}]\}$$

Equating corresponding indices on both sides

$$p + (3/2)\,q + t = 2$$

$$q/2 = 1$$

$$-2q - t = -2$$

and $p + (q/2) + t = 0$ (for μ)

Solving the first three equations gives

$$p = 1,\ q = 2,\ t = -2$$

and these values satisfy the fourth equation.

Hence, the torque expression is given by

$$T = K\, E^2\, M\, Z^{-2}$$

or

$$T = K\, M\, I^2$$

where I is the load current, equal to E/Z[1].

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[1]As dealt with later, the expression for torque of a dynamometer wattmeter is obtained to be

$$T = K\, I_1 \times I_2 \times \frac{dM}{d\theta}$$

in which I_1 represents the current through the fixed coil (or load current), I_2 the current through the moving coil (or current proportional to the supply voltage) and $dM/d\theta$ is the mutual inductance variation with deflection angle, θ.