1

D.C. CIRCUIT CONCEPTS AND CIRCUIT ELEMENTS - I

1.1 INTRODUCTION TO BASICS OF ELECTRICAL ENGINEERING

Electrical Engineering forms the foundation of Electrical, Electronics, Communications, Controls, Computers, Information, Instrumentation, etc. Hence a good grasp of the fundamentals of Electrical Engineering is an absolute necessity to become a good engineer in any discipline.

In this chapter we discuss the basics of Electrical Engineering like sources of electrical energy-voltage and current sources and their conversion, Ohm's law, calculation of electrical power and energy and DC circuit analysis using mesh and nodal analysis.

1.1.1. Circuit Concepts - Concepts of Networks

An Electrical Circuit or an Electrical Nework consists of one or more Electical Energy Sources connected to a number of circuit elements like active or passive elements or both in such a way that there is a connection between the different elements causing a current to flow through the elements. The electrical circuit should be a closed circuit so that current can flow through it. If an Electrical Circuit is open then no current can flow through the circuit.

1.2 CURRENT FLOW

1.2.1 Potential and Potential Difference

An electrically charged particle sets up an electric field around it. If the particle is stationary, then the field set up by it is said to be ***Electrostatic Field***. The electric field lines of a positive charge **+q** will be radial and directed away from the charge. The field set up by the negative charge **–q** will be radial and directed towards the negative charge. Like charges repel and unlike charges attract. The force of attraction or repulsion between two charges q_1 and q_2 will be governed by ***Coulomb's Law*** which states that the force will be proportional to product of the charges $q_1 \times q_2$ and inversely proportional to the square of the distance 'R' between them and depends upon the medium in which the charges are placed.

$$F = \frac{q_1 q_2}{4\pi \in R^2} \qquad \text{.......... (1.2.1)}$$

where F is the force in Newtons

q_1 and q_2 are the charges in coulombs

R is the distance in meters

$\in$ is the permittivity of the medium in Farads / meter

The Absolute Permittivity for free space or vacuum,

$$\in_o = 8.854 \times 10^{-12} = \frac{1}{36\pi} \times 10^{-9} \text{ Farads / meter}$$

For other media the Absolute Permittivity,

$$\in = \in_o \times \in_r$$

where $\in_r$ is the relative permittivity of the medium which is a mere number.

$\in_r = 1$ for Air and $\in_r = 1$ for Mica

The force will be attractive if q_1 and q_2 are opposite charges and repulsive if q_1 and q_2 are like charges.

If we want to place any charge from one point to another point in the electrostatic field work has to be done against the electrostatic force or coulomb force experienced by that charge.

The work done in bringing a unit positive charge from infinity upto a given point p in an electrostatic field is defined as the potential at that point in the electro-static field. The unit for potential will be Joules/Coulomb. The unit is also called Volts.

The work done in moving a unit positive charge from one point in the electric field to another field in the electric field is known as the potential difference between the two points and is measured in volts. If V_A is the potential at point A and V_B is the potential at point B then the potential difference between the two points A and B will be $V_{AB} = V_A - V_B$. If $V_A > V_B$ then V_{AB} will be positive and is known as potential drop from point A to point B.

If $V_A < V_B$ then V_{AB} will be negative and is known as voltage rise from point B to point A. The voltage rise from point B to point A is generally denoted by the letter E.

$$E_{AB} = (E_A - E_B) = (V_B - V_A) = V_{BA} = -V_{AB} \qquad \text{...... (1.2.2)}$$

1.2.2 Electric Current

An electron placed in an electric field will experience a force and move towards the positive potential of the field since it is negatively charged. Continuous flow of electrons constitute a current flow from negative potential to positive potential of the field. This current is known as ***electron current***. The conventional current flow is opposite to that of the electron current in direction. The conventional current flow which is in opposite direction to electron current flow, will be flowing from a point of higher potential to a point of lower potential.

In metals (conducting materials), a large number of free electrons are available which move from one atom to the other at random when a potential difference is applied between two points of the conducting material and the current starts flowing.

The rate of flow of charges through any cross-section of a conductor is called a ***current*** and is denoted as 'i'. Current is expressed in terms of ***amperes***. Ampere is denoted by A or sometimes by α.

$$i = \frac{dq}{dt} \text{ Amperes} \qquad \text{.......... (1.2.3)}$$

where *i* is the instantaneous value of the current (value at any particular instant of the current)

The ***steady current*** 'I' is given as,

$$I = \frac{Q}{t} \text{ Amperes} \qquad \text{.......... (1.2.4)}$$

where Q is the charge flowing through the cross section of the conductor in time 't', if the flow of the charges is uniform.

Otherwise,

$$Q = \int i \, dt \text{ Coulombs} \qquad \text{.......... (1.2.5)}$$

A wire is said to carry a current of ***one ampere*** when charge flows through it at the rate of one coulomb per second. Hence, one ampere is the current which flows when a charge of one coulomb moves across the cross-section of a conductor in one second.

1.3 ACTIVE AND PASSIVE ELEMENTS :

The elements of an electric circuit can be classified into active and passive elements. Active elements are the sources that supply electrical energy to the circuit causing current flow through it. The energy sources can be independent or dependent sources. They may be voltage or current sources.

1.3.1 Sources of Electrical Energy - Voltage Source

The ***voltage source*** is assumed to deliver energy with a specified ***terminal voltage*** V_T, if it is a steady voltage source or v(t) or simply v, if the voltage changes with respect to time. An ideal voltage source is expected to deliver a constant voltage to the outside circuit whatever be the amount of current drawn from the voltage source. The voltage of the source is called the ***Electro-Motive Force*** (**E.M.F.**) and is measured in Volts. It is denoted by the symbol E.

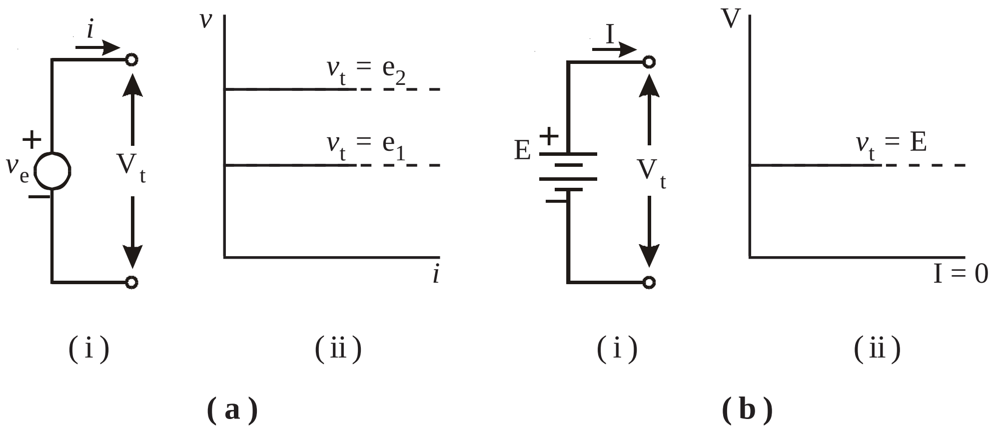
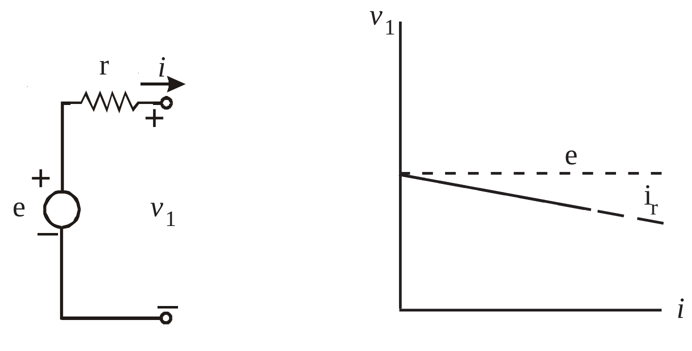


Fig. 1.1 (a) *For a Time-Varying Voltage Source,* ***(b)*** *For a Time-Invariant Voltage Source (represented by a Battery)*



(i) (ii)

(c)

Fig. 1.1 (c) *(i) Model for a Voltage Source in which 'r' represents Source Resistance. The internal resistance 'r' will be very low. For the model of (i), the Terminal Voltage depends on source current as shown in (ii) where $v_t = v \;\; ir$*

If the source has an internal resistance γ then,

$$V = v(t) - i\,r \qquad \text{.......... (1.3.1)}$$

or $$V_t = E - I\,r \text{ Volts} \qquad \text{.......... (1.3.1a)}$$

where I is the current drawn from the Voltage source in Amperes (A)

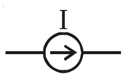
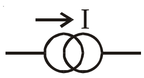
r is the internal resistance of the voltage source in Ohms (Ω).

In practice the terminal voltage of the voltage source will be decreasing as the current drawn from it is increased due to the voltage drop in the internal resistance of the voltage source. The internal resistance has to be very small in order that the voltage drop inside the source will be very small and maximum voltage may be available to the load.

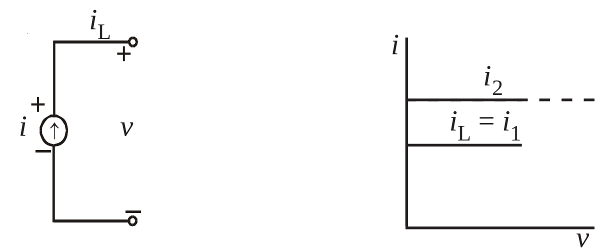
Note : *For any D.C. source, the polarities at the terminals will be same at all instants of time. For Time Varying Sources to polarities indicate the polarities at different terminals at any particular instant of time.*

1.3.2 Sources of Electrical Energy - Current Source

A current source is said to deliver a constant current i_2 = I to the circuit thorugh the terminals, if it is a steady current source or *i*(t) or simply *i*, if the current changes with respect to time. An ideal current source is expected to deliver a constant current to the outside circuit whatever be the circuit. An ideal current source can be

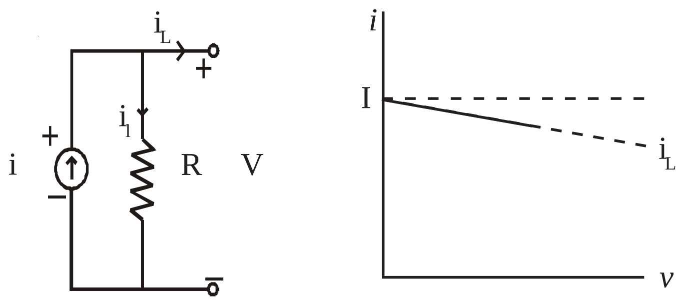
represented by the symbol  or 

In practice the current supplied by the current source will be decreasing as the voltage across the current source is increasing due to the internal resistance R of the current source, which is assumed to be across the current source. The internal resistance of the current source should be as high as possible so that maximum current will be delivered to the load connected across the current source with the current through the internal resistance being very very small.



(i) (ii)

Fig. 1.2 (a) *(i) Symbol for the Current Source for which i does not depend on v as shown in, (ii) Other lines may be drawn parallel to that shown for a specific current, i_1.*



(i) (ii)

Fig. 1.2 (b) *(i) Model for a Current Source in which R represents shunt resistance. For the model of 1.2 (i), the Terminal Current in 1.2 (ii) is given by $i_L = i - (1/R)v$ (ii) Other lines may be drawn parallel to that shown for a specific current, i_L.* The internal resistance 'R' will be very high.

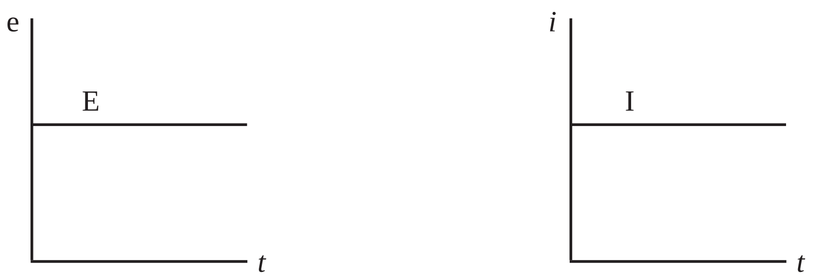
1.3.3 D.C. & A.C. Sources

If the voltage or current supplied by an electrical energy source is constant with respect to time as shown in Fig. 1.3 (a)(i) or Fig. 1.3 (a)(ii) then it is known as ***D.C. Voltage Source*** or ***Direct Current Source*** (or ***Steady Current Source***). D.C. stands for ***Direct Current***.

A D.C. Source has two terminals from which energy is supplied to the outside load. They are known as ***Positive Terminal*** which supplies the positive ions and ***Negative Terminal*** which receives the returning current or which can be assumed as supplying negative charges called electrons in the direction opposite to the conventional current direction. D.C. supply is provided by batteries or D.C. generators. The battery converts chemical energy into electrical energy. The generator converts mechanical energy into electrical energy.

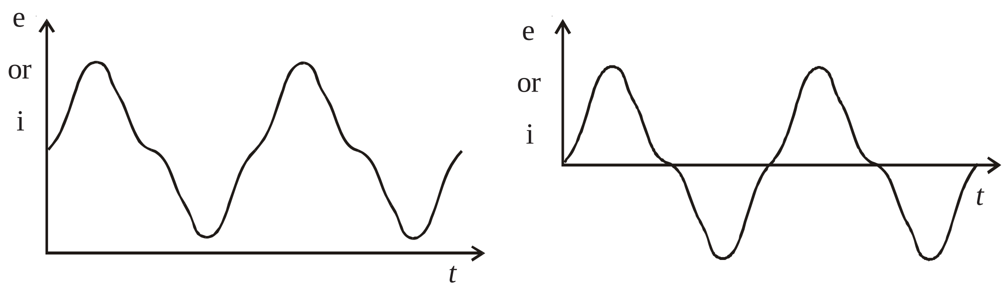
If the voltage or current is varying with respect to time but, has the same polarity as shown in Fig. 1.3(b)(i) then it is known as ***Unidirectional Source***. If the polarity is positive, it is known as ***Positive Source***. If the polarity is negative, it is known as ***Negative Source***.

If the voltage or current supplied by an electrical energy source varies in both magnitude and polarity with respect to time as shown in Fig. 1.3 (b)(ii) then it is known as ***A.C. Voltage Source*** or ***Alternating Current Source***. A.C. stands for ***Alternating Current***.



(i) D.C. Voltage Source (ii) D.C. Current Source

(a) D.C. Voltage & Current Sources



(i) Uni directional voltage or current (ii) A.C. Voltage or Current

(b) A.C. Uni-directional Voltage & Current Sources

Fig. 1.3 *A.C and D.C. Voltage & Current Sources*

The value of the voltage or current of an A.C. supply at any instant is called ***Instantaneous Value of Voltage or Current*** and is denoted as $v(t)$ or $i(t)$.

In general, the instantaneous values may also be denoted as v or i. **Generally no polarities will be marked for A.C. voltage or current. If at all polarities are marked for A.C. voltage or current, they mean the polarities of the voltage or current at the marked terminals, at any one particular instant and will be changing from time to time.**

1.3.4. Unilateral and Bilateral Elements :

Elements in which current flow is in only one direction are called unilateral elements. Eg. Diode. Elements in which current flow can be in both directions are called bilateral elements. Eg. Resistance, Inductance, Capacitance etc.,

i) Passive Elements - Resistance or Resistance Parameter

When a potential difference is applied across a conductor (or wire), the free electrons start moving in a particular direction. While moving through the material, these electrons collide with other atoms and molecules. They oppose this flow of electrons (or current) through it. This opposition is called ***Resistance***. Heat is produced because of the collisions of moving electrons with the other atoms and molecules. Thus whenever a current flows through a conductor, heat is produced in the conductor and this heat has to be dissipated fully. Otherwise, the insulation of the conductor (the Sheath made of insulating material covering the conductor) will get damaged.

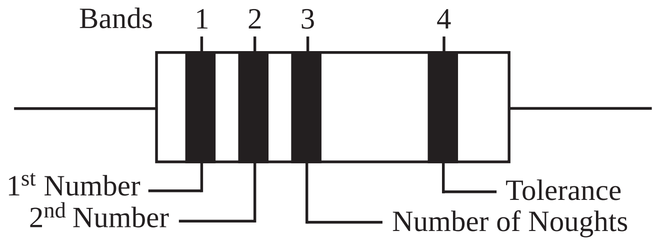
The opposition offered to the flow of current (free electrons) is called ***Resistance***. Resistance is denoted by R and is measured in ***ohms*** named after a German mathematician ***George Simon Ohm*** and is represented by the Greek symbol Ω. For very high resistance we use large units such as kilo-ohms ($k\Omega$ which is equal to 10^3 Ω) or Mega-ohms ($M\Omega$ which is equal to 10^6 Ω) while for small resistances we use smaller units such as milli-ohms ($m\Omega$ which is equal to 10^{-3} Ω) or micro-ohms ($\mu\Omega$ which is equal to 10^{-6} Ω).

In electronic circuits, the current will generally be very small in milli-Amperes (mA or 10^{-3}A), micro-Amperes (μA or 10^{-6}A) or nano-Amperes (nA or 10^{-9}A) and hence, the resistance or resistors used will be in kilo-ohms or Mega-ohms and will be denoted simply as k or M (Ω is understood). They will be made of carbon resistors and will have color codes for different digits. They will not be accurate and will have a tolerance limit denoted by another color band and the wattage of the resistors will also be specified as ½ Watt or 1 Watt. For higher wattages of 5 or 8 or 10 Watts etc., wire wound resistors will be used.

Color Code of Resistors :

There are resistors made from carbon mouldings or from metal-oxide film. Both are small, if not very small, and therefore we would find it most impossible to mark them with a rating such as 47,000Ω, ½ Watt and hence, color coding is used. 1 Watt resistance will be bigger in size than ½ Watt resistance.

In the case of carbon resistors, it is usual to identify the ratings by means of rings painted around the resistors, as shown in Fig. 1.4. One of the bands is always placed near to the end of the resistor and should be taken as the first band. The first, second and third bands are used to indicate the resistance of the resistor by means of a color code which is also given in Fig.1.4 which explains the colour coding clearly. In the code the first two bands are orange and blue which, from the table are 3 and 6 respectively. Therefore we are being told that the resistance has a numerical value of 36. The third band tells us how many zeroes to put after that number. In this case, the third band is green and there should be five zeroes, i.e., the resistance is 36,00,000Ω (3.6MΩ or 36 KΩ, simply called 3.6 M or 36 K).



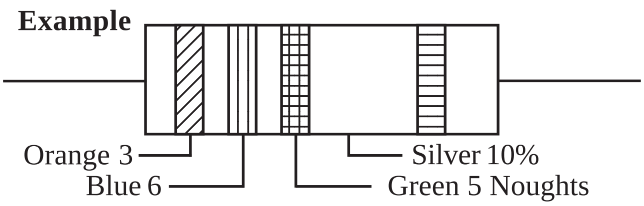


Fig. 1.4 *Color Code of Resistors*

In power electronics, the power electronic devices can carry very high currents like hundreds of Amperes.

The resistance R of a conductor depends on

1. *its length, in direct proportion*

2. *the area of cross section of the conductor, in inverse proportion*

3. *the material of the conductor, in direct proportion to the specific resistance of the material*

4. *the temperature, i.e. in direct proportion to the difference in temperature*

Usually, the resistance is given per unit cross-section and unit length. This is called ***specific resistance*** or ***resistivity*** of the material represented by the letter, ρ. Since Copper and Aluminium have good electrical conductivity and cheaper compared to Silver, they are used for conductors. The resistivity of copper is 0.0173 μΩ-m and that of aluminium is 0.0283 μΩ-m.

The colour codes for the bands are given in Table 1.1

Table 1.1 *Color Code*

Digit	Color
0	Black
1	Brown
2	Red
3	Orange
4	Yellow
5	Green
6	Blue
7	Violet
8	Grey
9	White

Tolerance	Color
5%	Gold
10%	Silver
20%	No Color Band

The color code can be remembered as,

BBROY of **G**reat **B**ritain has a **V**ery **G**racious **W**ife

If the resistance or resistivity is less the current flowing will be more. The resistance of a material is given by

$$R = \frac{\rho l}{a} \; \Omega \qquad \text{.......... (1.3.2)}$$

where ρ is the specific resistance of the material of the conductor in Ω-meters

l is the length of the conductor in meters

a is the area of cross-section of the conductor in square meters.

Not only conductors, but a coil wound with a conductor or any other electrical equipment offers resistance to current flow. A wire wound coil with two fixed terminals is called a ***resistor*** or ***resistance***. A coil with two fixed terminals and a variable contact terminal which makes contact with the body of the coil is called a ***Rheostat*** or ***Variable Resistor***. A rheostat can be connected in two ways as,

1. ***Series Resistance*** *(as shown in Fig. 1.5 (a))*

If the moving contact is very near the starting terminal the resistance offered by the rheostat will be minimum and if it is nearer to the farthest end terminal, the resistance offered by the rheostat will be maximum. Sometimes the moving contact and one of the end terminals will be connected together in which case, the resistance *offered will be the resistance of the remaining part of the winding. As the moving contact is varied the resistance offered by this part will be varying, as the moving contact is moved away from or towards the second terminal. If the moving contact is towards the starting terminal then the resistance offered by the rheostat will be less.*

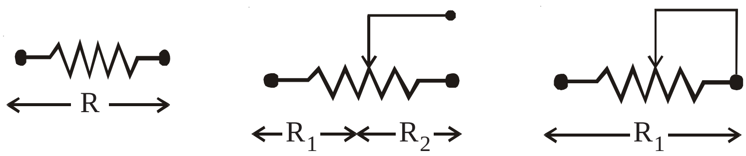


Fig. 1.5 (a) *Series Rheostat*

2. ***Potential Divider*** *(as shown in Fig.1.5 (b))*

The two ends of the rheostat are connected across a voltage source which constitutes the input to the potential divider. The output is tapped between the moving contact and one of the end terminals, in which case, part of the input voltage will be the output voltage. The voltage tapped is given by,

$$V_{out} = V_{in} \times \frac{R_t}{R} \text{ Volts} \qquad \text{.......... (1.3.3)}$$

where R is the total resistance of the rheostat and R_t is the reistance of the tapped part of the rheostat winding.

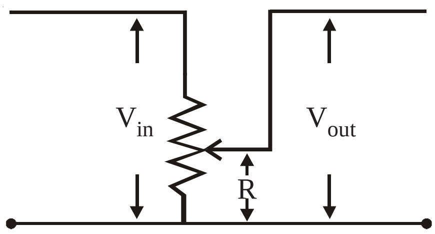


Fig. 1.5 (b) Rheostat as Potential Divider

Example 1.1 :

Find the resistance of a coil of mean diameter 4 cm containing 400 turns of manganese wire 0.05 cm in diameter. The resistivity of manganese is 42 $\mu\Omega$ - cm.

Solution :

Here $\rho = 42\ \mu\Omega\text{-cm}.$

$= 42 \times 10^{-6}\ \Omega\text{cm}$

$= 42 \times 10^{-8}\ \Omega\text{ - m}$

$$a = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} (\ 0.05\ )^2\ \text{cm}^2$$

$$= \frac{\pi}{4} (\ 0.05\ )^2 \times 10^{-4}\ \text{m}^2.$$

Number of turns (N) of the coil,

$N = 400$

Length per turn of the coil is $\pi \times D$

where D is the diameter of the coil.

Length (l) of the conductor of the coil,

$l = \pi \times D \times N$

$= \pi \times 4 \times 400$ cm

$= 16\ \pi$ m

Resistance of the coil,

$$R = \frac{42 \times 10^{-8} \times 16\pi \times 4}{\pi\left(0.05\right)^2 \times 10^{-4}} = 107.52\ \Omega.$$

1.3.5. Effect of Temperature on Resistance

As temperature increases the resistance of most of the conducting materials increase while for some material like Carbon, electrolytes, insulators the resistance decreases as the temperature increases. The change in resistance depends upon the temperature-coefficient of resistance, which will be positive if the resistance increases with temperature and negative if the resistance decreases with temperature. The change in resistance per ohm per degree temperature change is called ***temperature-coefficient of resistance*** and its symbol is α.

If a metallic conductor of resistance R_0 at 0^oC is heated to a temperature t_1, then the resistance R_1 at temperature t_1 is given by

$$R_1 = R_0\ (\ 1 + \alpha_0 t_1\ ) \quad \text{.......... (1.3.4)}$$

Since the temperature-coefficient itself varies with temperature, it does not have the same value at all temperatures. Thus if R_1 and R_2 are the resistances of a conductor at temperatures t_1 and t_2, we have

$$R_2 = R_1 (1 + \alpha_1 (t_2 - t_1)) \quad \text{.......... (1.3.5)}$$

where α_1 is the temperature-coefficient of resistance at $t_1{}^oC$.

Variation of α is obtained as

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_1 - t_2)$$

or
$$\alpha_2 = \frac{\alpha_1}{1+\alpha_1(t_2-t_1)} \quad \text{.......... (1.3.6)}$$

Example 1.2 :

The resistance of a coil decreases from 70Ω at 75^{o}C to 50Ω at 15^{o}C. Calculate the value of temperature-coefficient of resistance of the material of the coil at 0^{o}C. Find the resistance at 0^{o}C.

Solution :

Let α_0 be the temeperature-coefficient of resistance and R_0 be the resistance at 0^{o}C.

$$70 = R_0 (1 + 75\ \alpha_0) \quad \text{.......... (1)}$$

and
$$50 = R_0 (1 + 15\ \alpha_0) \quad \text{.......... (2)}$$

Dividing Eq. (1) by Eq. (2) we get,

$$\frac{70}{50} = 1.4 = \frac{1 + 70\alpha_0}{1 + 15\alpha_0}$$

$$54\ \alpha_0 = 0.4$$

or
$$\alpha_0 = \frac{0.4}{54} = 0.0074074$$

$$R_0 = \frac{70}{1+75\alpha_0} = \frac{70}{1.5555} = 45\ \Omega$$

1.3.6. Electrical Conductance

The reciprocal of resistance of a conductor is called ***Conductance*** of the conductor and is denoted by ‘G’ and is expressed in terms of Siemen abbreviated as “S” or mhos (℧).

$$G = \frac{1}{R}$$

$$= \frac{1}{\rho l / a}$$

$$= \frac{a}{\rho l} = \sigma \frac{a}{l} \text{ Siemens} \qquad \text{.......... (1.3.7)}$$

where σ is called specific conductance or conductivity and is measured in Siemens per meter (S/m).

If the conductance or conductivity is less the current flowing will be less.

1.4 OHM'S LAW

The relationship between the current flowing through a conductor and the potential difference across the conductor 47 is given by ***Ohm's Law***.

The *Ohm's Law* states that *the potential difference across a conductor is directly proportional to the current flowing through the conductor, the temperature of the conductor remaining constant*. The constant of proportionality is R, the resistance.

$$V = I \times R \text{ Volts} \qquad \text{.......... (1.4.1)}$$

or

$$V = R \times I \text{ Volts} \qquad \text{.......... (1.4.1a)}$$

Here, V is the voltage drop across the conductor.

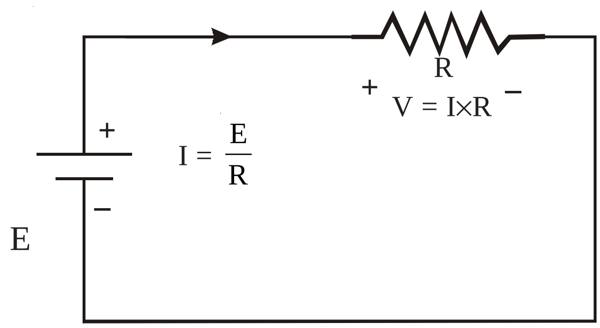


Fig. 1.6 *Circuit for Ohm's Law*

Note : *In the circuit, the voltage drop caused in the conducting wires connecting the battery and resistance are assumed to be zero as it will be negligible because the wire is a conducting material.*

Note : *While writing the equations for Volt-Ampere relationships in matrix form the second form i.e. Eq. (1.4.1a) will be applicable and can be expressed as,*

$$[\,V\,] = [\,R\,][\,I\,] \qquad \text{.......... (1.4.1b)}$$

Ohm's Law can also be expressed as

$$E = I \times R \text{ Volts} \qquad \text{.......... (1.4.1c)}$$

Here, E is the voltage rise across the conductor.

$$E_{AB} = -V_{AB}$$

The equation for Ohm's Law can also be written as,

$$I = \frac{V}{R} \text{ Amperes} \qquad \text{.......... (1.4.2)}$$

Ohm's Law also gives the Volt-Ampere relationship for an element. Ohm's Law can be applied to a part of a circuit or to the full circuit in which the current flows.

Ohm's Law can also be applied to A.C. Circuits or to circuits with Unidirectional Source in Laplace Transform domain for instantaneous values. For steady state conditions of A.C Ohm's Law using impedances and using RMS values for voltages and currents, all in complex form will be discussed later. However, for resistive circuits consisting of only resistances Ohm's Law can be written as,

$$v = i \times R \text{ Volts} \qquad \text{.......... (1.4.3)}$$

or

$$v = R \times i \text{ Volts} \qquad \text{.......... (1.4.3a)}$$

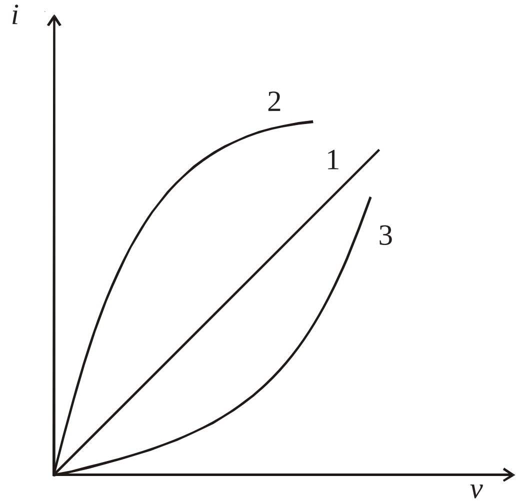


Fig. 1.7 *Linear & Non-Linear Resistances*

or $$i = \frac{v}{R} \text{ Amperes} \qquad \text{.......... (1.4.3b)}$$

where v and i are instantaneous values of Voltage and Current respectively.

1.4.1 Linear & Non-Linear Resistances

Those resistances in which the current flow changes in direct proportion with changes in the voltage applied across them are called ***Linear Resistances***. (curve 1 of fig. 1.7) The *v-i* characteristics for linear resistances will be current increasing as the voltage across increases. Ohm's Law is applicable as the resistance remains constant.

Those resistances for which the current through them does not vary in direct proportion are called ***Non-Linear Resistances***. For nonlinear resistances the *v-i* characteristics will be nonlinear. (curves 2 and 3 of fig.1.7)

In certain nonlinear resistances like Thyrite, the current increases more than proportionately with applied voltage with resistance decreasing rapidly like in curve 2 of Fig. 1.7. Hence, it is used in Lightning Arrestors.

In certain other nonlinear resistances like Semiconductors, Thermistors, the current decreases as the voltage across increases like in curve 3 of Fig. 1.7. Hence, thermistor is used in over current protection in Motors, etc.

Example 1.3 :

A current of 0.75A is passed through a coil of nichrome wire which has an area of cross-section of 0.01 cm^2. If the resistivity of the nichrome is $108 \times 10^{-6}\Omega$-cm and the potential difference across the ends of the coil is 81V. What is the length of the wire? What is the conductivity and conductance of the wire?

Solution :

Resistance,

$$R = \frac{\rho l}{a}$$

where, $$R = \frac{V}{I}$$

$$= \frac{81}{0.75}$$

$$= 108\ \Omega$$

$$a = 0.01\ \text{cm}^2 = 0.01 \times 10^{-4}\ \text{m}^2$$

$$l = \frac{R \times a}{\rho}$$

$$= \frac{108 \times 0.01 \times 10^{-4}}{108 \times 10^{-8}} = 100\ \text{m}$$

Conductivity,

$$\sigma = \frac{1}{\rho}$$

$$= \frac{1}{108 \times 10^{-8}}$$

$$= 92.59 \times 10^{4}\ \mho/\text{m}$$

Conductance,

$$G = \frac{1}{R}$$

$$= \frac{1}{108}$$

$$= 9.259 \times 10^{-3}\ \mho \text{ or Siemens}$$

1.5 ELECTRICAL POWER

Power is the rate of doing work and is expressed in Joules per second. When one coulomb of electrical charge moves through a potential difference of one volt in one second the work done is one Joule/sec and in electrical engineering it is expressed as one Watt and is denoted by the symbol P.

So Power supplied,

$$P = E \times I \text{ Watts} \qquad \text{.......... (1.5.1)}$$

where E is the source voltage.

Power expended,

$$P = V \times I \text{ Watts} \qquad \text{.......... (1.5.1a)}$$

where V is the voltage drop.

Applying Ohm's Law for V,

$$P = (I \times R) \times I \text{ Watts}$$

$$P = I^2 \times R \text{ Watts} \qquad \text{.......... (1.5.2)}$$

or Applying Ohm's Law for I,

$$P = V \times \left(\frac{V}{R} \right) \text{ Watts}$$

$$P = \frac{V^2}{R} \text{ Watts} \qquad \left(\because I = \frac{V}{R} \right) \qquad \text{.......... (1.5.3)}$$

For A.C. circuits or circuits with unidirectional source, the equations for electrical power can be written using instantaneous values as,

$$p = e \times i \text{ Watts} \qquad \text{.......... (1.5.4)}$$

where e is the source voltage.

Power expended,

$$p = v \times i \text{ Watts} \qquad \text{.......... (1.5.4a)}$$

where v is the voltage drop.

Applying Ohm's Law for v,

$$p = i^2 \times R \text{ Watts} \qquad \text{.......... (1.5.5)}$$

or Applying Ohm's Law for i,

$$p = \frac{v^2}{R} \text{ Watts} \qquad \text{.......... (1.5.6)}$$

The power expended is also known as ***Power Loss*** or ***Copper Loss*** since the power loss takes place in the conductor (generally made of copper). It is called copper loss, even though the conductor material is not made of copper.

The power loss or copper loss appears in the form of heat. This heat has to be dissipated properly or else the insulation of the conductor or the insulation coating (varnish)of the coil will get damaged and there will be short circuits between turns of the coil and the coil may get burnt away in the case of machines and other equipments using coils.

1.6 ENERGY CALCULATIONS

Energy is the work done in a given time to achieve the required state of heating, lighting, lifting weights, moving the objects, etc. As such energy calculations are very important. Of late to have good efficiency in getting the work done and to have good economy Energy Auditing is resorted to in Industries and because energy charges are recurring charges involving expenditure.

According to the *Law of Conservation of Energy*, energy can neither be created nor destroyed. As such energy can atmost be converted from one form of energy into another form like converting mechanical energy into electrical energy and vice versa, converting electrical energy into heat energy and vice versa, etc. In this process the efficiency of the equipment used in conversion plays an important role. Also the constants of conversion are to be considered.

1.7. ELECTRICAL ENERGY

Electrical Energy is the total amount work done and is expressed in Joules or in Watt-seconds in electrical engineering. It is denoted by W. If E is the voltage rise or electromotive force and I is the current then, the energy generated or the energy supplied for a time t seconds is given as,

$$W = P \times t = E \times I \times t \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.1)$$

or

$$W = I^2 \times R \times t \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.2)$$

or

$$W = \frac{E^2}{R} \times t \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.3)$$

For A.C. circuits or circuits with unidirectional source,

$$w = p \times dt = e \times i \times dt \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.4)$$

or

$$w = i^2 \times R \times dt \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.5)$$

or

$$w = \frac{e^2}{R} \times dt \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.6)$$

where p, v, i all stand for instantaneous values and dt is the differential time

So,

$$W = \int p\,dt \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.7)$$

or

$$W = \int e \times i\,dt \quad \text{Watt-sec} \qquad \text{..........} \ (1.7.8)$$

or $$W = \int \frac{e^2}{R} dt \quad \text{Watt-sec} \qquad \text{.......... (1.7.9)}$$

The energy expended canl be obtained by using the above equations substituting V for E or *v* for *e*.

If the power is supplied for time t_1 to t_2 seconds then, the total energy W will be given as,

$$W = \int_{t_1}^{t_2} p\,dt \quad \text{Watt-sec} \qquad \text{.......... (1.7.10)}$$

Since the Watt-sec is a small unit, for practical purposes, energy is expressed in terms of Kilo-Watt-Hour (KWH) or units.

1 unit of energy = 1 KWH

$$1 \text{ unit of energy} = \frac{\textit{Power in Watts} \times \textit{Time in seconds}}{1000 \times 60 \times 60}$$

Power distribution companies charge the electrical energy supplied to the consumer in terms of Standard Energy Units (Board of Trade Units) known as Kilo-Watt-Hours.

1.8 KIRCHOFF'S LAWS

There are two more important laws governing the performance of a circuit known as

1. *Kirchoff's Voltage Law (KVL)*

2. *Kirchoff's Current Law (KCL or KIL)*

where I stands for current in KIL.

1.8.1 Kirchoff's Voltage Law (KVL)

Kirchoff's Voltage Law states that, *in a closed electric circuit the algebraic sum of E.M.F.s and Voltage drops is zero.*

By convention, the E.M.F.s or Voltage rises are taken to be positive and Voltage drops are taken to be negative.

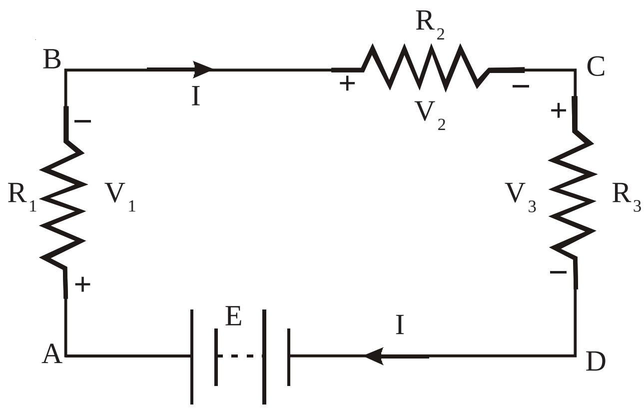


Fig. 1.8 *Circuit for Kirchoff's Voltage Law*

In the closed circuit ABCDA given in Fig. 1.8, applying Kirchoff's Voltage Law, we have,

$$E - V_1 - V_2 - V_3 = 0 \qquad \text{.......... (1.8.1)}$$

$$E - IR_1 - IR_2 - IR_3 = 0 \qquad \text{.......... (1.8.1a)}$$

or

$$IR_1 + IR_2 + IR_3 = E \qquad \text{.......... (1.8.1b)}$$

Sum of voltage drops = Sum of E.M.F.s or Voltage rises

Kirchoff's Voltage Law can be applied to any closed loop (closed circuit) even if there is no voltage source in which case the right hand side of Eq. (1.8.1b) will be zero for several loops. KVL in matrix form is given as,

$$[\,R\,][\,I\,] = [\,E\,] \qquad \text{.......... (1.8.1c)}$$

1.8.2 Kirchoff's Current Law (KCL or KIL)

Kirchoff's Current Law states that, *at any junction (or node) at which different elements are connected, the algebraic sum of the current at the junction is zero A junction or node is the meeting point of more than one element in a circuit. For eg. point B in the circuit given for Example 1.9 is a junction or node.*

By convention, a currents entering the junction are taken to be positive and currents leaving the junction are taken to be negative.

The currents of the current sources entering the junction are positive.

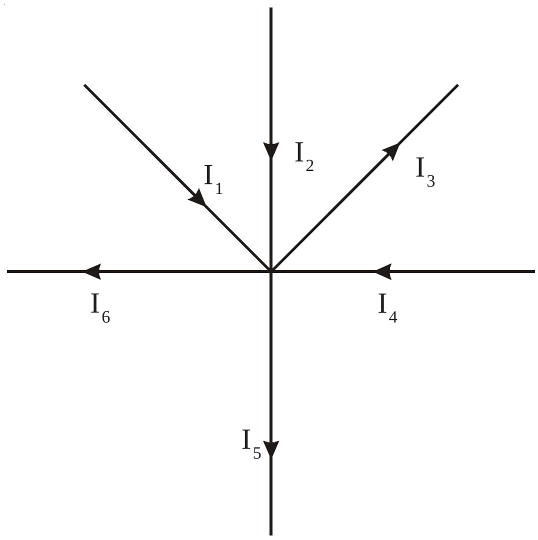


Fig. 1.9 *Kirchoff's Current Law*

In the Fig.1.9

applying Kirchoff's Current Law to the junction A, we have,

$$I_1 + I_2 + I_4 - I_3 - I_5 - I_6 = 0 \qquad \text{.......... (1.8.2)}$$

or $$I_3 + I_5 + I_6 = I_1 + I_2 + I_4 \qquad \text{.......... (1.8.2a)}$$

Sum of currents leaving the junctions = Sum of currents entering the junction

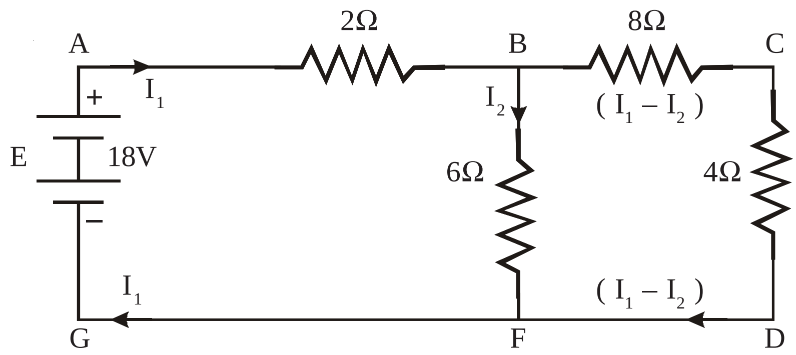
Note : Kirchoff's Voltage Law and Kirchoff's Current Law can also be applied to A.C. Circuits or to circuits with Unidirectional Source using instantaneous values for Voltages and currents. In A.C., for steady state values using impedances and using RMS values for voltages and currents, all in complex form will be discussed later. However, for resistive circuits consisting of only resistances Kirchoff's Voltage and Current Laws can be written as,

KVL : $$iR_1 + iR_2 + iR_3 = e \qquad \text{.......... (1.8.2c)}$$

KCL : $$i_3 + i_5 + i_6 = i_1 + i_2 + i_4 \qquad \text{.......... (1.8.2d)}$$

Example 1.4 :

Applying KCL and KVL, find the currents in the various elements of the circuit given in Fig. 1.9. Find the power delivered by the battery and the energy supplied by the battery for a period of half an hour. Also calculate the power loss in the 6Ω resistor.



Solution :

Let the current supplied by the battery to junction A be I_1. The same current I_1 flows through the 2Ω resistance towards junction B. At B a part of this current of I_1 flows through the 6Ω resistance towards junction F. Let it be I_2.

Applying Kirchoff's Current Law to junction A, the current 8Ω resistance will be $(I_1 - I_2)$ towards junction C.

Applying Kirchoff's Voltage Law to the loop ABFGA,

$$2 \times I_1 + 6 \times I_2 = 18$$

or $$2I_1 + 6I_2 = 18 \qquad \text{.......... (1)}$$

Applying Kirchoff's Voltage Law to the loop BCDFB,

$$8 \times (I_1 - I_2) + 4 \times (I_1 - I_2) - 6 \times I_2 = 0$$

i.e., $$12I_1 - 18I_2 = 0 \qquad \text{.......... (2)}$$

Solving Eq. (1) and (2) we obtain,

$$I_1 = 3\text{ A}$$

and $$I_2 = 2\text{ A}$$

Also, Eq. (1) and (2) can be solved using Cramer's Rule,

$$I_1 = \frac{\begin{vmatrix} 18 & 6 \\ 0 & -18 \end{vmatrix}}{\begin{vmatrix} 2 & 6 \\ 12 & -18 \end{vmatrix}}$$

$$= \frac{18\times(-18) - 6\times 0}{2\times(-18) - 6\times 12}$$

$$= \frac{-324}{-72} = 3\text{ A}$$

Similarly,

$$I_2 = \frac{\begin{vmatrix} 2 & 18 \\ 12 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & 6 \\ 12 & -18 \end{vmatrix}}$$

$$= \frac{2\times 0 - 18\times 12}{2\times(-18) - 6\times 12}$$

$$= \frac{-216}{-108} = 2\text{ A}$$

or $2I_1 + 6I_2 = 18$

Substituting the values of $I_1 = 3$ we obtain,

$$I_2 = 2 \text{ Amperes}$$

Current in 2Ω resistor,

$$I_1 = 3\text{ A}$$

Current in 6Ω resistor,

$$I_2 = 2\text{ A}$$

Current in 8Ω resistor,

$$I_1 - I_2 = 1\text{ A}$$

Current in 4Ω resistor,

$$I_1 - I_2 = 1\text{ A}$$

Current supplied by the battery,

$$P = EI_1$$

$$= 18\times 3 = 54\text{ Watts}$$

Energy, $W = P \times t$

Energy supplied by the battery for half an hour (1800 sec),

$$W = 54 \times 1800 = 97200 \text{ Watt-sec.}$$

$$= \frac{97200}{60 \times 60}$$

$$= 27 \text{ Watt-hours}$$

$$= \frac{27}{1000}$$

$$= 0.027 \text{ KWH}$$

Power Loss,

$$P = I^2R \text{ Watts}$$

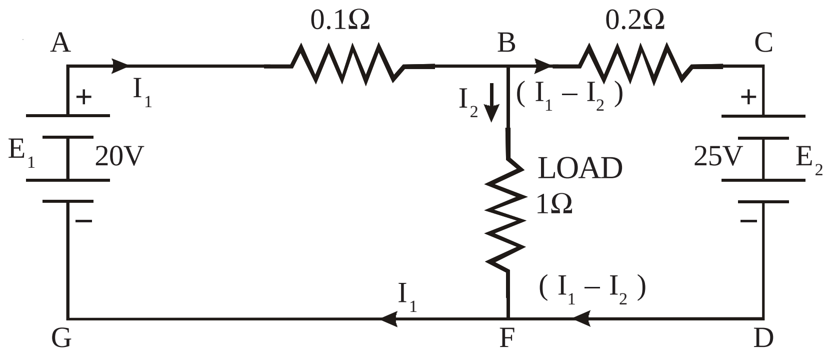
Power Loss in the 6Ω resistor $= (I_2)^2 \times 6$

$$= 2^2 \times 6$$

$$= 24 \text{ Watts}$$

Example 1.5 :

In the circuit of given figure, find the power supplied to the load. Find also the voltage at the load using KCL and KVL Equations. Also find the current through 0.2Ω resistance.



Solution :

Let the current entering node B be I_1 and let the current flowing through the resistance 1Ω resistance be I_2. Applying KCL for node B, the current through 0.2Ω resistance will be $(I_1 – I_2)$.

Writing KVL for loop ABFGA,

$$0.1I_1 + 1 \times I_2 = 20$$

i.e., $$0.1I_1 + I_2 = 20 \qquad \text{.......... (1)}$$

Writing KVL for loop BCDFB,

$$0.2x(I_1 - I_2) + 1x (- I_2) = -25$$

i.e., $$0.2I_1 - 1.2I_2 = -25 \qquad \text{.......... (2)}$$

Solving Eq. (1) and (2) we get,

$$I_1 = -3.125 \text{ A}$$

$$I_2 = -20.3125 \text{ A}$$

Hence, the current in the load is I_2

$$= 20.3125 \text{ A.}$$

Current through 0.2Ω resistance is,

$$(I_1 - I_2) = (-3.125 - 20.3125) = -23.4375 \text{ A}$$

The negative signs for I_1 , I_2, (I_1 - I_2) mean that the direction of the current flow is opposite to the assumed direction i.e., the current flows from B to A and not from A to B as assumed, I_2 from C to B and (I_1-I_2) from F to B.

Voltage at the load is

$$V = IR$$

$$= 20.3125 \times 1$$

$$= 20.3125 \text{ Volts}$$

Load Power,

$$P = I^2R$$

$$= (20.3125)^2 \times 1$$

$$= 412.598 \text{ Watts.}$$

1.9 RESISTANCES IN SERIES

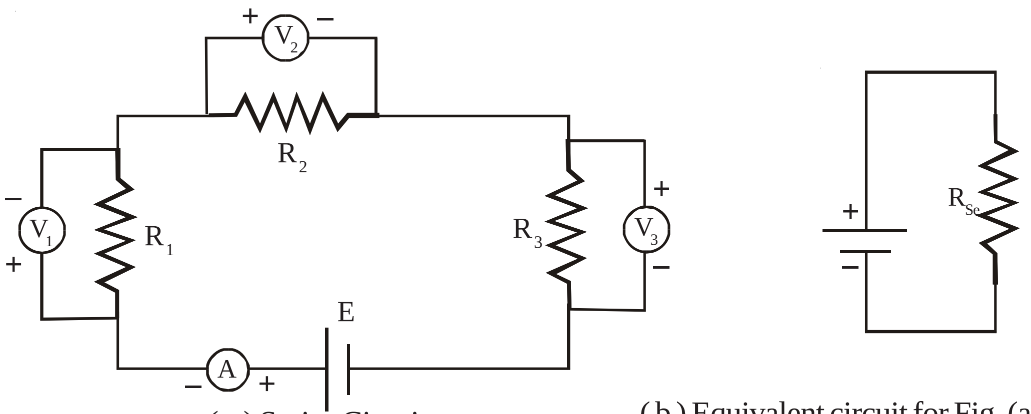
If the ending terminal of the resistance R_1 is connected to the beginning terminal of the resistance R_2 and the ending terminal of R_2 is connected to the beginning terminal of the resistance R_3 and so on then the resistances R_1, R_2, R_3, etc., are said to be connected in series.

In series circuits, the elements in the series can be connected in any order. For example, R_2, R_3, R_1, etc., instead of R_1, R_2 R_3 etc. In series circuits, the same current will flow through all the elements in series.

In D.C. series circuits, while connecting the elements in series, one should be very careful of the polarities of the meters used to measure the currents or voltages or the polarities of the equipment. Positive polarities of the meters or the equipments should always be connected to the positive of the supply point and the negative terminals should be connected to the negative of the supply point. While two equipments are connected in series, the positive of the first equipment should be connected to the positive terminal of the supply point. Negative terminal of the first equipment should be connected to the positive terminal of the second equipment and the negative terminal of the second equipment should be connected to the positive terminal of the third equipment and so on.

Ammeters are used to measure the currents. The ammeters should always be connected in series in the circuit so that the current to be measured flows through the ammeters. In order that, the voltage drop across the ammeter to be very very small so that full current flows through the circuit, the resistance of the ammeter should be very very small. Hence, if the ammeter is connected across the supply or across two points having large voltage drop, very heavy current will flow through the ammeter and the ammeter will get burnt.

Voltmeters are used to measure the voltage of the supply or voltage drop between two points inorder that the voltmeter does not draw more current so as to



(a) Series Circuit

(b) Equivalent circuit for Fig. (a)

Fig. 1.10 *Circuit with Resistances in Series*

allow full current in the circuit, the voltmeters should have very high resistance. If the voltmeter is connected in series, it causes high voltage drop across it and the voltage supplied to the remaining circuit will be less. Hence, voltmeters should be connected only in parallel and not in series.

In the closed circuit ABCDA given in Fig. 1.10 (a), applying Kirchoff's Voltage Law, we have,

$$E - V_1 - V_2 - V_3 = 0 \qquad \text{.......... (1.9.1)}$$

$$E - IR_1 - IR_2 - IR_3 = 0 \qquad \text{.......... (1.9.1a)}$$

or $$IR_1 + IR_2 + IR_3 = E \qquad \text{.......... (1.9.1b)}$$

or $$I(R_1 + R_2 + R_3) = E \qquad \text{.......... (1.9.1c)}$$

For the equivalent circuit of Fig. 1.10 (b),

$$IR_{Se} = E \qquad \text{.......... (1.9.2)}$$

Comparing Eq. (1.9.1c) and (1.9.2) we have,

$$R_{Se} = (R_1 + R_2 + R_3)$$

In general for n resistances in series,

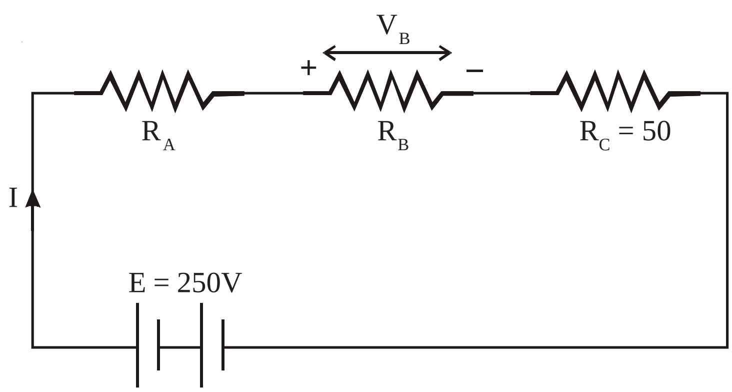
$$R_{Se} = \sum_{i=1}^{n} R_i \qquad \text{.......... (1.9.3)}$$

In terms of conductances for resistances in series,

$$\frac{1}{G_{Se}} = \sum_{i=1}^{n} \frac{1}{G_i} \qquad \text{.......... (1.9.4)}$$

Example 1.6 :

Fig. below shows three resistors R_A, R_B and R_C connected in series to a 250V source; Given R_C = 50Ω, and V_B = 80Volts when the current is 2 Amperes, calculate the total resistances, R_A and R_B.



Solution :

Since $I = 2$ Amperes

$V_B = IR_B$

$= 80V$

$R_B = 40\Omega$

Also, $I = \dfrac{E}{(R_A + R_B + R_C)}$

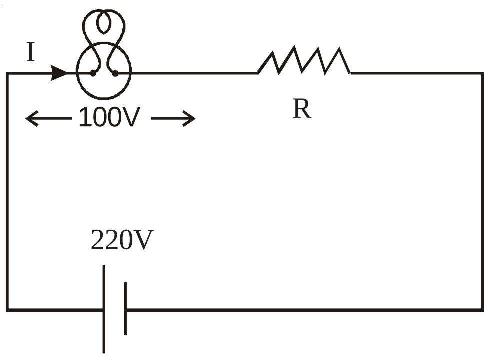
Therefore,

$$R_{Se} = R_A + R_B + R_C = \frac{E}{I} = \frac{250}{2} = 125\Omega$$

Therefore, $R_A = R_{Se} - (R_B + R_C) = 35\Omega.$

Example 1.7 :

A lamp rated 500W, 100V is to be operated from 220V supply. Find the value of the resistor to be connected in series with the lamp. What is the power lost in the resistance.



Solution :

$$\text{Current in the lamp} = \frac{W}{E}$$

$$= \frac{500}{100} = 5 \text{ Amperes}$$

Since the Voltage drop across the lamp is 100 V, Voltage to be dropped in the series resistor is 120V.

Therefore, Value of the resistor $= \dfrac{120}{5}$

$= 24\ \Omega$

Power Lost in this resistor $= I^2R$

$= 5^2 \times 24 = 600W$

1.10 RESISTANCES IN PARALLEL

If the starting terminal of two or more elements are connected together and the ending terminals of these elements are connected together then the elements are said to be connected in parallel. A parallel element may also be known as a Shunt Element.

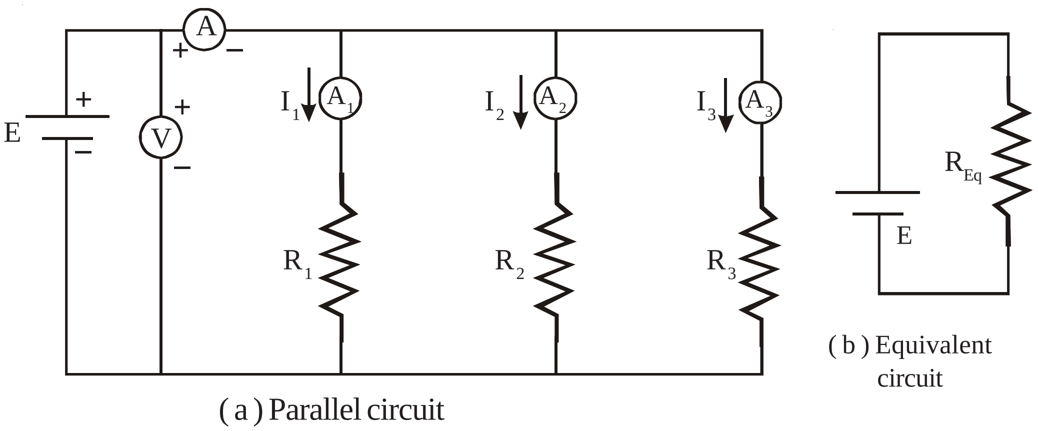


Fig. 1.11 *Circuit with Resistances in Parallel*

The voltage across all the elements that are connected in parallel will be same.

In D.C. circuits, if the elements of the meters or the equipments with polarities marked are connected in parallel then terminals of the same polarities should be connected together.

In the parallel circuit given in the Fig. 1.11(a), applying Kirchoff's Current Law to the junction A,

$$I = I_1 + I_2 + I_3 \quad \text{.......... (1.10.1)}$$

Applying Ohm's Law,

$$I = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \qquad \text{.......... (1.10.1a)}$$

For the equivalent circuit given in Fig. 1.14b,

$$I = \frac{E}{R_P} \qquad \text{.......... (1.10.2)}$$

Comparing Eq. (1.16.2) and (1.16.1a) we have,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{.......... (1.10.3)}$$

or In terms of conductances,

$$G_P = G_1 + G_2 + G_3 \qquad \text{.......... (1.10.4)}$$

In general for parallel circuits with n resistances in parallel,

$$\frac{1}{R_P} = \sum_{i=1}^{n} \frac{1}{R_i} \qquad \text{.......... (1.10.5)}$$

or

$$G_P = \sum_{i=1}^{n} G_i \qquad \text{.......... (1.10.6)}$$

If two resistances R_1 and R_2 are in parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{.......... (1.10.7)}$$

or

$$R_P = \frac{R_1 R_2}{R_1 + R_2} \qquad \text{.......... (1.10.7a)}$$

If the two resistances are equal and in parallel

i.e., $R_1 = R_2 = R$

then,

$$R_P = \frac{R}{2} \qquad \text{.......... (1.10.7b)}$$

If three resistances R_1, R_2 and R_3 are parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{.......... (1.10.8)}$$

or

$$R_P = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \text{.......... (1.10.8a)}$$

If three resistances are equal, i.e., $R_1 = R_2 = R_3 = R$ then,

$$R_P = \frac{R}{3} \qquad \text{.......... (1.10.9)}$$

In general, if n resistances, each of value R are in parallel the,

$$R_P = \frac{R}{n} \qquad \text{.......... (1.10.10)}$$

1.10.1 Division of Currents in Parallel Circuits

If two resistances R_1 and R_2 are connected in parallel and if the total current entering the parallel combination is I then this current I divides into two parts I_1 flowing through R_1 and I_2 flowing through R_2.

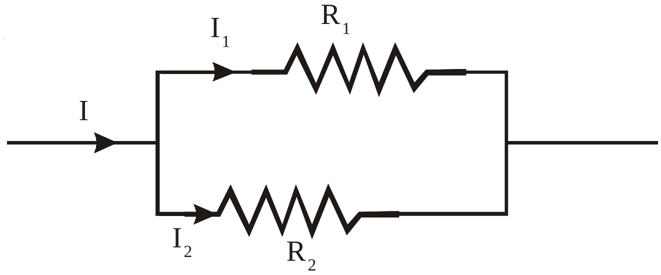


Fig. 1.12 *Division of Currents in Parallel Circuits*

The Voltage Drop across the parallel combination will be

$$V_P = I \times R_P = I \times \left(\frac{R_1 R_2}{R_1 + R_2} \right) \text{ Volts}$$

The current I_1 flowing through R_1 will be given as,

$$I_1 = \frac{V_P}{R_1} = \frac{1}{R_1} \times I \times \left(\frac{R_1 R_2}{R_1 + R_2} \right) \quad \text{Amps}$$

$$I_1 = I \times \left(\frac{R_2}{R_1 + R_2} \right) \quad \text{Amps} \qquad \text{.......... (1.10.11)}$$

When two resistances R_1 and R_2 are in parallel, Current through R_1 is given as,

$$I_1 = \text{Total Current} \times \frac{\text{Second Resistance}}{\text{Sum of the Two Resistances in Parallel}}$$

.... (1.10.12)

This form is used frequently in Electronic Circuits.

Similarly,

$$I_2 = \frac{V_P}{R_2}$$

$$= \frac{1}{R_2} \times I \times \left(\frac{R_1 R_2}{R_1 + R_2} \right) \quad \text{Amps}$$

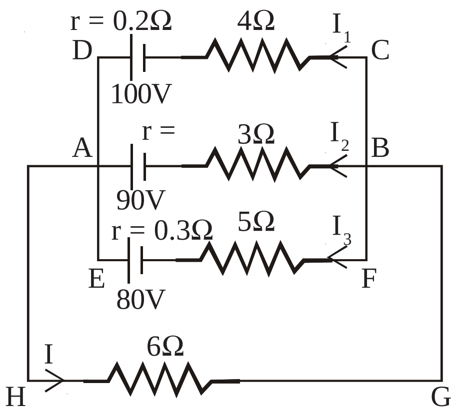
$$I_2 = I \times \left(\frac{R_1}{R_1 + R_2} \right) \quad \text{Amps} \qquad \text{.......... (1.10.13)}$$

Also, $\quad I_2 = (I - I_1)$ Amps

In general, *the current through any parallel path is given as the product of the total current and the parallel equivalent resistance divided by the resistance of that path.*

Example 1.8 :

Solve the network shown in the figure for the current through 6Ω resistor.



Solution :

Let the current flowing through various branches be as marked in the figure. Applying Kirchoff's Voltage Law to the following closed circuits,

Circuit CDAHGBC,

$$-4I_1 - 0.2I_1 + 100 - 6(I_1 + I_2 + I_3) = 0$$

or $$10.2\, I_1 + 6I_2 + 6I_3 = 100 \qquad \text{..........} \ (1)$$

Circuit BAHGB,

$$-3I_2 - 0.25I_2 + 90 - 6(I_1 + I_2 + I_3) = 0$$

or $$6I_1 + 9.25\, I_2 + 6I_3 = 90 \qquad \text{..........} \ (2)$$

Circuit FEAHGBF,

$$-5I_3 - 0.3I_3 + 80 - 6(I_1 + I_2 + I_3) = 0$$

or $$6I_1 + 6I_2 + 11.3\, I_3 = 80 \qquad \text{..........} \ (3)$$

Subtracting (2) from (1), we get,

$$4.2\, I_1 - 3.25\, I_2 = 10 \qquad \text{..........} \ (4)$$

Eqn.(2) x 11.3 - Eqn. (3) x 6 gives

$$31.8\, I_1 + 68.525\, I_2 = 537 \qquad \text{..........} \ (5)$$

Eqn.(5) x 4.2 - Eqn.(4) x 31.8 gives

$$391.51 I_2 = 1937.4$$

$$I_2 = 4.953\text{A}$$

From Eqn.(5) substituting for I_2

$$I_1 = 6.21A$$

From Eqn. (3), substituting for I_1 and I_2

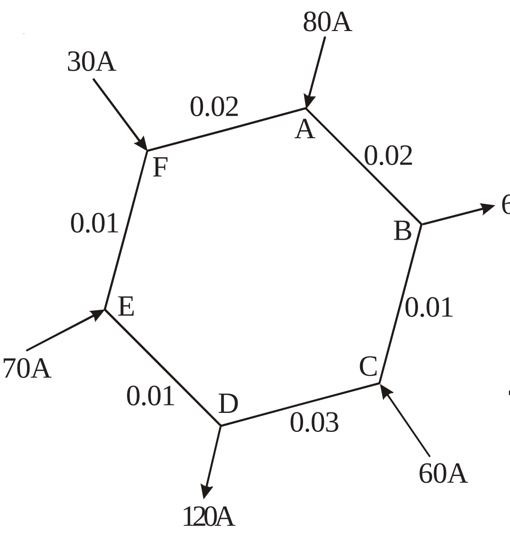
$$I_3 = 1.15A$$

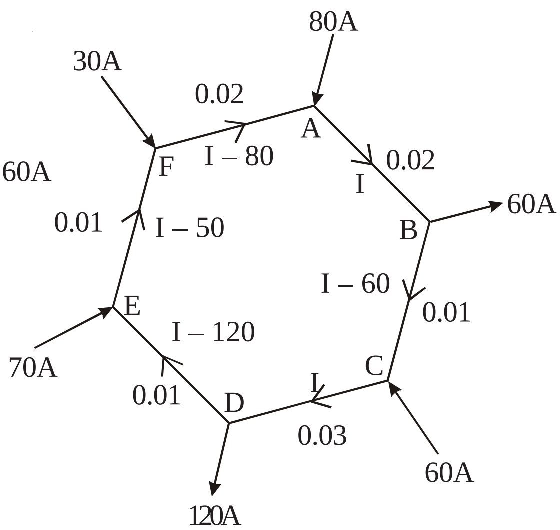
Current in 6Ω resistor,

$$I = I_1 + I_2 + I_3$$
$$= 6.21 + 4.953 + 1.15$$
$$= 12.313A$$

Example 1.9 :

Find the magnitude and direction of the currents in all branches of the circuit shown in the figure using Kirchoff's Laws. All resistances are in Ohms.





Solution :

Let current from A to B junctions be I Amps. Applying Kirchoff's First Law, let current flowing through various branches be as shown in the figure.

Applying Kirchoff's Current Law to current in each branch and Kirchoff's Voltage Law to a closed loop ABCDEFA, we get,

$$-0.02I - 0.01(I - 60) - 0.03I - 0.01(I - 120) - 0.01(I - 50)$$
$$- 0.02(I - 80) = 0$$

or $\quad 0.02I + 0.01I + 0.03I + 0.01I + 0.01I + 0.02I$

$= 0.6 + 1.2 + 0.5 + 1.6$

or $\quad 0.1I = 3.9$

or $\quad I = 39A$

Current in various branches is as under :

I_{AB}	$= 39A$			(i.e., A to B)
I_{BC}	$= I - 60$	$= -21A$		(i.e., C to B)
I_{CD}	$= I$	$= 39A$		(i.e., C to D)
I_{DE}	$= I - 120$	$= -81A$		(i.e., E to D)
I_{EF}	$= I - 50$	$= -11A$		(i.e., F to E)
I_{FA}	$= I - 80$	$= -41A$		(i.e., A to F)

1.11 SERIES-PARALLEL RESISTANCES

In the case of series-parallel resistances, the parallel equivalent of the resistances in parallel are obtained first as a single resistance which will be in series with the other resistances, thus bringing the circuit into a single series circuit. After finding the current flowing through this equivalent series circuit again the parallel equivalent resistance may be replaced with the corresponding parallel circuit and the current in the parallel paths are calculated as given in the Section 1.10, *the current through any parallel path is given as the product of the total current and the parallel equivalent resistance divided by the resistance of that path.*

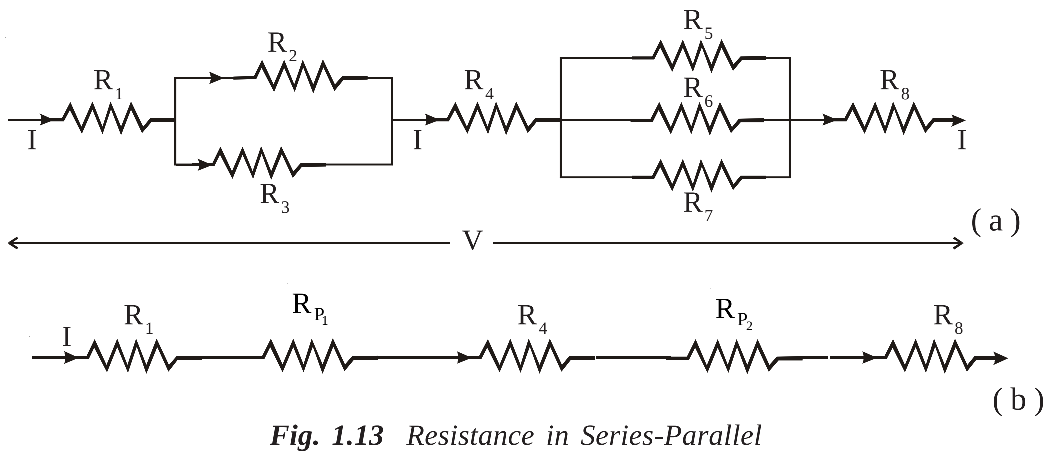


Fig. 1.13 *Resistance in Series-Parallel*

Typical values of currents are given below for the above circuit,

$$R_{P_1} = \frac{R_2 R_3}{(R_2 + R_3)}$$

$$R_{P_2} = \frac{R_5 R_6 R_7}{R_5 R_6 + R_6 R_7 + R_7 R_5}$$

$$I = \frac{V}{R_1 + R_{P_1} + R_4 + R_{P_2} + R_8}$$

$$I_2 = I \times \frac{R_3}{(R_2 + R_3)}$$

$$I_6 = \frac{I}{R_6}\left(\frac{R_5 R_6 R_7}{(R_5 R_6 + R_6 R_7 + R_7 R_5)}\right)$$

Example 1.10 :

A Wheatstone Bridge consists of AB = 4Ω, BC = 3Ω, CD = 6Ω and DA = 5Ω. A 2V Cell is connected between B and D and a Galvanometer of 10Ω resistance between A and C. Find the current through the Galvanometer.

Solution :

The circuit is shown in the figure. Applying Kirchoff's Current Law at junction B, A and C, the current in various branches is marked.

Applying Kirchoff's Voltage Law to various closed loops and considering loop BACB, we get,

$$-4I_1 - 10I_3 + 3I_2 = 0$$

or $$4I_1 + 10I_3 - 3I_2 = 0 \qquad \text{.......... (1)}$$

Considering loop ADCA, we get

$$-5(I_1 - I_3) + 6(I_2 + I_3) + 10I_3 = 0$$

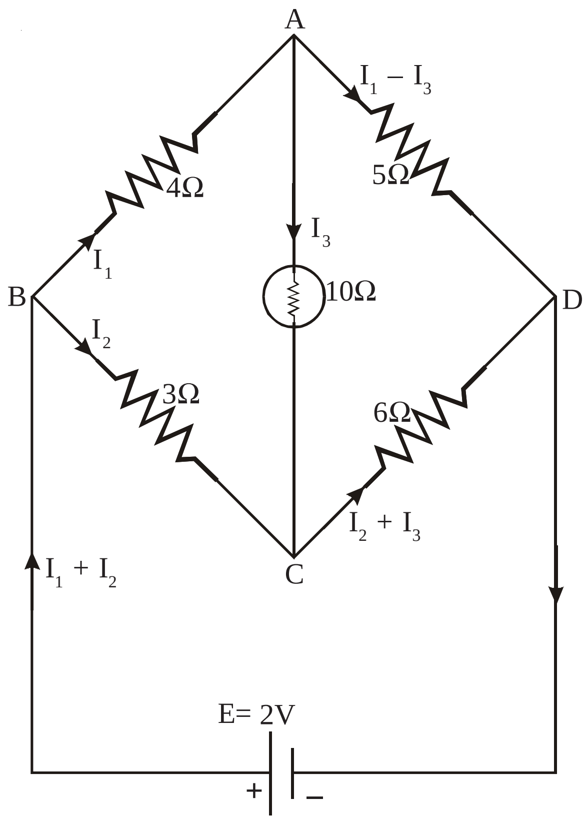
or $$5I_1 - 6I_2 - 21I_3 = 0 \qquad \text{.......... (2)}$$

Considering loop BADEB, we get,

$$-4I_1 - 5(I_1 - I_3) + 2 = 0$$

or $\quad 4I_1 - 5I_1 + 5I_3 = -2 \quad$ .... (3)

or $\quad 9I_1 - 5I_3 = 2$



Multiplying Eq. (1) by Eq. (2) and Subtracting from Eq. (2), we get,

$$5I_1 - 6I_2 - 21I_3 = 0$$

$$8I_1 - 6I_2 + 20I_3 = 0$$

$$-3I_1 \quad - 41I_3 = 0$$

or $\quad I_1 = -\dfrac{41}{3} I_3$

Substituting the value of I_1 in Eq. (3), we get,

$$9\left(-\frac{41}{3}I_3\right) - 5I_3 \quad = \quad 0$$

or $$-123I_3 - 5I_3 \quad = \quad 2$$

or $$I_3 \;=\; -\frac{1}{64}\text{ A}$$

Example 1.11:

A resistance of 15Ω is connected in series with two resistances each of 30Ω arranged in parallel. A voltage source of 30V is connected to this circuit.

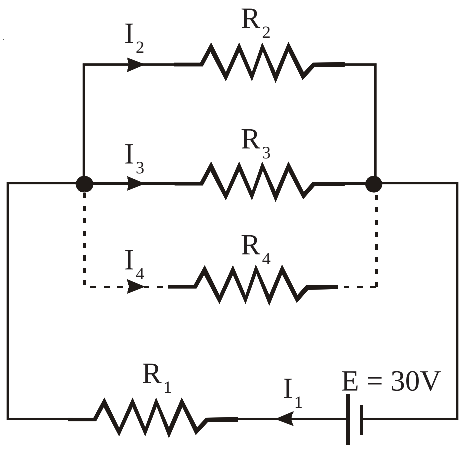
(a) What is the current drawn from the source.

(b) What resistance should be placed in shunt (parallel) with the parallel combination in order that the current drawn from the source is 1.2 A.

Solution:

The circuit is as shown in the figure given below.

(a) R_1 is in series with the parallel combination of R_2 and R_3.



Hence, Total Resistance R_t is,

$$R_t = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$= 15 + \frac{30\times 30}{30+30}$$

$$R_t = 30\Omega$$

Hence current,

$$I_1 = \frac{E}{R_t}$$

$$I_1 = \frac{30}{30} = 1\text{ A}$$

(b) With I_1 = 1.2A, Voltage across R_1

$$= I_1R_1 = 1.2 \times 15$$
$$= 18\text{ V}$$

Voltage Drop across parallel combination of R_2, R_3 and R_4 is,

$$V_P = 30 - 18$$
$$V_P = 12\text{ V}$$

Hence, Voltage Drop across each resistor R_2, R_3, R_4 is 12 V.

Hence, $$I_2 = \frac{V_P}{R_2} = \frac{12}{30}$$

$$I_2 = 0.4\text{ A}$$

$$I_3 = \frac{V_P}{R_3} = \frac{12}{30}$$

$$I_3 = 0.4\text{ A}$$
$$I_4 = I_1 - (I_2 + I_3)$$
$$= 1.2 - (0.4 + 0.4)$$
$$I_4 = 0.4\text{ A}$$

$$I_4 = \frac{V_P}{R_4}$$

$$0.4R_4 = 12$$

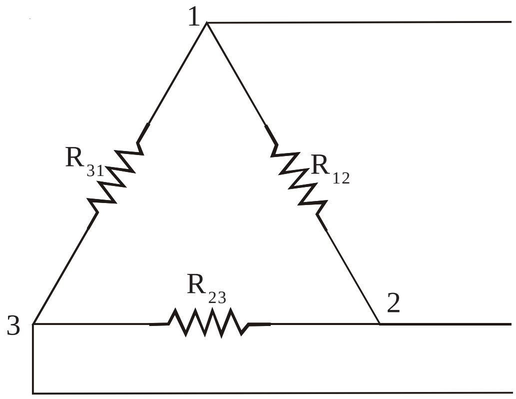
Hence, $$R_4 = \frac{12}{0.4}$$

$$R_4 = 30\ \Omega$$

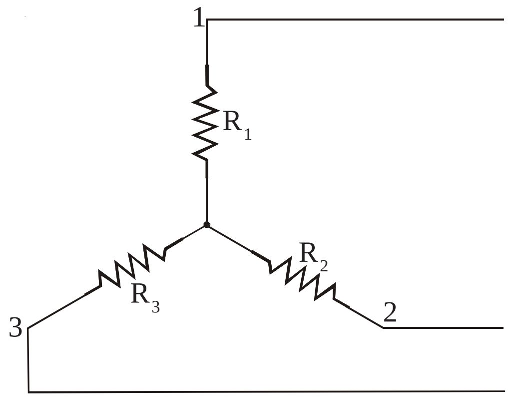
1.12 RESISTANCES IN STAR OR DELTA CONNECTIONS

If the end of R_1 is connected to beginning of R_2, the end of R_2 is connected to beginning of R_3 and the end of R_3 is connected to beginning of R_1 then, the resistances R_1, R_2, R_3 are said to be connected in Delta (Δ) and the three points common to any two resistances are connected to the remaining part of the circuit as given in the Fig. 1.14 (a).

If one end of the resistances R_1, R_2, R_3, are all connected together and the other ends of the three resistances are connected to different points of the circuits, as given in the Fig. 1.14 (b), then the resistances R_1, R_2, R_3 are said to be connected in Star or WYE (Y).



(a) Delta Connection



(b) Star Connection

Fig. 1.14 *Resistance in Star-Delta Connection*

In general, more than three resistances can be connected in Star.

In general, more than three resistances can also be connected in Mesh.

In some cases, the circuits cannot be solved by means of simple series, parallel or parallel combinations but by finding the equivalent Delta for the given Star connection or equivalent Star connection for the given Delta connection of resistances, the circuit can be resolved into simple Series, Parallel or Series-Parallel combinations.

1.12.1 The Equivalent Star Resistances For Given Delta Connected Resistances

Referring to Fig. 1.17 (a), in Delta Connection, we observe that between the terminals 1 and 2 the resistance R_{12} is in parallel with the series combination of R_{23} and R_{31}.

Referring to Fig. 1.17 (b), in Star Connection, we observe that between the terminals 1 and 2 the resistance R_1 is in series with R_2.

Equating the resistances between terminals 1 and 2 of Star and its equivalent Delta, we have R_{12} in parallel with (R_{23} + R_{31})

$$R_1 + R_2 = R_{12} + \frac{R_{23}R_{31}}{R_{23} + R_{31}}$$

Rewriting the above equation we have,

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{\sum R_{12}} \qquad \text{.......... (1.12.1)}$$

Similarly equating the resistances between terminals 2 and 3, we have,

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{\sum R_{12}} \qquad \text{.......... (1.12.2)}$$

Similarly equating the resistances between terminals 3 and 1, we have,

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{\sum R_{12}} \qquad \text{.......... (1.12.3)}$$

Subtracting Eq. (1.18.2) from Eq. (1.18.3), we have,

$$R_1 - R_2 = \frac{R_{12}(R_{31} - R_{23})}{\sum R_{12}} \qquad \text{.......... (1.12.4)}$$

Adding Eq. (1.18.1) and Eq. (1.18.4), we have,

$$2R_1 = \frac{R_{12}(2R_{31})}{\sum R_{12}}$$

Hence, $$R_1 = \frac{R_{12}R_{31}}{\sum R_{12}}$$

Rewriting the above equation, we have,

$$R_1 = \frac{R_{12}R_{13}}{\sum R_{12}} \qquad \text{.......... (1.12.5)}$$

Similarly, $$R_2 = \frac{R_{21}R_{23}}{\sum R_{12}} \qquad \text{.......... (1.12.6)}$$

$$R_3 = \frac{R_{31}R_{32}}{\sum R_{12}} \qquad \text{.......... (1.12.7)}$$

Note that from Eq. (1.12.5), Eq. (1.12.6) and Eq. (1.12.7), we observe that, *the equivalent star resistance connected to a given terminal is given by the product of the two Delta resistances that are connected to that terminal divided by the sum of the three Delta connected resistances.*

The same statement can be extended to more than three resistances connected in Mesh and its equivalent Star !! (check up)

1.12.2 The Equivalent Delta Resistances For Given Star Connected Resistances

Multiplying Eq. (1.12.5) and Eq. (1.12.6), we have,

$$R_1R_2 = \frac{R_{12}^2R_{23}R_{31}}{\left(\sum R_{12}\right)^2} \qquad \text{.......... (1.12.8)}$$

Multiplying Eq. (1.12.6) and Eq. (1.12.7), we have,

$$R_2R_3 = \frac{R_{12}R_{23}^2R_{31}}{\left(\sum R_{12}\right)^2} \qquad \text{.......... (1.12.9)}$$

Multiplying Eq. (1.12.7) and Eq. (1.12.5), we have,

$$R_3R_1 = \frac{R_{12}R_{23}R_{31}^2}{\left(\sum R_{12}\right)^2} \qquad \text{.......... (1.12.10)}$$

Adding Eq. (1.12.8), Eq. (1.12.9) and Eq. (1.12.10), we have,

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}\left(R_{12} + R_{23} + R_{31}\right)}{\left(\sum R_{12}\right)^2}$$

$$= (R_{12}R_{23}R_{31})\frac{\left(\sum R_{12}\right)}{\left(\sum R_{12}\right)^2}$$

$$= R_{12}\left[\frac{R_{23}R_{31}}{\sum R_{12}}\right]$$

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{12}R_3 \text{ by using}$$

R_3 from eqn.1.12.7 for the term inside the bracket

Hence, $$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

or $$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3} \qquad \text{.......... (1.12.11)}$$

Similarly, $$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1} \qquad \text{.......... (1.12.12)}$$

$$R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2} \qquad \text{.......... (1.12.13)}$$

Note that from Eq. (1.12.11), Eq. (1.12.12) and Eq. (1.12.13), we observe that, *the equivalent Delta resistance connected between any two given terminals is given by the sum of the two Star resistances that are connected between those two terminals plus the product of those two resistances connected between those two terminals divided by the third Star resistance.*

In terms of conductances the equivalent mesh element can be given in terms of the star elements for star or delta having three or more resistances as

$$G_{ij} = \frac{G_i G_j}{\sum G_i} \qquad \text{!! (check up)} \qquad \text{.......... (1.12.14)}$$

If $R_{12} = R_{23} = R_{31} = R$ (say), then, the equivalent star resistances will be,

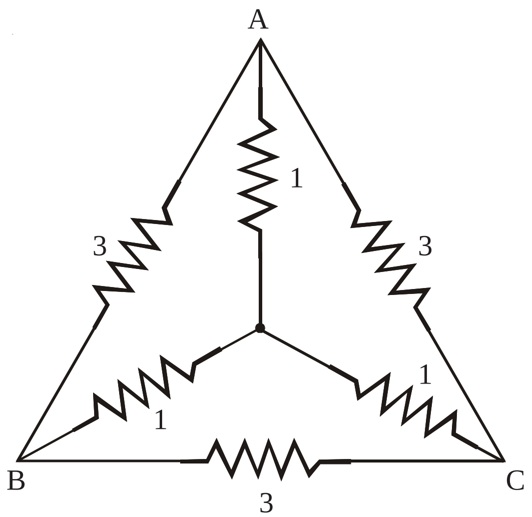
$$R_1 = R_2 = R_3 = \frac{R}{3} \qquad \text{.......... (1.12.15)}$$

If $R_1 = R_2 = R_3 = R$ (say), then, the equivalent delta resistances will be,

$$R_{12} = R_{23} = R_{31} = 3R \qquad \text{.......... (1.12.16)}$$

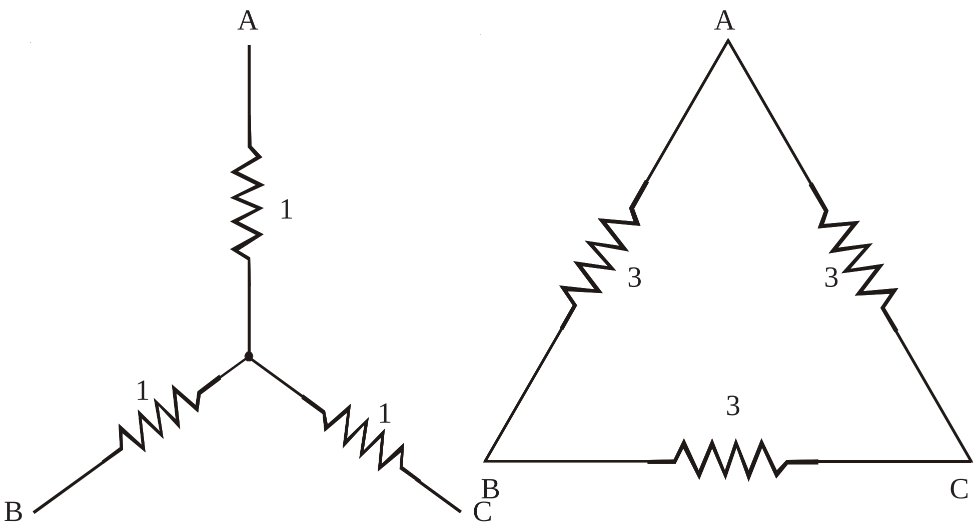
Example 1.12:

For the network shown in the figure below, find the equivalent resistance between the terminals B and C.

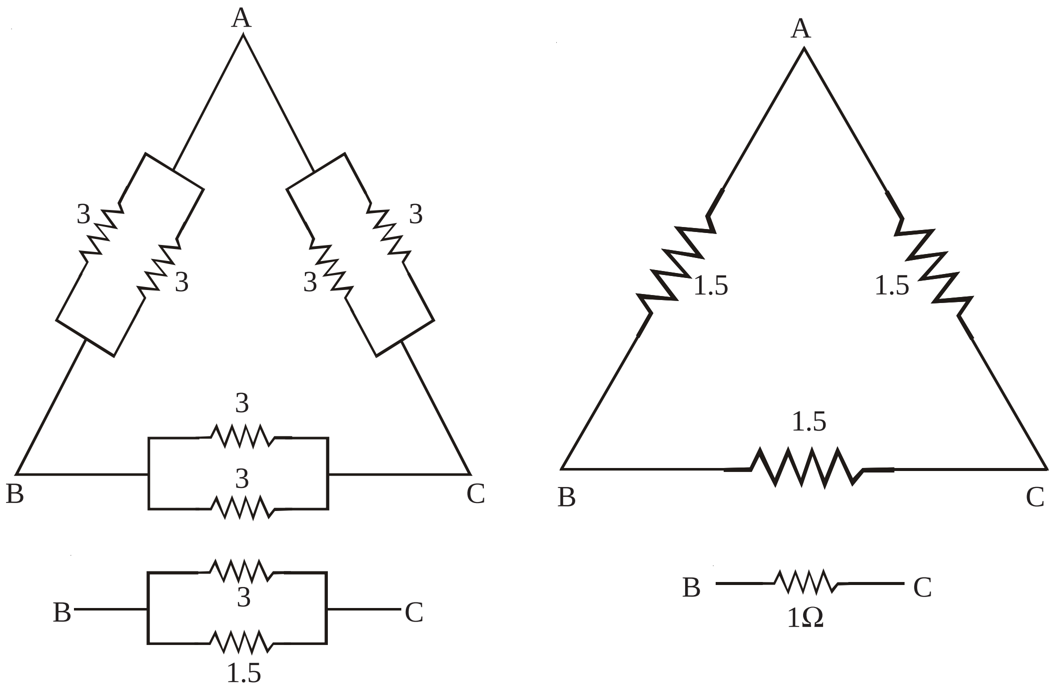


Solution:

The given combination of resistances between the terminals B and C is neither series combinations nor parallel combination. If, the star connection between A, B and C is converted into equivalent Delta, we will have a known combination which can be simplified by series parallel simplification.



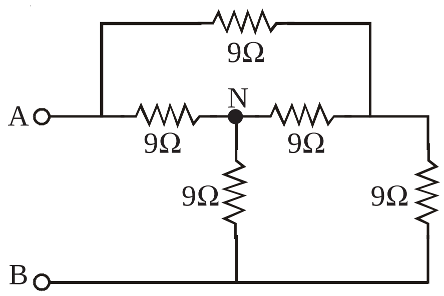
The given figure is redrawn after replacing the star by its equivalent Delta.



$$R_{BC} = \frac{3 \times 1.5}{3 + 1.5} = 1\Omega.$$

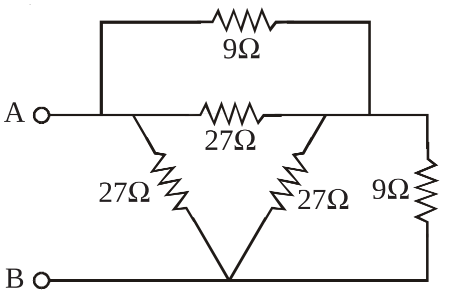
Example 1.13:

Determine the equivalent resistance between A and B.

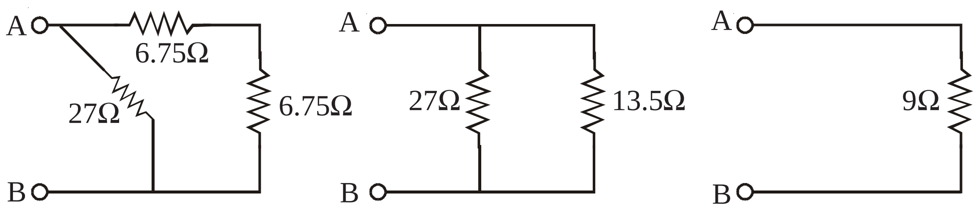


Solution:

The combination is neither series nor parallel. There is Delta and Star connection. Converting the star connection for which N is the star point and redrawing the circuit, we get,



$$\frac{27\times 9}{27+9} = \frac{27\times 9}{36} = 6.75\Omega$$



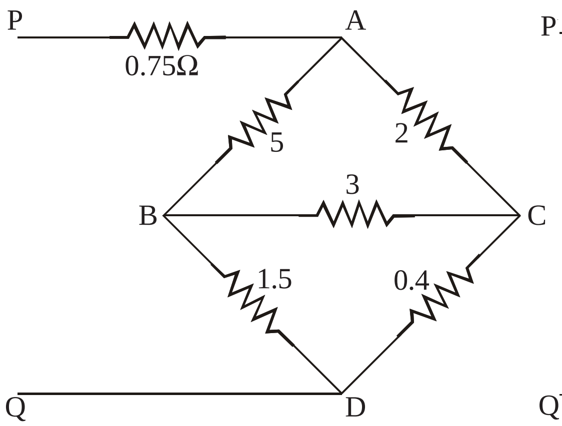
Therefore,

$$R_{AB} = 27 \,||\, 13.5$$

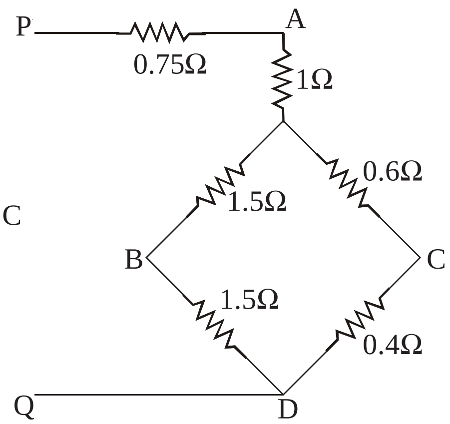
$$= \frac{27\times 13.5}{27+13.5} = 9\Omega.$$

Example 1.14 :

In the Wheatstone Bridge Circuit of figure (a) given below, find the effective resistance between PQ. Find the current supplied by a 10V Batter connected to PQ.



(a)



(b)

Solution :

Converting the Δ formed by 5, 2 and 3Ω in a star of figure (b), we have,

$$R_A = \frac{5\times 2}{5+3+2} = 1\Omega$$

$$R_B = \frac{5\times 3}{10} = 1.5\Omega$$

$$R_C = \frac{3\times 2}{10} = 0.6\Omega$$

The circuit then reduces to the following form.

The two 1.5 Ohm resistors are in series and this is in parallel with the 0.6 and 0.4 Ohm resistors which are in series.

Therefore, the total effective resistance between P and Q

$$= 0.75 + 1 + \frac{3\times 1}{3+1}$$

$$= 2.5\Omega$$

If a 10V battery is attached to PQ, current drawn,

$$= \frac{10}{2.5} = 4A.$$

Note : *The above methods of reducing the circuit using series, parallel, series-parallel or star-delta conversions are applicable to A.C. circuit and also circuits with Unidirectional sources using instantaneous values. For steady state calculations of A.C. circuits, the R.M.S. values of voltages and currents and the impedances, all in complex form have to be used.*

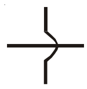
1.13 FEATURES OF A RESISTANCE

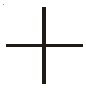
1. Resistance is used in both D.C. & A.C. circuits and it reduces the currents.
2. The current in resistance will be in phase with the voltage i.e., at an angle 0^{o}.
3. Resistance causes Power Loss and loss appears in the form of heat. Hence, sufficient provision should be made to dissipate this loss.
4. Resistance depends upon the dimensions and material of the resistance conductor.
5. In the case of resistance, the current is setup instantaneously when the voltage is applied and is cutoff instantaneously when the voltage is removed.
6. The resistance which is in the form of a coil, when used in an A.C. circuit will offer atleast some inductance and in precision work to avoid that inductance the resistance coil is split up into two halves which are wound in the opposite directions so as to have a pure resistance.
7. In precision work, the contact resistance, when the resistance is connected to a terminal should also be taken into account.
8. The resistance of the electrical transmission lines will have to be very small in order to cause negligible voltage drop across the resistance as it has to carry heavy currents. Thus, the resistance of the transmission lines is generally neglected in calculations.

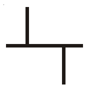
1.14 USES OF RESISTANCE

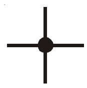
1. Resistance is used in any circuit to limit the current.
2. Resistance is used in series resonance circuits to limit the currents at resonance point.
3. Resistance will be used in Wave Shaping Circuits to obtain a different wave form from a given wave form.
4. Resistance can be used in analog circuits, to solve for other systems like Mechanical Systems, Hydraulic System, etc., in terms of Electrical Systems
5. Resistance will be used in Filter Circuits to select or reject certain frequencies.
6. Change of resistances as obtained in ***Resistance Strain Gauges***, which convert physical signals like temperature, strain, load, force etc., into electrical signals are used for measurement of the physical signals.
7. Resistance can also be used in Power Factor changing circuits but because it causes Power Loss and additional real power to be supplied, the power factor changing circuits generally do not employ change of resistances.
8. Resistances are used in quenching the arcs (extinguishing the arcs) in the case of circuit breakers, while breaking heavy current circuits in electrical transmission and distribution.
9. Resistance can be used in Measurement Circuits. For example, Variable Resistance is used to balance the bridge networks. Resistance shunt is used to extend the range of an ammeter or galvanometer, by diverting a major part of the circuit current through the shunt resistance so that only the allowable current flows through the ammeter or galvanometer. The actual current is calculated from the reading of the ammeter or galvanometer using a factor in terms of resistance of the galvanometer and the resistance of the shunt as in the case of parallel circuit. *A shunt is a very small pure resistance of the order 1Ω or less.*
10. Resistance shunt can also be used as a series resistance causing a small voltage drop across it, so that this voltage drop can be used in CRO (Cathode Ray Oscilloscope) to trace the current waveform which will be proportional to the voltage waveform across the resistance shunt.

1.15 DIFFERENCES BETWEEN ELECTRICAL & ELECTRONIC CIRCUITS

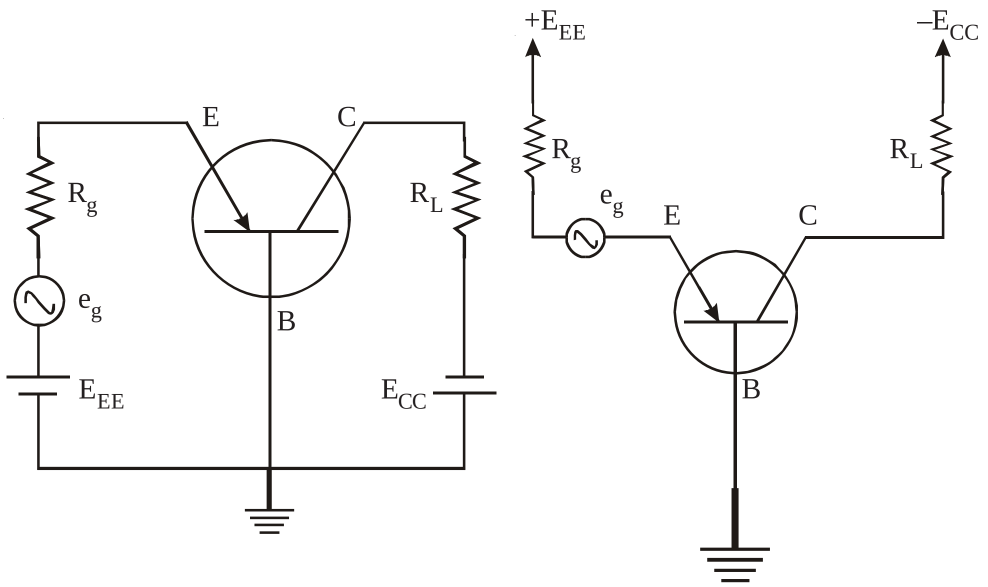
1. In electrical circuits, the cross over of two wires are shown as ,

   whereas in electronic circuits, the cross over is shown as .

2. In electrical circuits, the junction of wires is shown as , whereas

   in electronic circuits, the junction is shown is shown as .

3. In electrical circuits, the circuit is given completely including sources and elements whereas in electronic circuits, the circuit will not be shown completely. A pointing arrow with voltage along with its polarity marked indicates the connection to the voltage source of that polarity. The return path is assumed to be through ground by indicating the ground connections at the required places as shown in Fig.1.15. A common ground wire will be run to which all the grounding points will be connected.



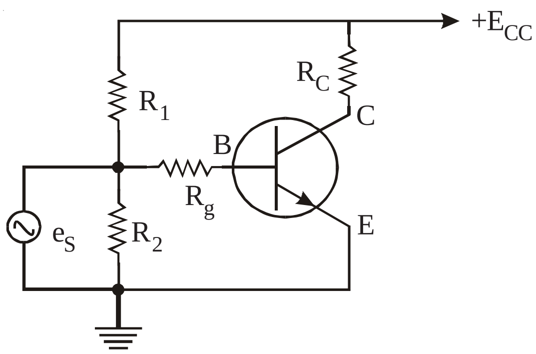
(a) As in Electrical Circuit **(b)** As in Electronic Circuit

Fig. 1.15 *Grounded-base Amplifier including Biasing of the PNP Transistor*

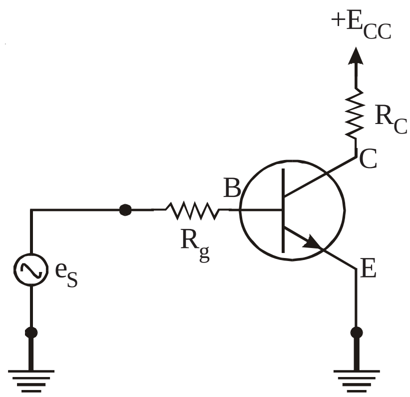
4. In electronic circuits like a transistor amplifier, the biasing of the transistor will not be shown separately. Only the input signal and the output load will be shown as in Fig. 1.16. In such cases, the biasing circuit is assumed to be present. Here resistances R_1 and R_2 are used for biasing to fix up the operating point in the characteristics of the transistor. The signal will make an excursion about the operating point.

   A separate wire for power supply to which power supply points will be connected and a separate grounded earthwire to which the grounding points will be connected, while rigging up the circuit have to be provided. While connecting the circuit the biasing circuit should also be connected.

5. The direction of electron current is denoted as "$\rightarrow$". The direction of the conventional current will be opposite to that of electron current. Conventional current can be treated as the direction of the movement of positive ions and is denoted as "$\leftarrow$"



(a) As in Electronic Circuit with Biasing Circuit



(b) As in Electronic Circuit without Biasing Circuit

Fig. 1.16 Grounded-emitter Amplifier using NPN Transistor

D.C. CIRCUIT CONCEPTS AND CIRCUIT ELEMENTS - II

1.16. INTRODUCTION TO ENERGY STORING ELEMENTS

Earlier we have studied ahout Resistance Parameter. Resistance is a bilateral element and it can be linear or non-linear. Generally linear resistances are widely used. Resistance is an element which will be used in both D.C. and A.C. circuits.

Now we discuss about Capacitance and Inductance Parameters which are known as ***Energy Storing Elements***. Capacitance and Inductance elements are generally used in A.C. Circuits. They are used in D.C. circuits only for the purpose of making use of transients (transient voltage or transient current) for special purposes like wave shaping, differentiation, integration circuits. They are also used in filter circuits for allowing or rejecting currents or voltages of certain frequencies.

Inductance and capacitance can be used in resonance circuits to select or reject the particular frequency voltage or current. They can also be used in narrow band filters based on resonance to select or reject a narrow band of frequencies.

In the case of resistance parameter, the resistance reduces the current and at the same time causes *Power Loss* in the resistance.

Just like resistance, the capacitance and also the inductance reduce the current, but they store energy in them. The energy stored in the case of capacitance will be in the form of Electrostatic Energy and the energy stored in the case of inductance will be in the form of Electromagnetic Energy.

For an ideal capacitor and for an ideal inductor there will be no power loss. Just like the resistance, the capacitance and also the inductance depend upon the dimensions and geometry.

Capacitance also depends on the dielectric medium between the capacitance plates and the inductance depends up the core material of the inductor.

1.17. CAPACITANCE PARAMETER

Two conducting plates of equal and opposite charges, separated by a distance in air or any other dielectric material constitutes a capacitor and there will be force of attraction between the plates given by Coulombs Law.

The electric field between the plates will be parallel and having a value,

$$E = \frac{F}{Q} \qquad \text{.......... (1.17.1)}$$

where Q is the total charge of the *capacitance* or *capacitor*.

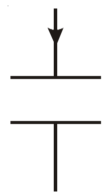


Fig. 1.17 *Parallel Plate Capacitor*

Capacitor is also called ***Condenser***.

The electric field E will exists between charged conductors of any arbitrary shape also. But the parallel plate capacitors will have a charge density of Q/A, where A is the area of the parallel plate. According to ***Gauss's Law***, the total normal electric flux emanating from a closed surface enclosing a point charge Q or distributed charge of total value Q will be equal to Q.

The electric flux,

$$Q = D \times A \qquad \text{.......... (1.17.2)}$$

where D is the electric flux density and

A is the area of the capacitor plate.

$$D = \varepsilon E \qquad \text{.......... (1.17.3)}$$

where E is the electric field intensity and the permittivity, $\varepsilon = \varepsilon_o \varepsilon_r$

The *Voltage Gradient*,

$$E = \frac{V}{d} \qquad \text{.......... (1.17.4)}$$

where d is the distance between the two capacitor plates and

V is the voltage across the capacitor plates.

$$V = Ed = \left(\frac{d}{\varepsilon A}\right) Q$$

From Eq. (2.2.2) and Eq. (2.2.3), we have,

$$E = \frac{D}{\varepsilon}$$

and the electric flux density,

$$D = \frac{Q}{A}$$

$$Q = V\frac{\varepsilon A}{d} \qquad \text{.......... (1.17.5)}$$

$$Q = C \times V$$

where C is the capacitance of the parallel plate capacitor

and d is the distance between the two capacitor plates

$$C = \frac{\varepsilon A}{d} \qquad \text{.......... (1.17.6)}$$

1.17.1 The Current Flowing Through A Capacitor

Since, the dielectric medium between the parallel plates is an insulator, the steady state current flowing through the capacitor, when connected in a D.C. circuit will be equal to zero. If the capacitor is connected in an A.C. circuit, the instantaneous current flowing through the capacitor

$$i = \frac{dQ}{dt} \qquad \text{.......... (1.17.7)}$$

Hence, $$Q = \int i\,dt = \int_{-\infty}^{t} i\,dt$$

This equation may be written in a different form by separating the integral into two parts.

$$Q = \int_{-\infty}^{0} i\,dt + \int_{0}^{t} i\,dt$$

$$= Q_0 + \int_{0}^{t} i\,dt$$

$$i = \frac{dQ}{dt}$$

$$= \frac{d}{dt}(CV)$$

$$i = C\frac{dV}{dt} \qquad \text{if C is constant.} \qquad \text{.......... (1.17.8)}$$

If C is also varying with respect to time,

$$i = \frac{dQ}{dt}$$

$$= \frac{d}{dt}(CV)$$

$$= C\frac{dV}{dt} + V\frac{dC}{dt} \qquad \text{.......... (1.17.9)}$$

Hence, the voltage across the capacitor or capacitance,

$$V = \frac{1}{C}\int i\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{0} i\,dt + \frac{1}{C}\int_{0}^{t} i\,dt$$

$$= v_0 + \frac{1}{C}\int_{0}^{t} i\,dt \qquad \text{.......... (1.17.10)}$$

where v_0 is the initial voltage across the capacitor.

$\frac{1}{C}$ is also known as ***Elastance*** and is denoted by D and is measured in Daraf (Farad written in reverse direction).

Note : *Current can flow through the capacitance only so long as the charge flow is varying with respect to the time since,*

$$i = \frac{dQ}{dt}$$

This is possible only in A.C. circuits since current varies with respect to time. In D.C., when the voltage across the capacitance is steady the capacitor gets charged fully to the supply voltage and there will be no flow of current in the capacitor since,

$$I = \frac{dQ}{dt} = 0$$

1.17.2 Energy Stored In A Charged Capacitance

Capacitance stores energy between its plates to the form of Electrostatic field since electrostatic field is established between the two charged plates

The instantaneous value of power to the capacitor,

$$p = V \times i \text{ Watts}$$

$$= V \times C\frac{dV}{dt} \text{ Watts}$$

The total energy supplied to the capacitor when potential difference across the capacitor is increased from 0 to V Volts is given as,

$$W_C = \int p\,dt = \int_0^V vC\,\frac{dv}{dt}\,dt$$

From Eq. (2.2.8), we have,

$$W_C = \int_0^V Cv\,dv$$

$$= \tfrac{1}{2}C v^2 \Big|_0^V$$

$$= \tfrac{1}{2}CV^2 \text{ Joules or Watt-sec.}$$

also $$W = \tfrac{1}{2}C\left[\frac{Q}{C}\right]^2$$

$$= \tfrac{1}{2}\frac{Q^2}{C} \qquad \because v = \frac{Q}{C} \qquad \text{.......... (1.17.11)}$$

For a capacitor with dielectric of thickness d meters and area A square meters, energy per cubic meter is,

$$\frac{1}{2}\frac{CV^2}{Ad} = \frac{1}{2}\frac{\in A}{d}\frac{V^2}{Ad}$$

$$= \frac{1}{2}\in\left(\frac{V}{d}\right)^2 = \frac{1}{2}\in E^2 \text{ Watt-sec} \quad \text{........ (1.17.12)}$$

$$W_C = \frac{1}{2}DE$$

$$= \frac{1}{2}\frac{D^2}{\in} \text{ Joules or Watt-sec.} \quad \text{.......... (1.17.13)}$$

where D is the Flux Density and

E is the Electric Field Intensity.

Note :

1. *In a system altered instantaneously by the closing or opening of a switch, the voltage across the capacitance cannot change immediately and is equal to v_0 the initial voltage across the capacitance since,*

$$V_C = V_o + \int_0^t i\,dt$$

Hence, the voltage across a capacitance cannot change all on a sudden. This principle is made use in designing of Pulse and Digital Circuits.

2. *The Capacitors used in electronic circuits will have Paper or Ceramic or an electrolyte as the dielectric medium. The Ceramic capacitors will be of the order micro-Farads ($\mu F = 10^{-6}$ Farads) or pico-Farads ($pF = 10^{-12}$ Farads)and will have a color coding painted on them to indicate their values. The electrolytic capacitors will have polarities marked and they have to be connected to the same polarities of the supply else the capacitor will burst. The symbol for the electrolytic capacitor is given in Fig. 1.18.*

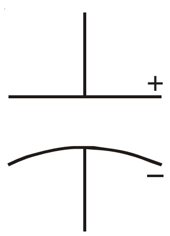


Fig. 1.18 *Capacitor*

3. *A pure capacitance will have no resistance. The capacitance gets charged to its full value and the voltage across the capacitance remains the same when the capacitor circuit is opened, since, the charge of the capacitance cannot be discharged. Hence the capacitance gives an electric shock when touched after the circuit is opened. To prevent the shock, the capacitor has to be discharged by connecting a wire across the two terminals of the capacitor. In some capacitors like power capacitors used in electrical power circuits, a high resistance will be connected across the capacitor to discharge the capacitor when the circuit is opened.*

4. *The unit of capacitance is Farads and is denoted by F. Farad is a large unit and hence, capacitance have values of micro-Farads. The capacitor used in electronic circuits may be denoted by the manufacturer as MF. It should not be mistaken as Mega-Farads, since, Farad itself is a big unit and it should be taken as micro-Farads only.* ***A Farad may be defined as the capacitance of a capacitor between the plates of which there appears a potential difference of one volt when it is charged by one coulomb of electricity.***

5. *The electric power lines have charged conductors and hence, there will be capacitance between any two conductors and between a conductor and earth. Since, it is a long line the capacitance will be distributed capacitance and expressed in micro-Farads per kilometer length of line.*

6. *For precision work, the capacitance between the two ends of a coil and also the capacitance between two turns of the coil should be considered.*

Example 1.15 :

A 1μF parallel plate capacitor with air as the dielectric medium is charged to a potential of 200V. The distance between the two plates is 1 cm, Calculate

(i) *The electric stress with in the dielectric*

(ii) *The electric stress on the plate surface*

(iii) *The charges on the plates*

Calculate the values (i), (ii) and (iii) if air dielectric is replaced by paper of relative permittivity 4.

Solution :

Capacitance of capacitor,

$$C = 1\mu F = 1 \times 10^{-6} F$$

Distance between plates,

$$d = 1 \text{ cm} = 0.01\text{m}$$

Potential Difference across pates,

$$V = 200V$$

(i) Electric Stress of Intensity in the Dielectric,

$$E = \frac{V}{d}$$

$$= \frac{200}{0.01}$$

$$= 20 \times 10^3 \text{ V/m}$$

$$= 20\text{KV/m}$$

(ii) Electric Stress on the plate surface is same as in the Dielectric Medium, i.e.,

$$E = 20\text{KV/m}$$

(iii) Charge on the plates

$$Q = CV$$

$$= 1 \times 10^{-6} \times 200$$

$$= 2 \times 10^{-4}\text{C}$$

When air is replaced by paper of relative permittivity,

$$\varepsilon_r = 4$$

Electric stress in the medium = Electric Stress on the plates,

$$\frac{V}{d} = \frac{200}{0.01}$$

$$= 20\text{KV/m}$$

Note :

Capacitance of capacitor,

$$C_P = \varepsilon_r C$$

$$= 4 \times 1 = 4\mu\text{F} = 4 \times 10^{-6}\text{F}$$

Therefore, Charge on the plates,

$$Q = C_P V$$
$$= 4 \times 10^{-6} \times 200$$
$$= 8 \times 10^{-4} C$$

Example 1.16 :

Calculate the capacitance and energy stored in a Parallel Plate Capacitor which consists of two metal plates each $60 cm^2$ separated by a dielectric of 1.5mm thickness and $\varepsilon_r = 3.5$ if Potential Difference of 1000V is applied across it.

Given $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

Solution :

Area of each plate,

$$A = 60 cm^2 = 60 \times 10^{-4} m^2$$

Thickness of Dielectric,

$$d = 1.5 mm = 1.5 \times 10^{-3} m$$

Relative Permittivity,

$$\varepsilon_r = 3.5$$

Potential Difference across Capacitor,

$$V = 1000V$$

Capacitance of the capacitor,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 3.5 \times 60 \times 10^{-4}}{1.5 \times 10^{-3}}$$

$$= 123.96 \text{ pF}$$

$$\text{Energy Stored} = \tfrac{1}{2} CV^2$$
$$= \tfrac{1}{2} \times 123.96 \times 10^{-12} \times (1000)^2$$
$$= 61.98 \times 10^{-6} \text{ Joules}$$

1.17.3 Parallel Plate Capacitor with Guard Ring

One difficulty encountered in practice with a parallel plate capacitor is due to the fringing of flux which occurs at the edges. This difficulty is overcome by the provision of a ***Guard Ring*** around one electrode, as shown in the Fig. 1.19

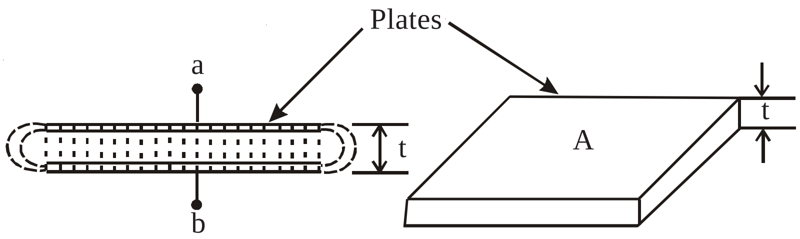


Fig. 1.19 *Parallel - Plate Capacitor*

The potential of the guard ring is maintained at the same level as that of the central electrode, but from a separate source. Fringing of flux is now confined to the outer edge of the guard ring, while the flux density over the central electrode is uniform. That is what we require as then the charge that actually reaches the electrodes through the lead L is accurately given by the equation charge density,

$$D = \sigma = \frac{Q}{A} \quad \text{Coulombs/m}^2$$

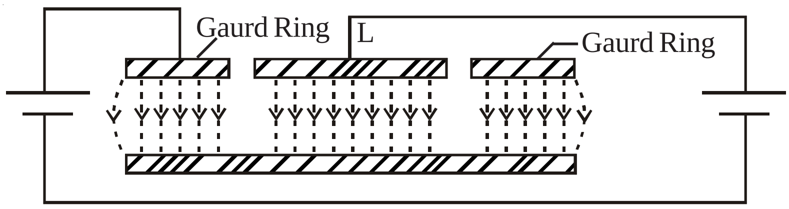


Fig. 1.20. *Parallel - Plate Capacitor with Gaurd Ring*

1.17.4 Capacitances in Series or Series Capacitors

Applying Kirchoff's Voltage Law,

$$V = V_1 + V_2 + V_3$$

$$= \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \frac{1}{C_3}\int i\,dt$$

$$V = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)\int i\,dt \qquad \text{.......... (1.17.14)}$$

For an equivalent capacitance,

$$V = \left(\frac{1}{C_{Se}}\right)\int i\,dt \qquad \text{.......... (1.17.15)}$$

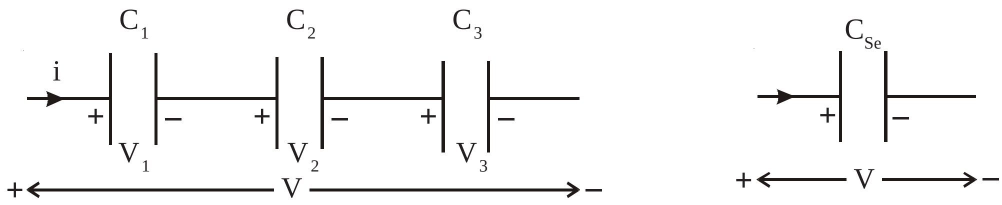


Fig. 1.21. *Capacitance in Series*

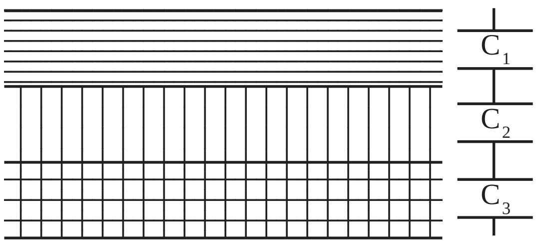


Fig. 1.22. *Composite Dielectric Capacitors*

Comparing Eq. (2.2.14) and Eq. (2.2.15), we get,

$$\frac{1}{C_{Se}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \qquad \text{.......... (1.17.16)}$$

For the Capacitance in Series,

$$\frac{1}{C_{Se}} = \sum_{i=1}^{n}\frac{1}{C_i} \qquad \text{.......... (1.17.17)}$$

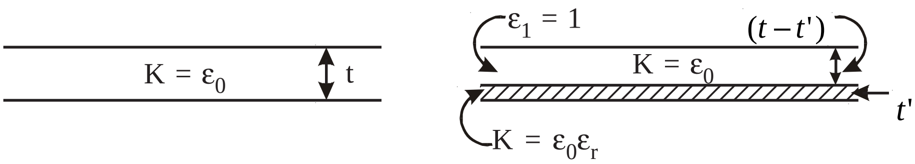
Example 1.17 :

Deduce an expression for the capacitance of a parallel-plate capacitor having two dielectric media.

A parallel plate capacitor has a plate separation t. The capacitance with air only between the plates is C. When a slab of thickness t and relative permittivity ε_r is placed on one of the plates, the capacitance is C . Show that,

$$\frac{C'}{C} = \frac{\varepsilon_r t}{t' + \varepsilon_r (t - t')}$$

Solution :



With air alone as dielectric,

$$C = \varepsilon_0 \frac{A}{t} \qquad (\text{ as } \varepsilon_r = 1) \qquad \text{.......... (1)}$$

Now, with the introduction of a slab of thickness t', the thickness of air-film is reduced to $(t - t')$ Let E_1 and E_2 be the field intensities in the air-film and slab, respectively and V_1 and V_2 the potential differences across them.

If V is the total potential difference across the capacitor, then

$$V = V_1 + V_2 \qquad \text{.......... (2)}$$

$$V_1 = E_1 (t - t') \qquad \text{.......... (3)}$$

$$V_2 = E_2 t' \qquad \text{.......... (3a)}$$

If Q is the charge on the plates, then, from the boundary relations, the normal components of electric flux densities being equal,

$$D = \frac{Q}{A}$$

we have, $\quad E_1 = \dfrac{D}{\varepsilon_0}$

and $$E_2 = \frac{D}{\varepsilon_0 \varepsilon_r}$$

Thus substituting for E_1 and E_2, we obtain from Eq. (3) and Eq. (3a),

$$V = \frac{Q}{A}\left[\frac{1}{\varepsilon_0}(t - t') + \frac{1}{\varepsilon_0 \varepsilon_r} t'\right]$$

$$= \frac{Q}{A\varepsilon_0}\left[\frac{1}{\varepsilon_r} t' + (t - t')\right] \qquad \text{.......... (4)}$$

The new value of capacitance is then,

$$C = \frac{Q}{V}$$

$$= \frac{A\varepsilon_0}{\frac{t'}{\varepsilon_r} + (t - t')}$$

$$= \varepsilon_0 \frac{A\varepsilon_r}{t' + \varepsilon_r (t - t')} \qquad \text{.......... (5)}$$

From Eq. (1) and Eq. (5), we get,

$$\frac{C'}{C} = \frac{\varepsilon_r t}{t' + \varepsilon_r (t - t')} \qquad \text{.......... (6)}$$

Example 1.17 :

A parallel plate capacitor has plates of area $2m^2$ spaced by the three slabs of different dielectric materials. The relative permittivities are 2, 3 and 6 and the thicknesses are 0.4, 0.6 and 1.2 mm respectively. Calculate the combined capacitance and the electric stress in each material when the applied voltage is 1000V.

Solution :

Area for the plates,

$$A = 2m^2$$

Relative Permittivity of three dielectric materials,

$$\varepsilon_{r1} = 2; \quad \varepsilon_{r2} = 3; \quad \varepsilon_{r3} = 6$$

Thickness of each dielectric material,

$$d_1 = 0.4\text{mm} = 0.0004\text{m}$$

$$d_2 = 0.6\text{mm} = 0.0006\text{m}$$

$$d_3 = 1.2\text{mm} = 0.0012\text{m}$$

Combined capacitance of capacitor,

$$C = \frac{\varepsilon_0 A}{\sum \frac{d}{\varepsilon_r}}$$

$$= \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}}$$

$$= \frac{8.854 \times 10^{-12} \times 2}{\frac{0.0004}{2} + \frac{0.0006}{3} + \frac{0.0012}{6}}$$

$$= 0.0295\mu\text{F}$$

Charge on the plates,

$$Q = CV$$

$$= 0.295 \times 10^{-16} \times 1000$$

$$= 29.5 \times 10^{-6}\text{C}$$

Electric Flux Density,

$$D = \frac{Q}{A}$$

$$= \frac{29.5 \times 10^{-6}}{2}$$

$$= 14.75 \times 10^{-6} \text{ c/m}^2$$

Electric Intensity of First Layer,

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}}$$

$$= \frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 2}$$

$$= 832.956 \text{KV/m}$$

Electric Intensity of Second Layer,

$$E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r2}}$$

$$= \frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 3}$$

$$= 555.304 \text{KV/m}$$

Electric Intensity of Third Layer,

$$E_3 = \frac{D}{\varepsilon_0 \varepsilon_{r3}}$$

$$= \frac{14.75 \times 10^{-6}}{8.854 \times 10^{-12} \times 6}$$

$$= 277.652 \text{KV/m}$$

Example 1.18 :

A parallel plate paper capacitor has 17 plates each having an effective area of 5cm^2 and each separated by a paper sheet of 0.005mm thick. Find the capacitance. Take the relative permittivity of paper as 4.

Solution :

Number of parallel plates,

$$n = 17$$

Area of each plate,

$$A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{m}^2$$

Thickness of paper sheet,

$$d = 0.005\,mm = 5 \times 10^{-6}\,m$$

Relative Permittivity of Paper,

$$\varepsilon_r = 4$$

Capacitance of Multi-plate Capacitor (Series Capacitors),

$$C = (n-1)\frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$= (17-1)\frac{8.854 \times 10^{-12} \times 4 \times 5 \times 10^{-4}}{5 \times 10^{-6}}$$

$$= 56.66 \times 10^{-9} F$$

Example 1.19 :

Consider that three capacitors having capacitances 5, 10, 12 microfarads are connected in series across 600V D.C. main. Determine :

(i) *Equivalent Capacitance of the Combination*
(ii) *Charge on each capacitor*
(iii) *Potential across each capacitor*
(iv) *Stored energy in each of the capacitor*

Solution :

Capacitance of each capacitor connected in series,

$$C_1 = 5\mu F; \quad C_2 = 10\mu F; \quad C_3 = 12\mu F;$$

Voltage across the grouping,

$$V = 600V$$

(i) Let C be the total or effective capacitance of the grouping, then,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{12}$$

$$= \frac{23}{60}$$

(ii) Charge on each capacitor,

$$Q = CV = 2.608 \times 600$$
$$= 1565\mu C$$

(iii) Potential across each capacitor,

$$V_1 = \frac{Q}{C_1}$$
$$= \frac{1565 \times 10^{-6}}{5 \times 10^{-6}}$$
$$= 313V$$
$$V_2 = \frac{Q}{C_2}$$
$$= \frac{1565 \times 10^{-6}}{10 \times 10^{-6}}$$
$$= 156.5V$$
$$V_3 = \frac{Q}{C_3}$$
$$= \frac{1565 \times 10^{-6}}{12 \times 10^{-6}}$$
$$= 130.5V$$

(iv) Energy stored in each capacitor,

$$W_1 = ½C_1V_1^2$$
$$= ½ \times 5 \times 10^{-6} - (313)^2$$
$$= 0.245J$$
$$W_2 = ½C_2V_2^2$$
$$= ½ \times 10 \times 10^{-6} - (156.5)^2$$
$$= 0.122J$$
$$W_3 = ½C_3V_3^2$$
$$= ½ \times 12 \times 10^{-6} - (130.5)^2$$
$$= 0.102J$$

1.17.5 Capacitances in Parallel or Parallel Capacitors

Parallel plate capacitance is obtained in the case of composite dielectric capacitors.

Applying Kirchoff's Current Law,

$$I = i_1 + i_2 + i_3$$

$$= C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + C_3 \frac{dV}{dt}$$

$$= (C_1 + C_2 + C_3) \frac{dV}{dt} \qquad \text{.......... (1.17.18)}$$

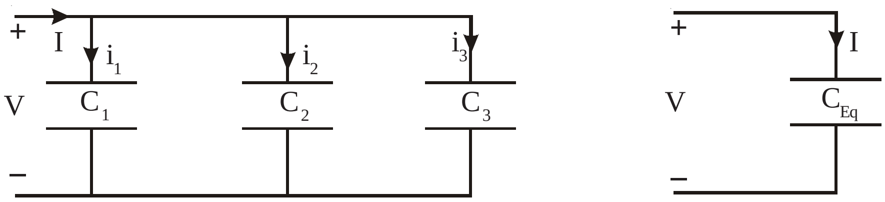


Fig. 1.23. *Capacitance in Parallel*

For the Equivalent Capacitance,

$$I = C_{Eq} \frac{dV}{dt}$$

Hence, $$C_{Eq} = C_1 + C_2 + C_3 \qquad \text{.......... (1.17.19)}$$

For the Capacitance in Parallel,

$$C_{Eq} = \sum_{i=1}^{n} C_i \qquad \text{.......... (1.17.20)}$$

Table 1.2 *Important Insulating Materials*

Material	Relative Permittivity
Vacuum	1.0
Air	1.0006
Paper (dry)	2.0 to 2.5
Polythene	2.0 to 2.5
Insulating Oil	3.0 to 4.0

Bakelite	4.5 to 5.5
Glass	5.0 to 10.0
Rubber	2.0 to 3.5
Mica	3.0 to 7.0
Porcelain	6.0 to 7.0
Distilled Water	80.0
Barium Titanate	6000+

Table 1.3 *Dielectric Strengths (MV/m)*

Material	**Thickness (mm)**	**Dielectric Strength (MV/m)**
Air (at normal pressure and temperature)	0.2	5.75
	0.6	4.92
	1.0	4.46
	6.0	3.27
	10.0	2.98
Mica	0.01	200
	0.1	176
	1.0	61
Glass (density 2.5)	1.0	28.5
	5.0	18.3
Ebonite	1.0	50
Paraffin-waxed paper	0.1	40 to 60
Transformer Oil	1.0	200
Ceramics	1.0	50

Example 1.20 :

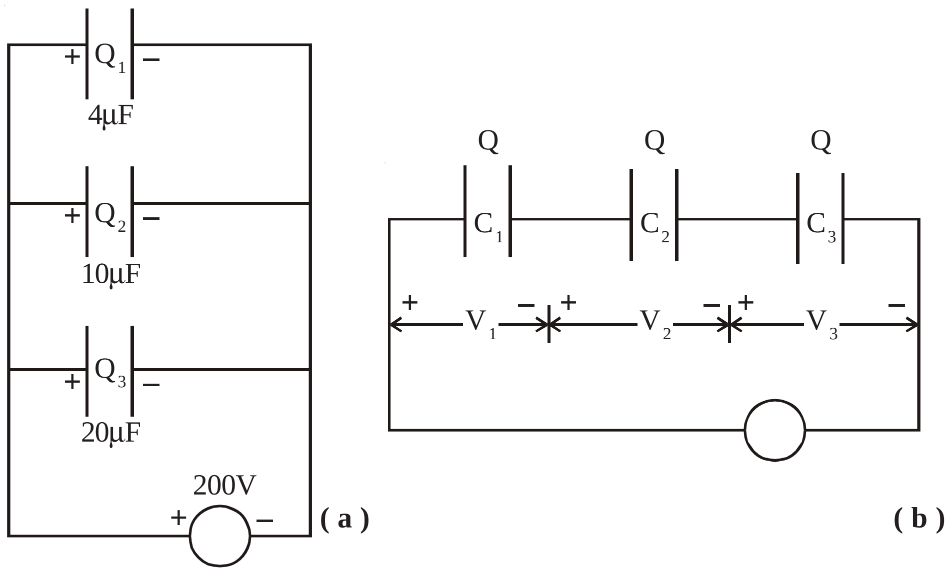
Three capacitors have capacitances of C_1 = 4μF, C_2 = 10μF and C_3 = 20μF. Find the total capacitance and the charge on each capacitor when connected in parallel to 200V supply. Calculate the total capacitance and voltage across each capacitor when connected in series to the same 200V supply.

Solution :

Fig. (a), shows the parallel circuit connection.

Then,

$$Q_1 = C_1 V$$
$$= 4\mu F \times 200V = 800\mu C$$
$$Q_2 = C_2 V$$
$$= 10\mu F \times 200V = 2000\mu C$$



(a) (b)

$$Q_1 = C_3V$$
$$= 20\mu F \times 200V = 4000\mu C$$
$$C = C_1 + C_2 + C_3 = 4 + 10 + 20 = 34\mu F$$

Fig. (b), shows the series circuit connection. The charge on each capacitor is the same.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Hence, $$C = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

$$= \frac{4 \times 10 \times 20}{(4 \times 10) + (10 \times 20) + (20 \times 4)}$$
$$= 2.5\mu F$$
$$Q = CV$$
$$= 2.5 \times 10^{-6} \times 200C = 500\mu C$$
$$Q = C_1V_1 = C_2V_2 = C_3V_3$$

Hence,

$$V_1 = \frac{Q}{C_1}$$

$$= \frac{500\mu C}{4\mu F} = 125V$$

$$V_2 = \frac{Q}{C_2}$$

$$= \frac{500\mu C}{10\mu F} = 50V$$

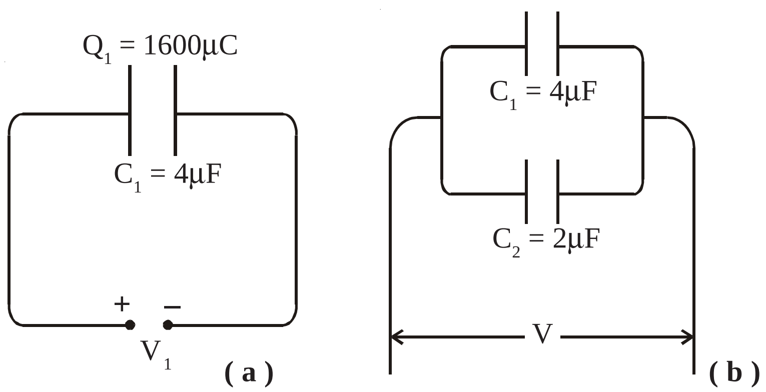
$$V_3 = \frac{Q}{C_3}$$

$$= \frac{500\mu C}{20\mu F} = 25V$$

Example 1.21 :

A capacitor of 4μF capacitance is charged to a potential difference of 400V and then connected in parallel with an uncharged capacitor of 2μF capacitance. Calculate the Potential Difference across the parallel capacitors.

Solution :



Capacitance of first capacitor,

$$C_1 = 4\mu F$$
$$= 4 \times 10^{-6} F$$

Potential Difference across first capacitor,

$$V_1 = 400V$$

Charge on the capacitor,

$$Q_1 = C_1 V_1$$
$$= 4 \times 10^{-6} \times 400$$
$$= 1600 \times 10^{-6}$$

Capacitance of second capacitor,

$$C_2 = 2\mu F$$
$$= 2 \times 10^{-6} F$$

When capacitor C_1 and C_2 are connected in parallel as shown in Fig. (b).

Total Capacitance,

$$C = C_1 + C_2$$
$$= 4 + 2 = 6\mu F$$

The charge on capacitor C_1 will be shared by the capacitor C_2.

Therefore, total charge on the two capacitors when connected in parallel,

$$Q = Q_1$$
$$= 16 \times 10^{-4} C$$

Potential Difference across the Parallel Capacitor,

$$V = \frac{Q}{C}$$
$$= \frac{16 \times 10^{-4}}{6 \times 10^{-6}}$$
$$= 266.67V$$

1.17.6 Variable Capacitors

These require two set of rigid plates which can be moved between one another as indicated in Fig. 1.24. The plates must be rigid so that they can move between each other without touching. It follows that the dielectric between the plates in air. Normally one set of plates is fixed and the other made to rotate. The greater the insertion of the movable plates then the greater the capacitance. Most of us know this type of capacitor because it is the device used to tune radios.

This type of capacitor is known as ***Gang Condenser***.

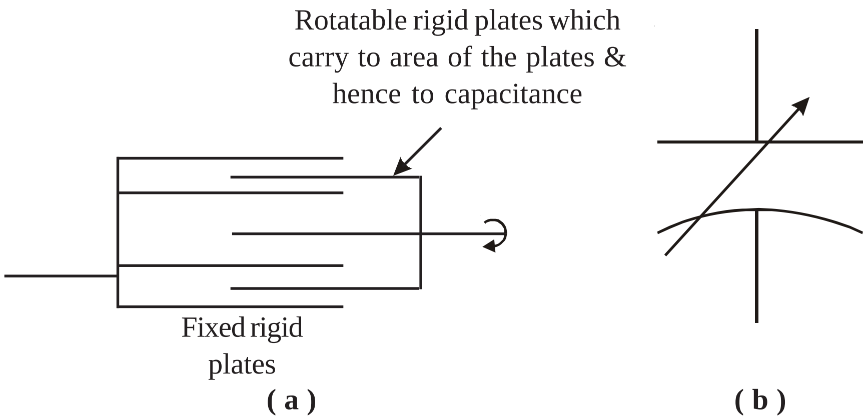


Fig. 1.24. *(a) Cross-section of a Gang Condensor*
(b) Variable Capacitor

Example 1.22 :

A variable air capacitor has 15 movable plates and 16 fixed plates. The area of each plate is 15 cm^2 and separation between opposite plates is 0.2mm. Determine

(i) *The maximum capacitance of this variable capacitor*

(ii) *The capacitance when ¼ area of movable plates is overlapping the fixed plates*

Solution :

Number of movable plates,

$$n_1 = 15$$

Number of fixed plates,

$$n_2 = 16$$

Area of each plate,

$$A = 15cm^2 = 15 \times 10^{-4} m^2$$

Distance between opposite plates

$$d = 0.2mm = 2 \times 10^{-4} m$$

Total Number of plates,

$$n = n_1 + n_2$$

$$= 15 + 16 = 31$$

(i) Capacitance will be maximum when movable plates are rotated so that two sets of plates are completely overlapping each other.

$$C_m = (31-1)\frac{8.854 \times 10^{-12} \times 1 \times 15 \times 10^{-4}}{2 \times 10^{-4}}$$

$$= 19.92 \times 10^{-2} F$$

(ii) Capacitance of capacitor, when ¼ area of movable plates is overlapping the fixed plates.

$$C_1 = (31-1)\frac{8.854 \times 10^{-12} \times 1 \times 15 \times 10^{-4}}{2 \times 10^{-4} \times 4}$$

$$= 4.08 \times 10^{-10} F$$

1.17.7 Capacitances in Series-Parallel

The equivalent capacitances for the parallel combinations of capacitances are found. These equivalent capacitances are treated to be in series with the remaining capacitances. After solving the circuit, the current in the parallel branches are calculated from the total current.

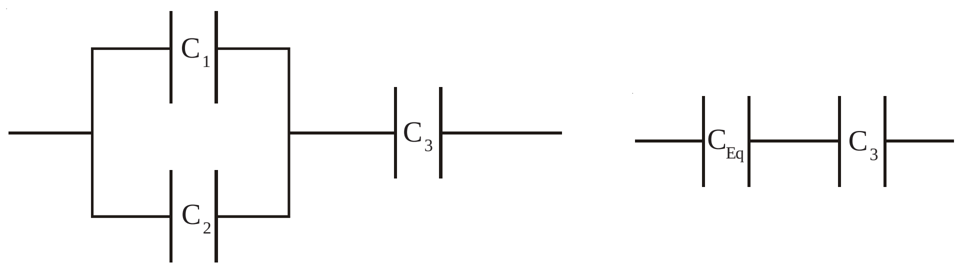
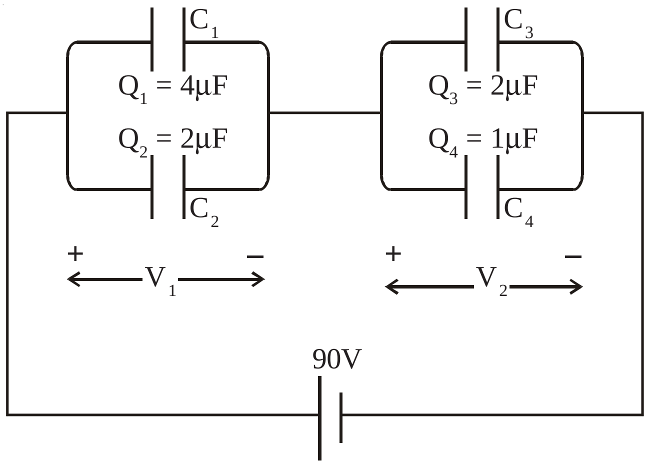


Fig. 1.25 Capacitance in Series-Parallel

Example 1.23 :

Calculate the charge and potential difference across each capacitor in the given circuit.



Solution :

C_1 and C_2 in parallel are equivalent to

$$C_{12} = C_1 + C_2 = (4 + 2) = 6\mu F.$$

C_3 and C_4 in parallel are equivalent to

$$C_{34} = C_3 + C_4 = (2 + 1) = 3\mu F$$

Capacitance of the whole circuit consisting of C_{12} and C_{34} in series is equivalent to C_t given by,

$$C_t = \frac{C_{12}C_{34}}{C_{12}+C_{34}}$$

$$= \frac{6\times 3}{6+3}\mu F = 2\mu F$$

Charging on the combination is

$$Q_t = C_t V$$

$$= 2\mu F \times 90V$$

$$= 180\mu C.$$

$$V_1 = \frac{Q_t}{C_{12}}$$

$$= \frac{180\mu C}{6\mu F} = 30V$$

$$V_2 = 90 - 30 = 60V$$

$$Q_1 = C_1V_1$$

$$= 4\mu F \times 30V = 120\mu C$$

$$Q_2 = C_2V_2$$

$$= 2\mu F \times 30V = 60\mu C$$

$$Q_3 = C_3V_3$$

$$= 2\mu F \times 60V = 120\mu C$$

$$Q_4 = C_4V_4$$

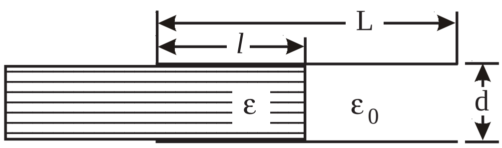
$$= 1\mu F \times 60V = 60\mu C$$

Example 1.24 :

A parallel-plate capacitor comprising two square plates, L m on a side, separated by a distance d m, is connected to a constant D.C. supply. A dielectric slab of width L and thickness d is inserted between the plates, as shown in the figure. Neglecting edge effects, find expressions for the following:

i. *The capacitance of the capacitor.*
ii. *The total energy stored in the capacitor.*
iii. *The force tending to draw the dielectric slab into the capacitor.*

Solution :



The solution of this problem proceeds in a straight forward manner. Each part of the problem is examined in order.

i. To find the capacitance of the capacitor, it is noted that the total charge on the capacitor plates is given by expression,

$$Q = \sigma l L + \sigma_0 (L - l)L$$

where σ is the charge density in the region of the dielectric slab

σ_0 is the charge density in the region of space.

Noting, however, that

$$\sigma = D = \varepsilon E = \frac{\varepsilon V}{d}$$

and $$\sigma_0 = D_0 = \varepsilon_0 E_0 = \frac{\varepsilon_0 V}{d}$$

then the total charge on the plates becomes,

$$Q = \frac{\varepsilon V}{d} lL + \frac{\varepsilon_0 V}{d} (L - l)L$$

It follows directly from this, since by definition,

$$C = \frac{Q}{V},$$

that

$$C = \frac{L}{d}\left[\varepsilon l + \varepsilon_0 (L - l)\right] \text{ Farads} \qquad \text{.......... (1)}$$

ii. To find the total energy in the capacitor, use is made of the expression for energy density. Thus,

$$W_e = \tfrac{1}{2}\varepsilon E^2 lLd + \tfrac{1}{2}\varepsilon_0 E_0^2 (L - l)Ld$$

But Since, $$E = E_0 = \frac{V}{d}$$

then this becomes,

$$W_e = \frac{1}{2}\frac{V^2 L}{d}\left[\varepsilon l + \varepsilon_0 (L - l)\right] \text{ Joules} \qquad \text{.......... (2)}$$

which may be written, by Eq. (1) as,

$$W_e = \tfrac{1}{2} C V^2$$

iii. To find the force tending to draw the dielectric slab into the capacitor, the work done in moving the slab a distance dl will be calculated. This work must equal the change in energy in the field. Thus, from (ii),

$$\mathrm{d(\ work\ )} = f\,\mathrm{d}l = \frac{1}{2}\frac{V^2L}{d}(\varepsilon - \varepsilon_0)\,\mathrm{d}l$$

Hence, the net force on the dielectric slab is,

$$f = \frac{(\varepsilon - \varepsilon_0)LV^2}{2d} \text{ Newtons} \qquad \text{.......... (3)}$$

Note : *We observed that the dielectric material experiences a force and gets pulled into the region of capacitor plates. This is the region where we observe a small bit of paper which is a dielectric material and when brought nearer to Nylon or Terylene clothes gets attracted towards the clothes because the cloth gets charged due to the friction with the body.*

1.17.8 Dielectric Strength and Breakdown

One of the important properties of a dielectric is its capability to withstand the electric stress (viz., the potential gradient) without rupture when the potential difference is raised. The material will cease to be a nonconductor or dielectric once the field produced exceeds a certain value. The maximum value of field-intensity or potential-gradient that the material can withstand without disruption may be referred to as the Dielectric Strength of the material. According to Maximum Stress Theory, if the potential difference across the material is raised to such an extent that the electric stress exceeds the above limit, breakdown is sure to occur. Breakdown may also occur due to presence of impurities, lack of homogeneity, surface-irregularities, etc.

The high electric field intensity surrounding high voltage power lines accounts for an additional energy loss in the transmission of power. The high voltage gradient at the surface of a wire sometimes accelerates electrons in the air sufficiently to ionize air molecules by collision. If the voltage gradient at the wire exceeds a certain critical value, the process of ionization becomes cumulative and results in appreciable loss of energy. The ionization is characterized by a faint glow surrounding the wire and is called ***Corona***.

The dielectric strength of air at barometric pressure of 76cm at 25^{o}C is 30KV/Cm (Peak Value) or 21.1KV RMS value / Cm.

1.17.9 Electrostatic Induction

A positively charged conductor induces negative charges on a nearby conductor. This is called ***Electrostatic Induction***. The charges tend to get distributed at the outer surface of the conductor. The charges get concentrated at sharp points of the conductor. Hence, in the case of lighting arrestors the charged clouds induce opposite charges in the sharp end of arrestor rod which passes the charges to ground, thus, saving the damage to the structure.

1.17.10 Features of a Capacitance

1. Capacitance is generally used in A.C. circuits and it reduces the current.
2. The current in a capacitor leads the voltage by 90^o.
3. Capacitance stores energy in the form of Electrostatic Energy between the plates.
4. For an ideal capacitor there will be no Power Loss.
5. Capacitance depend upon the dimensions and geometry and also the dielectric medium between the plates.
6. In the case of D.C. supply, the capacitance initially gets charged to the full voltage and there will be no further charging and hence, no current will be flowing through the capacitance. Thus, the capacitance initially acts as a Short circuit and as an Open Circuit finally.
7. The voltage across a capacitance cannot change all on a sudden.
8. The charged capacitance retains its charge and voltage even after removal of the supply and hence causes an electric shock if touched. Hence, one should be very careful in dealing with capacitance. The charged capacitance has to be discharged by external means before touching.
9. Large Commercial Power Capacitor will have a discharging resistance connected across it.
10. A lossy capacitance (due to the loss in the dielectric) can be represented as an ideal capacitor in parallel with a resistance to account for the loss in the capacitor.

1.17.11 Uses of Capacitors

1. In resonance circuits to tune and select particular frequency signals as in Radio and TV., etc.
2. In Filter Circuits to select or reject certain frequencies

3. In Operational Amplifiers which is used for integration, differentiation, etc.
4. In Wave Shaping Circuits to obtain a desired wave form from a given waveform
5. In analog circuits, to solve for other systems like Mechanical Systems, Hydraulic System, etc., in terms of Electrical Systems
6. In ***Transducers***, which convert physical signals into electrical signals as in microphone.
7. In Power Factor Changing Circuits
8. To improve the Power Factor of Load.
9. To improve the voltage profile of transmission lines.
10. In High Voltage Impulse Generators.
11. In Measurement Circuits.

1,18 INDUCTANCE

Inductance is also an energy storing element like capacitor. The energy stored in the inductance is in the form of Electromagnetic Field. Both Inductance can also be used in wave shaping circuits. Inductance is also called as Inductor.

Inductance exhibits delay in the rise and fall of currents through it. Hence, it is used to represent a mass possessing inertia in Electrical Analog Circuits used to represent a Mechanical System.

A current flowing through a conductor sets up an electromagnetic field around the conductor. The direction of the magnetic field setup is given by, ***Maxwell's Right Hand Cork-Screw Rule*** or ***Right Hand Threaded Screw Rule*** which states that, *the direction of the magnetic field setup by a current flowing through a conductor will be given by the direction of rotation of a right hand cork-screw if the screw is to advance in the direction of the current flow.* Hence, it will be circular around the conductor as given in the Fig 1.26

Note : *The dot inside the conductor of Fig. (1.26) represents the arrow head and indicates that the current is directed towards us i.e., away from the plane of the paper. The cross inside the conductor of Fig. (1.26) represents the tail of an arrow and indicates that the current is directed away from us i.e., into the plane of the paper.*

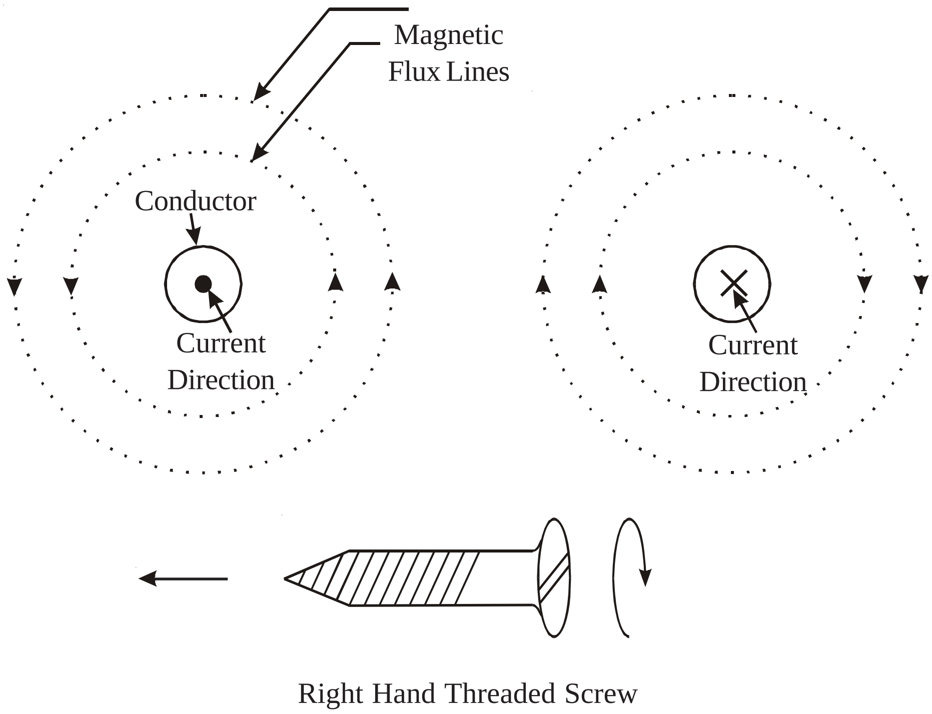


Fig. 1.26 *Magnetic Field Setup by a Current Carrying Conductor*

1.18.1 Magnetic Field of a Solenoid

In the Fig 1.27 (b) and Fig. 1.27 (c), the conductor is shown by circular cross-section with a dot or cross inside to indicate the direction of the current. The circle around the conductor along with its direction indicates the direction of the magnetic field setup by the current flowing through this conductor.

If a coil is wound on a soft iron rod, as in Fig. 1.27 and connected to a battery, the steel becomes magnetized and behaves like a temporary magnet. This magnetic field of the electromagnet is represented by the dotted lines and its direction by the arrow heads.

The direction of the magnetic field produced by a current in a solenoid may be deduced by applying either the Right Hand Threaded Screw or the Right Hand Grip Rule.

If the axis of the screw is placed along that of the solenoid and if the screw is turned in the direction of the current, it travels in the direction of the magnetic field inside the solenoid, namely towards the right in Fig. 1.28

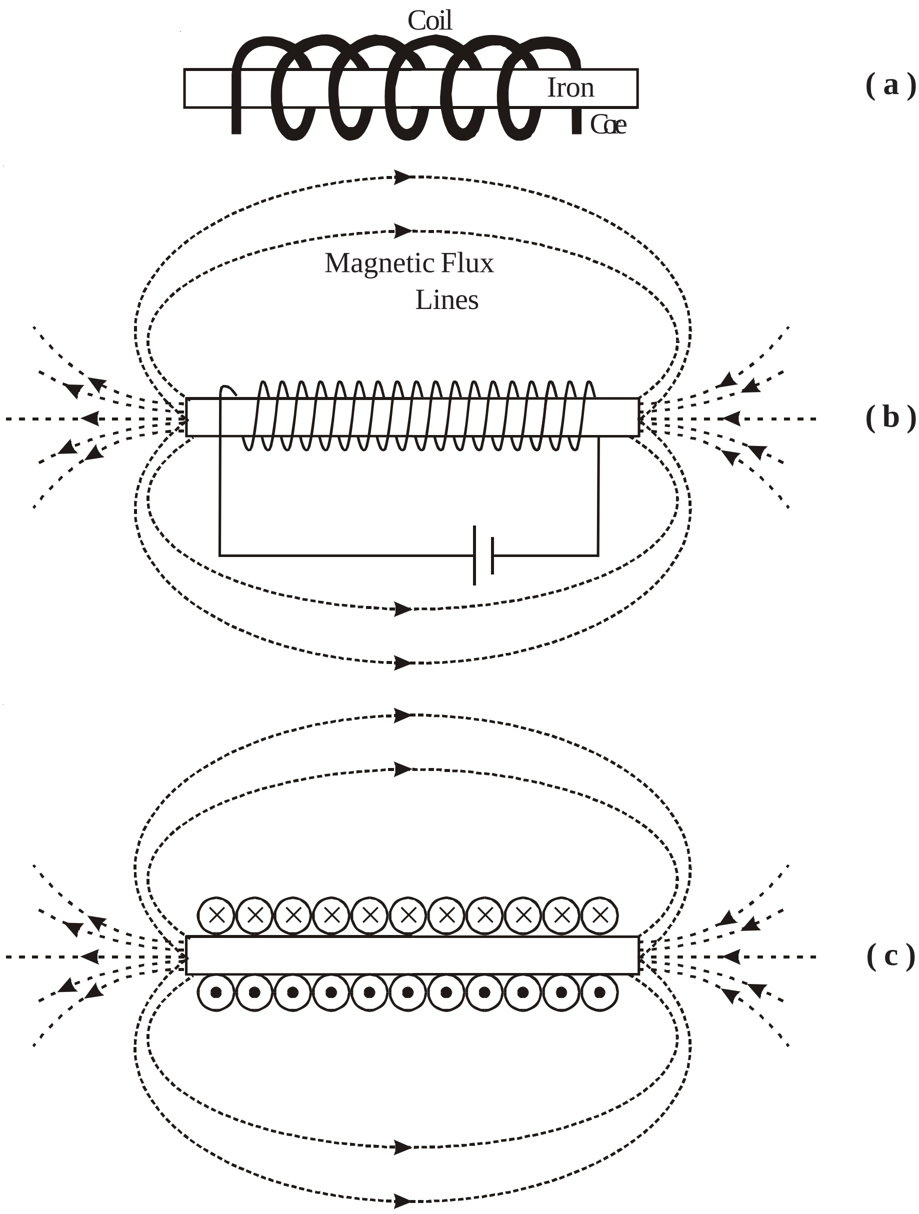


Fig. 1.27 *Solenoid and its Magnetic Field Setup by the Solenoid*

The ***Right Hand Grip Rule***, can be expressed thus : *if the solenoid is gripped with the right hand, with the fingers pointing in the direction of the current, i.e., conventional current, then the thumb outstretched parallel to the axis of the solenoid points in the direction of the magnetic field inside the solenoid.*

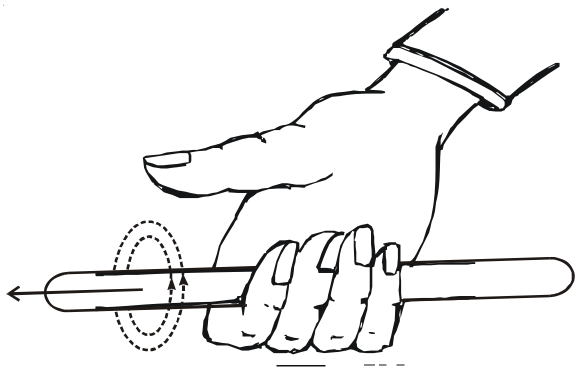
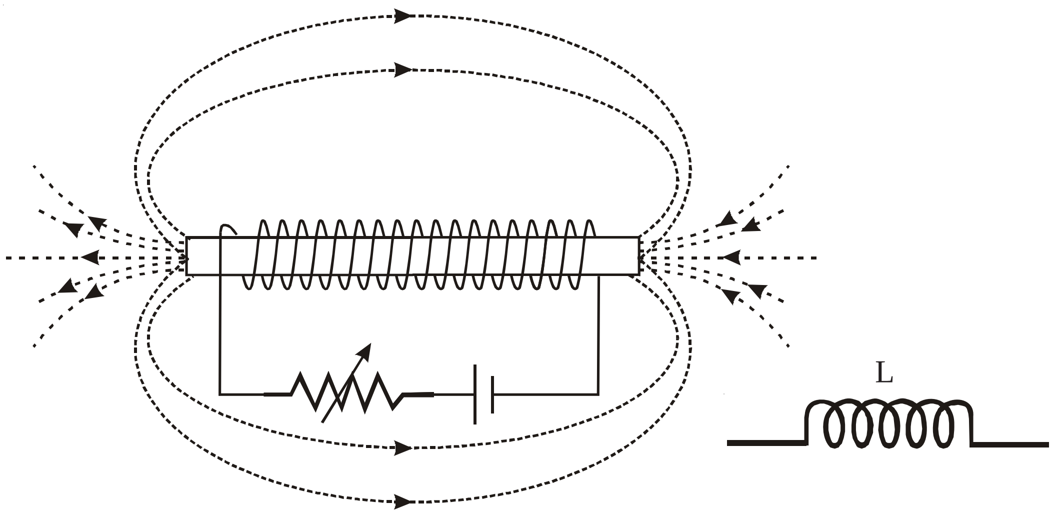


Fig. 1.28 *Right Hand Grip Rule*

1.18.2 Self Inductance of a Coil

If the current flowing through a coil is A.C. current then, the electromagnetic field setup by the current will be time varying i.e., varying with respect to time. According to Faraday's Law, this time varying magnetic field will induce an alternating voltage called induced E.M.F. (E) in the coil or conductor.



(a) Setting up of Self Inductance **(b)** Symbol for Self Inductance

Fig. 1.29 *Self Inductance of a Coil*

According to Lenz's Law, the E.M.F. induced in the conductor will be opposing the cause that produces it. That means it will try to reduce the current flowing through the conductor which sets up the magnetic field. This reduction in current can be treated as reduction in current due to a parameter called ***Inductance Parameter*** (L) just as the resistance (R) reduces the current. The inductance is denoted by L and is measured in Henry.

A circuit has an inductance of one henry (1 H) if an E.M.F. of one volt is induced in the circuit when the current varies uniformly at the rate of one ampere per second. Henry is also a big unit and hence, the coils will generally have inductance of the order of milli-Henrys.

In this case, the E.M.F. induced in the coil is due to the magnetic field setup by the current flowing in the same conductor. Hence is known as ***self induced E.M.F.*** and also the inductance is known as ***self inductance*** or ***coefficient of self induction***.

The self inductance of a coil is defined as the flux linkages linking that coil for unit current flowing through that coil. If the flux produced by the current i flowing through that coil is is ϕ webers and if the coil has N turns, then the flux linking that coil will be $N\Phi$ weber-turns. Then the self inductance,

$$L = \frac{N\Phi}{i} \text{ henrys or Weber-turns/ampere.} \qquad \text{.......... (1.18.1)}$$

This is one way of calculating the Self Inductance.

Hence, a coil is said to have a self-inductance of one henry if a current of one ampere flowing through it produces flux linkages of one wb-turns in it.

Alternatively, for a Solenoid or Toroid of N turns, the magnetization H is given as

$$H = \frac{NI}{l} \text{ AT / m} \qquad \text{.......... (1.18.2)}$$

where l is the length of the magnetic path = $2\pi r$ meters

r is the radius of the toroidal ring of the magnetic material in meters

a is the area of cross-section of the magnetic ring in square meters.

The magnetic flux,

$$\Phi = B \times a \ \ Wb \quad = \mu_o \mu_r H a \ Wb$$

$$\Phi = \frac{Ni}{l/\mu_o \mu_r a} \ wb \qquad (\because \ H = \frac{NI}{l}) \qquad \text{........ (1.18.3)}$$

where B is Flux density in wb/m^2 or Tesla 'T', a μ_o is the permeability of free space with μ_o equals $4\pi 10^{-7} H/m$, μ_r is relative permeability with μ_r=3000 for Iron, a is area of cross section of the magnetic path in square meters

$$\Phi = \frac{\text{Magnetomotive Force}}{\text{Reluctance}}$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \ \text{or} \ \frac{F}{S} \ wb \qquad \text{.......... (1.18.4)}$$

The magnetomotive force,

$$\mathcal{F} \ \text{or} \ F = NI \ \ \text{amp-turns} \qquad \text{.......... (1.18.4a)}$$

$$\text{Reluctance, } \mathcal{R} \text{ or } S = \frac{1}{a\mu_0\mu_r} AT/wb$$

The *self inductance* of a coil is the flux linkages with that coil per unit current flowing through that coil. Hence,

$$L = \frac{N\Phi}{i} \quad \text{Wb Turns / Amp. or H} \qquad \text{.......... (1.18.5)}$$

This is another way of calculating the Self Inductance.

$$\text{Also,} \qquad L = \frac{N\Phi}{i} = \frac{N}{i} \frac{N \times i}{l/\mu_o \mu_r a}$$

$$L = \frac{N^2}{l/\mu_o \mu_r a} \ \text{henry} \qquad \text{.......... (1.18.6)}$$

$$L = \frac{N^2}{S} \qquad \text{.......... (1.18.7)}$$

This is another way of calculating the Self Inductance and is in terms of the geometry of the coil.

Note : *Henry is denoted by symbol H* ***and should not be confused with magnetization force H****.*

1.18..3 Voltage Drop a cross the Inductance

According to Faraday's Law and Lenz's Law, the self induced E.M.F. in a coil is the negative of the rate of change of flux linkages in that coil.

$$e_l(t) = -\frac{d\lambda}{dt} \text{ or } -\frac{d\psi}{dt} = -\frac{d}{dt}(N\Phi)$$

$$= -\frac{d}{dt}(Li) \qquad \because \; L = \frac{N\Phi}{i}$$

$$e_l(t) = -L\frac{di}{dt} \qquad \text{.......... (1.18.8)}$$

The voltage drop across the inductance,

$$v_l(t) = -e_l(t)$$

$$= L\frac{di}{dt} \text{ Volts} \qquad \text{.......... (1.18.8a)}$$

Hence, $L = \dfrac{v_l}{di/dt}$ Henry

or $L = -\dfrac{e_l}{di/dt}$ Henry .... (1.18.9)

This is another way of calculating the Self Inductance.

Hence, we can also say that a coil or a circuit has an inductance of one henry (1 H) if an E.M.F. of one volt is induced in the coil or circuit when the current varies uniformly at the rate of one ampere (1 A) per second.

From Eq. (2.3.8a), the Volt-Ampere relationship of the inductance parameter is given as

$$v_l(t) = L\frac{di(t)}{dt}$$

or $v_l = L\dfrac{di}{dt}$

where v_l and i stand for instantaneous values or values at any particular instant of voltage and current respectively.

$$i = \frac{1}{L}\int v\,dt$$

$$= \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$i = \Gamma \int_{-\infty}^{t} v\,dt \qquad \text{.......... (1.18.10)}$$

where Γ is the Greek Capital Letter Gamma = $\frac{1}{L}$

$$= \frac{1}{L}\int_{-\infty}^{0} v\,dt + \frac{1}{L}\int_{0}^{t} v\,dt$$

$$i = i_l(0^+) + \frac{1}{L}\int_{0}^{t} v\,dt \qquad \text{.......... (1.18.10a)}$$

1.18.4 Energy Stored in an Inductor or Coil or Inductance

The instantaneous power p(t) of a coil is

$$p(t) = v(t) \times i(t) \text{ Watts}$$

or $$p = v \times i \text{ Watts}$$

$$= L\frac{di}{dt} i$$

The instantaneous energy w(t) stored in a coil is given as

$$w(t) = p(t)\,dt$$

or $$w = p\,dt$$

$$w = L\frac{di}{dt} i\,dt$$

$\therefore$ $$w = Li\,di \qquad \text{.......... (1.18.11)}$$

The total energy absorbed by the magnetic field when the current increases from 0 to I Amperes is

$$W_L = L\int_0^I i\,di$$

$$= L\,\frac{1}{2}i^2\Big|_0^I$$

$$\therefore \quad W_L = \frac{1}{2}LI^2 \text{ Joules or Watt-sec.} \qquad \text{.......... (1.18.12)}$$

From Eq. (2.3.5), the Self Inductance,

$$L = \frac{N^2}{l/a\mu}$$

$$= N^2\mu\frac{a}{l}$$

for a homogeneous magnetic circuit of uniform cross-sectional area a and length *l*.

Therefore energy per cubic meter ω_t is

$$\omega_t = \frac{1}{2}\,N^2\mu\,\frac{a}{l}\,I^2 \text{ x } \left(\frac{1}{al}\right) = \frac{1}{2}N^2I^2\frac{\mu}{l^2}$$

Since, $\quad H = \dfrac{NI}{l}$,

$$\omega_t = \frac{1}{2}\mu H^2 \text{ Joules or Watt-sec.} \qquad \text{.......... (1.18.13)}$$

Since the magnetic flux density,

$$B = \mu H \text{ wb/m}^2$$

$$\therefore \quad \omega_t = \frac{1}{2}HB \text{ Joules or Watt-sec.}$$

$$\therefore \quad \omega_t = \frac{1}{2}\,\frac{B^2}{\mu_o\mu_r} \text{ Joules / cubic meter} \qquad \text{.......... (1.18.14)}$$

Example 1.25 :

The iron core of a choke has mean length of 25cm with an air gap of 1mm. The choke is designed for an inductance of 15H when operating at a flux density of $1Wb/m^2$. The iron core has a relative permeability of 3000 and $8cm^2$ area of cross-section. Determine the required number of turns of the coil.

Solution :

From Eq. (2.3.5), Inductance of the coil,

$$L = \frac{N^2}{S_T}$$

where S_T is the total reluctance of the magnetic circuit.

$$S_T = \text{Reluctance of Iron Core + Reluctance of Air Gap}$$

$$S_T = \frac{l_i}{a\mu_0\mu_r} + \frac{l_g}{a\mu_0}$$

$$= \frac{0.25}{8\times10^{-4}\times4\pi\times10^{-7}\times3000} + \frac{1\times10^{-3}}{8\times10^{-4}\times4\pi\times10^{-7}}$$

$$= 1077612 \text{ Ampere-Turns/Wb}$$

Now, $N = \sqrt{LS_T}$

$$= \sqrt{15\times1077612} = 4020.5 \text{ turns.}$$

Example 2.12 :

A Solenoid of 1m in length and 10cm in diameter has 5000 turns. Calculate the energy in the magnetic field when a current of 2A flows in the Solenoid.

Solution :

Inductance of the Solenoid,

$$L = \frac{N^2}{l/\mu_o\mu_r a}$$

$$= \frac{N^2\mu_o\mu_r a}{l} = N^2\mu_o\mu_r a \qquad (\text{ Since } l = 1\text{m })$$

where $N = 5000,$ $a = \pi\dfrac{d^2}{4} = 25\pi \times 10^{-4} m^2$

$l = 1m;$ $\mu_r = 1,$ $\mu_0 = 4\pi \times 10^{-7}$ Henry/m

$$L = (5000)^2 \times 25\pi \times 10^{-4} \times 4\pi \times 10^{-7} \times 1 = 0.2467H$$

$$\text{Energy Stored} = ½LI^2$$

$$= ½ \times 0.2467 \times (2)^2 = 0.4934 \text{ Joules}$$

Example 1.26 :

The field winding of a D.C. Electromagnet has 1000 turns and resistance of 40Ω. With exciting voltage of 200V, the magnetic flux linking the winding is 0.006Wb. Calculate the Self Inductance of the winding and the energy stored in the magnetic field.

Solution :

$$\text{Current,} \quad I = \frac{200\text{Volts}}{40\Omega} = 5A$$

$$\Phi = 0.006Wb$$

$$N = 1000$$

Hence, Self Inductance,

$$L = \frac{N\Phi}{I}$$

$$= \frac{1000 \times 0.006}{5} = 1.2 \text{ Henry}$$

$$\text{Energy Stored} = ½LI^2 = ½ \times 1.2 \times 5 = 15 \text{ Joules}$$

Example 1.27 :

A coil of 200 turns carries a current of 4A. The magnetic flux linkage with the coil is 0.02Wb. Calculate the inductance of the coil. If this current is uniformly reversed in 0.02 second, calculate the induced E.M.F.

Solution :

$$\text{Inductance, } L = \frac{N\Phi}{I} = \frac{200 \times 0.02}{4}$$

$$= 1 \text{ H}$$

Induced Voltage,

$$e = L\frac{dI}{dt}$$

But, $dI = 4 - (-4) = 8\ A$

$dt = 0.02sec.$

Hence, $e = 1 \times \frac{8}{0.02} = 400\ V$

Example 1.28 :

A coil of 2000 turns is wound on a toroidal magnetic core having reluctance of 5×10^5 Ampere-Turns/Wb. If the current in the coil is 8A and is increasing at the rate of 300A/sec. Determine

(i) *Energy stored in the magnetic circuit*
(ii) *Voltage applied across the coil.*

Repeat the above calculation if the coil resistance is 4Ω.

Solution :

From Eq. (2.3.5),

$$L = \frac{N^2}{S}$$

$$= \frac{(2000)^2}{5 \times 10^5} = 8\ \text{Henry}$$

(i) Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2} \times 8 \times (8)^2$

$= 256$ Joules

(ii) Voltage applied across the coil = Self Induced E.M.F. in the coil.

Hence, $V = L\frac{dI}{dt}$

$$= 8 \times \frac{300}{1} = 2400\ V$$

If the coil resistance is 4Ω, there results additional loss of $(8)^2 \times 4 = 256J$ in the coil resistance. But the energy stored in the coil remains the same. However the Voltage across the coil will increase by $8A \times 4\Omega = 32V$ and will now becomes 2432V.

1.18.5 Inductances in Series or Series Inductors without Mutual Induction

Applying Kirchoff's Voltage Law,

$$V = V_1 + V_2 + V_3$$

$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$V = (L_1 + L_2 + L_3) \frac{di}{dt} \qquad \text{.......... (1.18.15)}$$

For the Equivalent Series Inductance,

$$V = L_{Se} \frac{di}{dt} \qquad \text{.......... (1.18.16)}$$

Comparing Eq. (2.3.15) and Eq. (2.3.16), we get,

$$L_{Se} = L_1 + L_2 + L_3 \qquad \text{.......... (1.18.17)}$$

In general,

$$L_{Se} = \sum_{i=1}^{n} L_i \qquad \text{.......... (1.18.17a)}$$

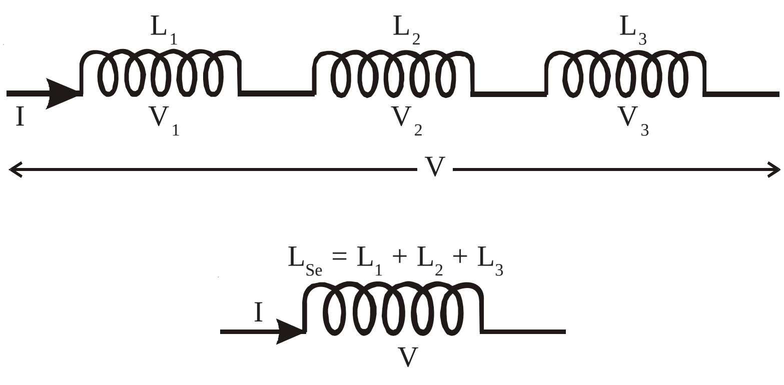


Fig. 1.30 *Inductance in Series without Mutual Induction*

Example 1.29 :

A coil of 250 turns is wound on a magnetic circuit of reluctance 1,00,000 Ampere-Turns/Wb. If a current of 2A flowing in the coil is reversed in 5milli-sec, find the average E.M.F. induced in the coil.

Solution :

$$\phi = \frac{\text{M.M.F.}}{\text{Reluctance}} = \frac{NI}{S}$$

where, $N = 250,\quad I = 2A,\quad S = 1{,}00{,}000$ Ampere-Turns/Wb

$$\phi = \frac{250 \times 2}{100000} = 5 \text{ m-Wb}$$

Average Induced E.M.F.,

$$e = N\frac{d\phi}{dt}$$

where $d\phi = 5 - (-5) = 10$ m-Wb

$$e = \frac{250 \times 10 \times 10^{-3}}{5 \times 10^{-3}} = 500 \text{ V}$$

1.18.6 Inductances in Parallel or Parallel Inductors without Mutual Induction

Applying Kirchoff's Current Law,

$$i = i_1 + i_2 + i_3$$

$$= \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt + \frac{1}{L_3}\int v\,dt$$

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int v\,dt \qquad \text{.......... (1.18.18)}$$

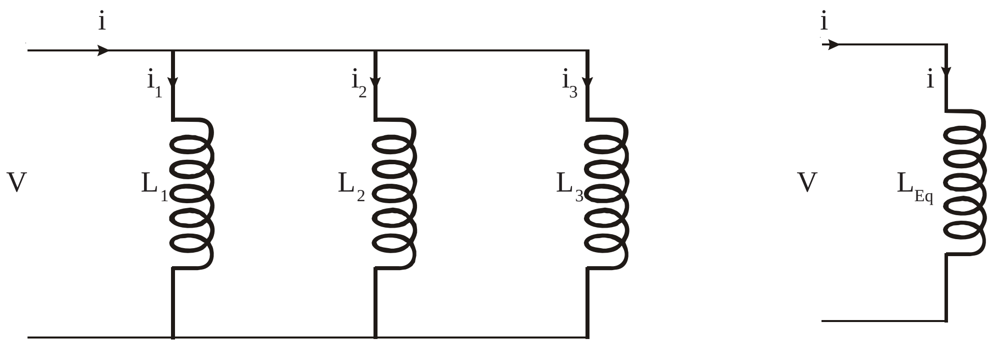


***Fig. 1.31** Inductance in Parallel without Mutual Induction*

For the Equivalent Parallel Inductance,

$$i = \frac{1}{L_{Eq}} \int v \, dt \qquad \text{.......... (1.18.19)}$$

Comparing Eq. (1.14.4) and Eq. (1.14.5), we get,

$$\frac{1}{L_{Eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \qquad \text{.......... (1.18.20)}$$

In general,

$$\frac{1}{L_{Eq}} = \sum_{i=1}^{n} \frac{1}{L_i} \qquad \text{.......... (1.18.20a)}$$

1.18.7 Inductances in Series-Parallel Without Mutual Induction

The equivalent inductances for the parallel combinations of inductances are found. These equivalent inductances are treated to be in series with the remaining series inductances. After solving the circuit, the current in the parallel branches are calculated from the total current. Fig.1.32 gives two inductances in parallel and one inductance in series, non of them having any mutual induction.

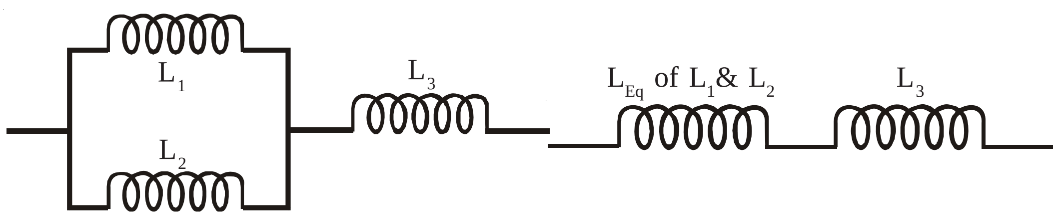


***Fig. 1.32** Inductance in Series-Parallel without Mutual Induction*

1.18.8 Mutual Induction Between Coils

If the E.M.F. is induced in a coil due to the magnetic field setup by another nearby coil then the E.M.F. induced in that coil is called ***Mutual E.M.F***. and the

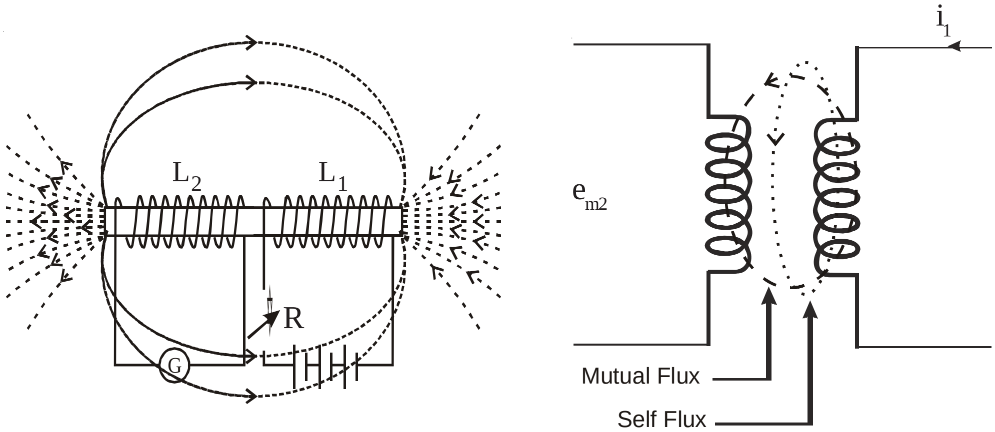


Fig. 1.33 Mutual Inductance

inductance induced in the first coil gets changed and this additional change in inductance is called ***Mutual Inductance***.

The direction of the induced E.M.F. depends upon the direction of the current in the other coil or flux setup by the other coil. If the mutual E.M.F. adds to the self induced E.M.F. of the first coil, then the mutual inductance adds to the self inductance of the first coil. If the induced E.M.F. opposes self induced E.M.F. of the first coil, then the mutual inductance reduces the self inductance of the first coil.

Mutual inductance may be defined as the ability of one coil (or circuit) to produce an E.M.F. in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an E.M.F. in the first when current in the second coil changes. This ability of reciprocal induction is measured in terms of the ***coefficient of mutual induction*** M.

If a current i_1 flowing through a coil 1 of turns N_1 produces a flux Φ_1 wb and if the flux Φ_1 links with coil 2 of turns N_2 then the mutual flux linking the coil 2 will be $N_2 \times \Phi_1$ and hence, the mutual inductance of coil 2 due to current in coil 1 is

$$M_{21} = \frac{N_2 \Phi_1}{i_1} \qquad \text{.......... (1.18.21)}$$

The first subscript of M indicates response and the second subscript indicates the excitation of source. M_{21} denotes the mutual inductance of coil 2 due to current i_1 in coil 1.

Similarly, if a current i_2 flowing through a coil 2 of turns N_2 produces a flux Φ_2 wb and if the flux Φ_2 links with coil 1 of turns N_1 then the mutual flux linking the coil 1 will be $N_1 \times \Phi_2$ and hence, the mutual inductance of coil 1 due to current in coil 2 is

$$M_{12} = \frac{N_1\Phi_2}{i_2} \qquad \text{.......... (1.18.22)}$$

In general,

$$M_{12} = M_{21} = M \qquad \text{.......... (1.18.23)}$$

where M is called the mutual inductance or coefficient of mutual inductance.

Alternatively,

The *mutual inductance* of a coil of N_1 turns is the flux linkages with that coil per unit current flowing through the neighboring coil of N_2 turns. Hence,

$$M_{12} = \frac{N_1\Phi_2}{i_2}$$

Since, $\Phi_2 = B_2 a = \mu_0\mu_r H_2 a = \mu_0\mu_r\left(\frac{N_2 i_2}{l}\right)a$

We have,

$$M_{12} = \frac{N_1 N_2}{l/\mu_o\mu_r\, a}$$

$$M_{12} = \frac{N_1 N_2}{S} \text{ henry} \qquad \text{.......... (1.18.24)}$$

Alternatively,

$$M = \frac{\mu_o\mu_r\, a\, N_1 N_2}{l} \text{ Henry} \qquad \text{.......... (1.18.25)}$$

The mutually induced E.M.F. e_{m2} in coil 2 due to current i_1 flowing in coil 1 is given as,

$$e_{m2} = L_{21} \times \frac{di_1}{dt}$$

$$= M_{21} \times \frac{di_1}{dt}$$

$$e_{m2} = M \times \frac{di_1}{dt} \qquad \text{.......... (1.18.26)}$$

where M_{12} is the mutual inductance between the coils 1 and 2 when i_1 flows through the coil 1.

The mutually induced E.M.F. e_{m1} in coil 1 due to current i_2 flowing in coil 2 is given as,

$$e_{m1} = L_{12} \times \frac{di_2}{dt}$$

$$= M_{12} \times \frac{di_2}{dt}$$

$$e_{m1} = M \times \frac{di_2}{dt} \qquad \text{.......... (1.18.27)}$$

The self induced E.M.F. e_{s2} in coil 2 due to current i_2 flowing in coil 2 is given as,

$$e_{s2} = L_{22} \times \frac{di_2}{dt} \qquad \text{.......... (1.18.28)}$$

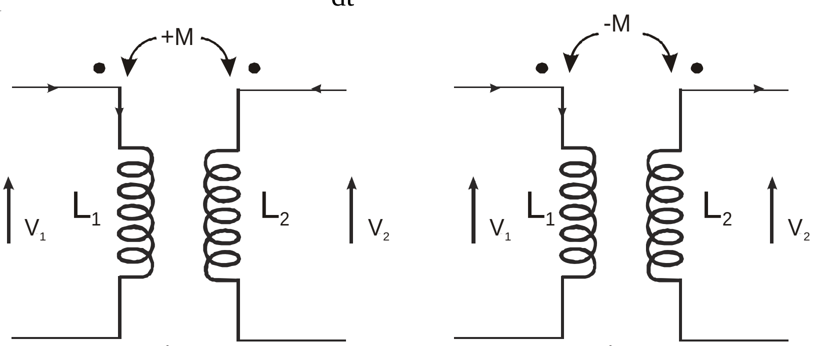


Fig 1.34 Mutual Inductances

The self induced E.M.F. e_{s1} in coil 1 due to current i_1 flowing in coil 1 is given as,

$$e_{s1} = L_{11} \times \frac{di_1}{dt} \quad \text{.......... (1.18.29)}$$

$$V_1 = e_{s1} \pm e_{m1}$$

$$V_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \quad \text{.......... (1.18.30)}$$

$$V_2 = e_{s2} \pm e_{m2}$$

$$V_2 = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} \quad \text{.......... (1.18.31)}$$

1.18.9 Dot Convention for Mutual Inductances

When two current carrying coils are located nearby, if the magnetic flux setup by the coil 1 which links coil 2 aids the magnetic flux setup by coil 2 then, the net flux in coil 2 will be increased and the total E.M.F. induced in coil 2 (sum of self and mutually induced E.M.F) will increase. Then the mutual inductance M is considered to be positive. Similarly, when two current carrying coils are located nearby, if the magnetic flux setup by the coil 2 which links coil 1 aids the magnetic flux setup by coil 1 then, the net flux in coil 1 will be increased and the total E.M.F. induced in coil 1 (sum of self and mutually induced E.M.F.) will increase. Then the mutual inductance M is considered to be positive.

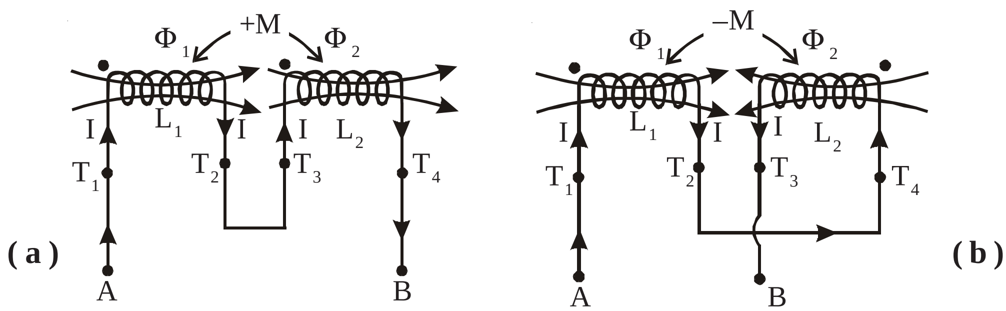


Fig. 1.35 Dot Convention of Mutual Inductance

When two current carrying coils are located nearby, if the magnetic flux setup by the coil 1 which links coil 2 opposes the magnetic flux setup by coil 2 then, the net flux in coil 2 will be decrease and the total E.M.F. induced in coil 2 (sum of self and mutually induced E.M.F). will decrease. Then the mutual inductance M is considered to be negative. Similarly, when two current carrying coils are located nearby, if the magnetic flux setup by the coil 2 which links coil 1 aids the magnetic flux setup by coil 1 then, the net flux in coil 1 will be decrease and the total E.M.F. induced in coil 1 (sum of self and mutually induced E.M.F.) will decrease. Then the mutual inductance M is considered to be negative.

This phenomenon is represented by dot convention as shown in Fig. 1.35.

Referring to Fig. 1.35 in dot convention, if the current in the two coils enter the terminals marked with a dot then it is assumed that the self flux of a coil and the mutual flux due to the other coil will aid and hence, the total E.M.F. induced in that coil will increase.

$$e_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad \text{.......... (1.18.32)}$$

$$e_2 = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}$$

$$e_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \qquad \text{.......... (1.18.33)}$$

$$\because \qquad M_{12} = M_{21} = M$$

Referring to Fig. 1.35, in dot convention, if the current in one coil enters the dotted terminal marked with a dot and the current in the other coil leaves the dotted terminal marked with a dot then it is assumed that the self flux of a coil and the mutual flux due to the other coil will be opposing and hence, the total E.M.F. induced in that coil will decrease.

$$e_1 = L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt}$$

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad \text{.......... (1.18.34)}$$

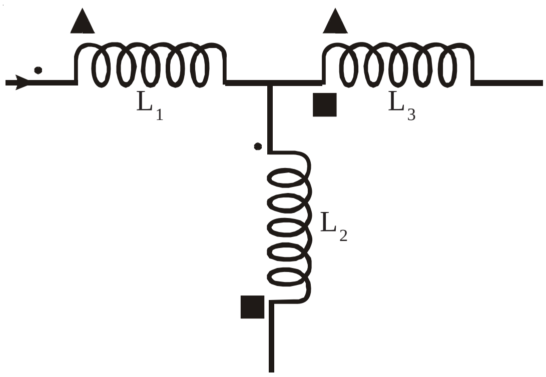


Fig. 1.36 *Dot Convention of Mutual Inductance*

$$e_2 = L_2 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt}$$

$$e_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \qquad \text{.......... (1.18.35)}$$

$$\because \qquad M_{12} = M_{21} = M$$

If there are more than two coils which have mutual inductance then, for a set of two coils with mutual inductance, dots may be indicated for their mutuals. For the second set of coils with mutuals to avoid confusion, instead of dots small triangles or small squares may be used to indicate the mutual inductance as given in Fig. 1.36.

In general, when two coils mutual coupling are connected in series, the total E.M.F. induced across the two coils which are in series with current i flowing through them will be,

$$e_t = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \pm 2M \frac{di}{dt} \qquad \text{.......... (1.18.36)}$$

The positive sign is used for 2M (+2M), if the current enters into both the coils and the negative sign is used for 2M (–2M), if the current enters into one coil and leaves away from the second coil as given by the dot convention.

In the Fig. 1.36.

Between L_1 & L_2, the Mutual Induction is Positive

Between L_2 & L_3, the Mutual Induction is Negative

Between L_3 & L_1, the Mutual Induction is Positive

1.18.10 Expression for Mutual Inductance M in Terms of Dimensions of Two Coils

The flux in the first coil (primary coil) is,

$$\Phi_1 = \frac{N_1 I_1}{l/\mu_o \mu_r a} \text{ Wb} \qquad \text{.......... (1.18.37)}$$

$$L_{11} = L_1 = \text{Flux/Ampere} = \frac{\Phi_1}{I_1}$$

$$L_{11} = L_1 = \frac{N_1}{l/\mu_o \mu_r a} \text{ Henry} \qquad \text{.......... (1.18.38)}$$

We assume that the whole of this flux F_1 links with the other coil having N_2 turns. Then the Wb-Turns in the second coil due to this flux/ampere in the first coil are:

$$M = \frac{N_2 \Phi_1}{I_1}$$

$$= \frac{N_1 N_2}{l/\mu_o \mu_r a} \text{ Henry}$$

Thus, $$M = \frac{N_1 N_2}{S} \text{ Henry}$$

where S is the reluctance and equals $l/\mu_o\mu_r a$.

Alternatively,

$$M = \frac{\mu_o \mu_r a N_1 N_2}{l} \text{ Henry} \qquad \text{.......... (1.18.39)}$$

1.18.11 Coefficient of Magnetic Coupling or Magnetic Coupling between two Coils

Normally only a part of the flux produced by one coil will be linking with the other coil due to leakage flux which will not be linking with the other coil. Two coils are said to be magnetically coupled if full or part of the flux produced by one links with the other.

Let L_1 and L_2 be the self inductances of the two coils, M their mutual inductance, and k be the coefficient of mutual coupling between the two coils. ϕ_1 and ϕ_2 are the flux setup by coils 1 and 2 respectively. N_1 and N_2 are the number of turns of coils 1 and 2 respectively. S is the reluctance of the magnetic path.

The mutual inductance of coil 2 due to flux setup by coil 1,

$$M_{21} = N_2 \frac{k\phi_1}{I_1}$$

$$= kN_2 \frac{N_1}{S}$$

Similarly, the mutual inductance of coil 1 due to flux setup by coil 2,

$$M_{12} = N_1 \frac{k\phi_2}{I_2}$$

$$= kN_1 \frac{N_2}{S}$$

$$M_{12} = M_{21} = M$$

Hence,
$$M^2 = k^2 \frac{N_2^2}{S} \times \frac{N_1^2}{S}$$

$$= k^2 L_1 L_2$$

$$k^2 = \frac{M^2}{L_1 L_2}$$

Therefore,

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad \text{.......... (1.18.40)}$$

When all the flux produced by one coil links with the other, then mutual inductance between the two coils is maximum and with k = 1, M is given by

$$M = \sqrt{L_1 L_2} \qquad \text{.......... (1.18.41)}$$

When there is no common flux between the two coils, they are said to be magnetically isolated. Since, in that case, M = 0, k = 0 also.

Hence, *coefficient of coupling may be defined as the ratio of actual mutual inductance present between the two coils to the maximum possible value of the mutual inductance between the two coils.*

Since the A.C. current passing through a coil sets up an inductance in addition to the resistance, in order to have a pure resistance offered by the coil, the coil is split up into two parts and each part is wound in the opposite direction in which case, the magnetic flux of the two parts cancel each other and the inductance offered by the two parts of the coil will be come zero, resulting only in ***pure resistance***. A pure resistance is denoted by the symbol .

Example 1.30 :

A wooden ring has mean diameter of 150mm and a cross-sectional area of 250mm^2. It is wound with 1500 turns of insulated wire. A second coil of 900 turns in wound on the top of the first. Assuming that all flux produced by the first coil links with the second, calculate the Mutual Inductance and the coefficient of coupling.

Solution :

Mutual Inductance,

$$M = \frac{N_1 N_2}{l} a\mu_o\mu_r$$

where $N_1 = 1500$, $N_2 = 900$, $l = \pi D = 0.15\pi$–meter

$a = 250 \times 10^{-6} m^2$, $\mu_r = 1$

$$M = \frac{1500 \times 900}{0.15\pi} \times 250 \times 10^{-6} \times 4\pi \times 10^{-7} \times 1 = 0.9 \text{ mH}$$

$$L_1 = \frac{N_1^2}{l}\mu_0\mu_r a$$

$$= \frac{1500^2}{0.1 \times \pi} \times 250 \times 10^{-6} \times 4 \times \pi \times 10^{-7} \times 1$$

$$L_1 = 1.5 \text{ mH}$$

$$L_1 = \frac{N_2^2}{l}\mu_0\mu_r a$$

$$= \frac{900^2}{0.1 \times \pi} \times 250 \times 10^{-6} \times 4 \times \pi \times 10^{-7} \times 1$$

$$= 0.54 \text{ mH}$$

Coefficient of Coupling,

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.9}{\sqrt{1.5 \times 0.54}}$$

$$= \frac{0.9}{\sqrt{0.81}} = \frac{0.9}{0.9} = 1$$

Note : *Here there is perfect coupling between the two coils since one coil is wound fully over the other coil.*

Example 1.31 :

Two coils have Mutual Inductance of 0.6H. If current in one coil is varied from 4A to 1A in 0.2sec, calculate

(i) *The average E.M.F. induced in the other coil*

(ii) *The change of flux linking the later assuming that it is wound with 150 turns.*

Solution :

Mutually Induced E.M.F.,

$$e_m = M \frac{dI_1}{dt}$$

where $M = 0.6H$, $dI_1 = 4 - 1 = 3A$, $dt = 0.2$ sec.

$$e_m = 0.6 \times \frac{3}{0.2}$$

$$= 9V$$

Now, $$e_m = N_2 \frac{d\Phi_{12}}{dt}$$

Change of flux with second coil,

$$d\Phi_{12} = \frac{e_m \times dt}{N_2}$$

$$= \frac{9 \times 0.2}{150} = 12\text{m-Wb}$$

Example 1.32 :

Two coils A and B having 50 and 500 turns respectively are wound side-by-side on a closed iron circuit of cross-sectional area 120 Sq. cm and mean length 240 cm. Find the mutual inductance between the two coils if the relative permeability of iron core is 800. If current in coil A grows from zero to 10 Amperes in 0.01 second, calculate the E.M.F. induced in coil B.

Solution :

$$M = \frac{N_1 N_2}{l} a\mu_o\mu_r \text{ Henry}$$

where $N_1 = 50$, $N_2 = 50$, $B = 120 \times 10^{-4} m^2$, $l = 2.4m$

$\mu_r = 800$

Hence, $$M = \frac{4\pi \times 10^{-7} \times 800 \times 120 \times 10^{-4} \times 50 \times 500}{2.4} = 0.126H$$

$dI_1 = 10 - 0 = 10A$

$dt = 0.01sec.$

Hence, $$e_m = M\frac{dI_1}{dt} = 0.126 \times \frac{10}{0.01} = 126V$$

Example 1.33 :

Two coils P and S each having 1200 turns are placed near each other. Coil S is open circuited. Coil P carries a current of 4A resulting in flux 0.3Wb in it and 50% of this flux links with the turns of coil S. Determine

(a) *Mutual Inductance between the coil and*

(b) *The Voltage induced in open circuited coil S when the current in coil P changes at the rate of 1.5A/sec.*

Solution :

Mutual Inductance,

$$M = \frac{N_2 \Phi_2}{I_1}$$

$N_1 = N_2 = 1500$ turns

Flux linked with coil,

$$S = 0.5 \times 0.3 = 0.15\text{Wb}$$

Hence, $$M = \frac{1200 \times 0.15}{4} = 45\text{Henry}$$

Hence, Mutually Induced E.M.F. in coil S is,

$$e_m = M\frac{dI_1}{dt} = 45 \times 1.5 = 67.5\text{V}$$

1.18.12 Principle of Constant Flux Linkages

Current through an inductance,

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$i_L = \frac{1}{L}\left(\int_{-\infty}^{0} v\,dt + \int_{0}^{t} v\,dt\right) \qquad \text{..........} (1.18.42)$$

$$i_L = i_L(0^+) + \frac{1}{L}\int_{0}^{t} v\,dt \qquad \text{..........} (1.18.43)$$

where $i_L(0^+)$ is the initial current in the inductance, i.e., at the time $t = (0^+)$ just when the inductance is switched on.

$i_L(0^+) = i_L(0^-)$, the current in the inductance at time $t = (0^-)$ just before the inductance is switched on.

Hence, the current through out the inductance cannot change all on a sudden.

The magnetic flux linkage ($\psi = NI$)

$$\psi = \int_{-\infty}^{t} v\,dt$$

$$\psi = \int_{-\infty}^{0} v\,dt + \int_{0}^{t} v\,dt \qquad \text{..........} (1.18.44)$$

$$\psi = \psi_0 + \int_0^t v\,dt \qquad \text{.......... (1.18.45)}$$

In a system altered instantaneously like the closing of a switch, the flux linkages must be the same before and after the system is altered, but only over very small interval of time ($t = 0^-$ to 0^+) during which

$$\psi = \psi_0 = \text{Constant}$$

Hence, the flux linkages cannot change instantaneously.

1.18.13 Inductances in Series (or Series Inductors) with Mutual Inductance

Consider Fig. 1.37, two coils magnetically coupled having self-inductance of L_1 and L_2 respectively and a Mutual Inductance of M Henry.

The two coils, in an electrical circuit, may be connected in different ways giving different values of resultant inductances as given below:

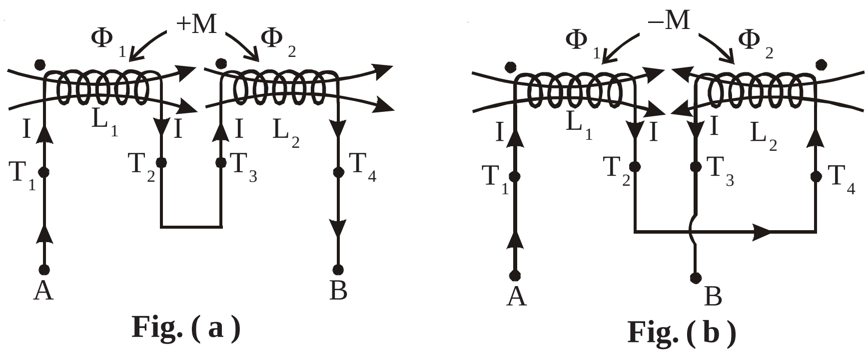


Fig. 1.37 *Mutual Inductance in Series*

The two coils may be connected in series in the following two ways :

(i) *As in Fig. 1.37 (a), when their fields (or M.M.F.s) are* ***additive*** *i.e., their fluxes are setup in the same direction as shown in Fig. 1.37 (a). In this case, the inductance of each coil is* ***increased*** *by* M_1 *i.e.,*

$$\text{Total Inductance, } L_T = (L_1 + M) + (L_2 + M)$$

$$= L_1 + L_2 + 2M \qquad \text{.......... (1.18.46)}$$

(ii) *As in Fig. 1.37 (b), when their fields (or M.M.F.s) are* ***subtractive*** *i.e., their fluxes are setup in the opposite direction as shown in Fig. 1.32. In this case, the inductance of each coil is decreased by M, i.e.,*

$$\text{Total Inductance, } L_T = (L_1 - M) + (L_2 - M)$$

$$= L_1 + L_2 - 2M \qquad \text{.......... (1.18.47)}$$

Note : *It may be noted that direction of field produced by a coil is denoted by a Dot placing it at the side of which current enters (or flux enters the core).*

Example 1.34 :

The combined inductance of two coils connected in series is 0.6H and 0.1H depending upon the relative direction of currents in the coils. If one of the coils when isolated has a self inductance of 0.2H, calculate the mutual inductance of the coils and the self inductance of the other coil.

Solution :

The combined inductance of the two coils when connected in series

(a) having their fields additive $= L_1 + L_2 + 2M = 0.6$ .... (1)

(b) having their fields subtractive $= L_1 + L_2 - 2M = 0.1$ .... (2)

Subtracting Eq. (2) from Eq. (1), we get,

$$4M = 0.5$$

or

$$M = 0.125\text{H}$$

From Eq. (2),

$$L_1 + L_2 - 2 \times 0.125 = 0.1$$

or

$$L_1 + L_2 = 0.35\text{H}$$

Self Inductance of First Coil,

$$L_1 = 0.2\text{H}$$

Self Inductance of Second Coil,

$$L_2 = 0.35 - 0.2 = 0.15\text{H}$$

1.18.14 Inductances in Parallel (or Parallel Inductors) with Mutual Inductance

When the flux in the two mutually coupled coils connected in parallel are aiding or adding then, considering the effect of mutual coupling we have, (L_1 + M) in parallel with (L_2 + M).

The total equivalent,

$$L_{Eq} = \frac{(L_1 + M)(L_2 + M)}{(L_1 + M) + (L_2 + M)}$$

$$= \frac{L_1 L_2 + M(L_1 + L_2) + M^2}{L_1 + L_2 + 2M} \qquad \text{.......... (1.18.48)}$$

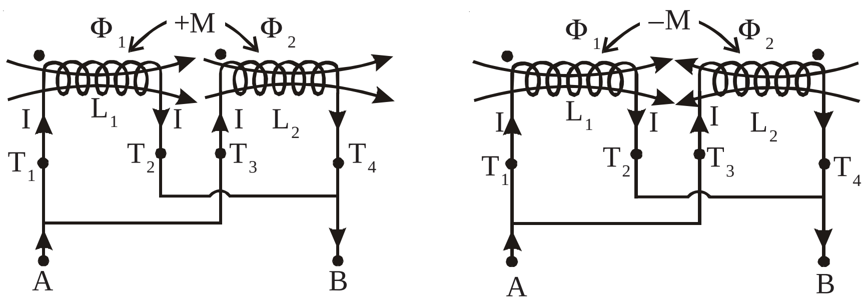


Fig. (a) **Fig. (b)**

Fig. 1.38 *Mutual Inductance in Parallel*

When the two fluxes are opposing,

$$L_{Eq} = \frac{(L_1 - M)(L_2 - M)}{(L_1 - M) + (L_2 - M)}$$

$$= \frac{L_1 L_2 - M(L_1 + L_2) + M^2}{L_1 + L_2 - 2M} \qquad \text{.......... (1.18.49)}$$

Example 1.35 :

Two coils of self inductance 120mH and 250mH and Mutual Inductance of 100mH are connected in parallel. Determine the equivalent inductance of combination if

(i) *Mutual Flux helps the individual fluxes and*

(ii) *Mutual flux opposes the individual fluxes*

Solution :

When Mutual Flux helps the individual fluxes,

$$L_T = \frac{(L_1 + M)(L_2 + M)}{(L_1 + M) + (L_2 + M)}$$

$$= \frac{(120 + 100)(250 + 100)}{(120 + 100) + (250 + 100)}$$

$$= 135.09 \text{ mH}$$

When Mutual Flux opposes the individual fluxes,

$$L_T = \frac{(L_1 - M)(L_2 - M)}{(L_1 - M) + (L_2 - M)}$$

$$= \frac{(120 - 100)(250 - 100)}{(120 - 100) + (250 - 100)}$$

$$= 17.65 \text{ mH}$$

1.18.15 Inductances in Series-Parallel with Mutual Inductances

The equivalent inductances for the parallel combinations of inductances with mutual inductances are found. These equivalent inductances are treated to be in series with the remaining inductances along with their mutuals if any. After solving the circuit, the current in the parallel branches are calculated from the total current.

L_{Eq1} = L_1 in parallel with L_2 with Mutual Flux Opposing

$$L_{Eq1} = \frac{(L_1 - M_{12})(L_2 - M_{12})}{(L_1 - M_{12}) + (L_2 - M_{12})}$$

L_{Eq2} = L_3 in series with L_4 with Mutual Flux Aiding

$$L_{Eq2} = L_3 + L_4 + 2LM_{34}$$

Total Inductance,

$$L_T = L_{Eq1} + L_{Eq2} + L_5$$

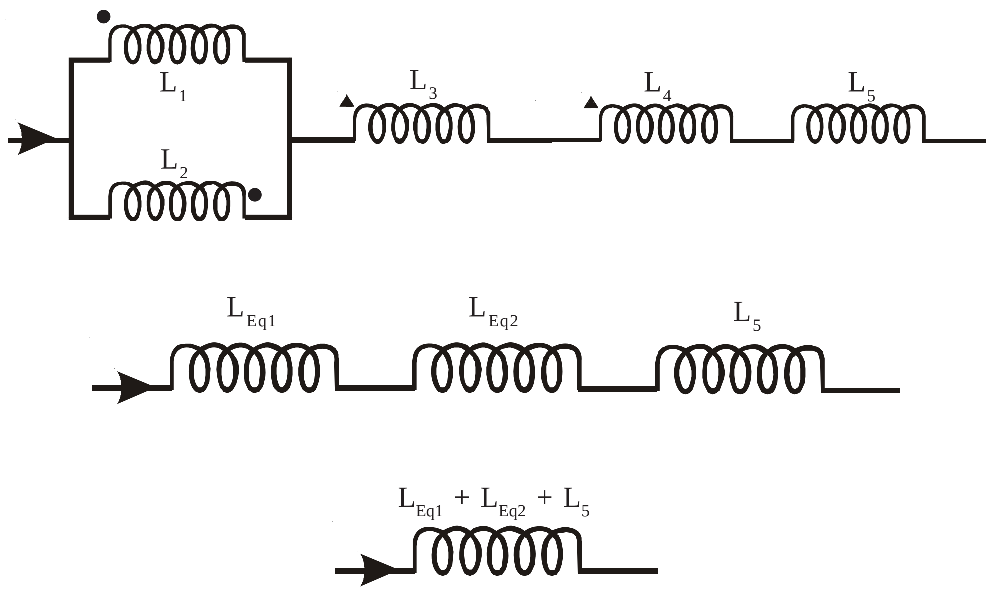


Fig. 1.39 *Inductance in Series-Parallel with Mutual Induction*

1.18.16 Variable Inductance

It is known that the inductance of a coil depends upon the flux linkages linking with that coil. If the core material is a magnetic material of high permeability then, the flux linking with the coil will be maximum and hence, the inductance (self or mutual) will be maximum. For that purpose, an iron core is used instead of air core for the coil. As we draw the core inside into the coil or away from the coil by means of linear motion or screwed motion by means of a dial or wheel, the inductance of the coil will increase or decrease accordingly. Now the inductance consists of two parts which are equivalent two partial coils, one with air core and the other with magnetic core. By varying the length of the magnetic core, the inductance is varied.

1.18.17 Magnetic Hum

The movement of the magnetic core or the vibrations of the laminated parts of the core of the electrical machinery, if they are not bound tightly due to the forces on them setup by the magnetic field, a humming noise will be produced. To reduce the noise the magnetic core should be fixed tightly.

1.19. FEATURES OF AN INDUCTANCE

1. Inductance is generally used in A.C. circuits and it reduces the current.
2. The current in a inductor lags the voltage by 90^o.
3. Inductance stores energy in the form of Electromagnetic Energy in the core. If the current in an inductive circuit like a coil, motor or fan is broken by opening a switch, the stored magnetic energy gets dissipated in the form of an arc across the switch. Hence special care has to be taken in opening of inductive circuits like opening of the circuit breakers of electric power circuits. The arc should be quenched by proper means if necessary.
4. For an ideal inductor there will be no Power Loss.
5. Inductance depend upon the dimensions, geometry and material of the magnetic core.
6. In the case of D.C. supply, the inductance initially acts as an Open Circuit and current builts up slowly and finally the inductance acts as Short Circuit allowing full current to flow through. If D.C. current is flowing through a conductor or a coil then, the magnetic field setup by the D.C. current will be steady and does not vary with time and hence no E.M.F. will be induced in it and it does not offer any inductance. If the resistance of the coil is less as is generally the case, the coil will get burnt away.
7. The current through an inductance cannot change all on a sudden. So, for a system with constant inductance the current through the inductance, before the change will be the same just immediately after the change like closing of a switch or opening of a switch. This principle is used in Pulse and Digital Circuits.
8. The Unit of inductance Henry is a large unit. Hence, inductance is expressed in milli-Henries.
9. Normally, the inductance should have very low resistance. But the coils used in electronic circuits because of their low current capacity and smaller area of cross-section will have fairly good resistance.
10. The core of an inductance can be an air core or iron core. For large inductances iron core is implied so that the inductance will be more.

11. The magnetic core of the inductance should be fixed very tightly otherwise there will be magnetic hum creating noise.
12. Inductance is setup by a coil when A.C. current passes through it. A coil will also have a resistance equal to the resistance of the wire constituting the coil. However, if this resistance is negligible then the coil exhibits only inductance and is called a pure inductance.
13. The principle of constant flux linkages states that the flux linkages cannot change instantaneously in a given system.
14. The inductance should have very less or negligible resistance or on the other hand, the coils used in electronic circuits will have more resistance since the area of cross-section of the conductor of the coil will be very small.
15. In electrical transmission lines, because of the time varying magnetic field setup by the A.C. currents passing through them, the transmission lines will have inductance and hence, inductive reactance even though the transmission conductors are stationary. The electric power lines will also exhibit inductance and since it is a long line the inductance will be distributed and is expressed as milli-Henries per kilometer length of line.
16. The Unit of inductance Henry is a large unit. Hence, inductance is expressed in milli-Henries.

1.20. USES OF INDUCTORS

1. Inductors are used in the form of Solenoids for operating Gate Valves to control the fluid flow etc.,
2. The inductance coils are used in Electromagnetic relays for opening and closing the control circuits.
3. The principle of Mutual Induction is used in Transformers in which Electrical Energy at certain voltage in one circuit can be transferred magnetically to another circuit at a different voltage with out actual connection between the two circuits. Transformers are used in Electric Power Transmission and Distribution.
4. Extra High Voltage (EHV) Lines will have large capacitance due to its long length and will cause higher receiving and voltage than the sending end voltage of the transmission line. It will also cause High Voltage Oscillations when the line is switched off. To overcome this, Large Inductors are used between the transmission lines and the Earth to compensate for the line capacitance.

5. Inductors can also be used in tuning. But since the inductance will be heavy, instead of inductors, capacitors are used to tune and select particular frequency signals as in Radio and TV., etc. The coils used in Electronic Circuits (Ferrite Rods) for tuning will have resistance also.
6. Inductors can also be used in Filter Circuits to select or reject certain frequencies.
7. Inductors can also be used in Wave Shaping Circuits to obtain a desired wave form from a given waveform
8. Because of its inertia, Inductors can be used in analog circuits, to solve for other systems like Mechanical Systems, Hydraulic System, etc., in terms of Electrical Systems
9. In ***Transducers***, which convert physical signals like displacement, etc., into electrical signals as in the case of Linear Variable Differential Transformer (LVDT).
10. Inductors can also be used in Power Factor Changing Circuits
11. Inductance can be used in Measurement Circuits.

* * * * * *

COMPREHENSION - 1

Note : By answering all the questions given in this comprehension one can answer any objective type of questions easily whatever be the form of the question. Answers for these questions can be obtained by going through the topics in this chapter carefully.

1. State the difference between Electrical and Electronic Currents.
2. State Coulomb's Law.
3. What is meant by Absolute Permittivity and Relative Permittivity? Give the value for Absolute Permitivity of Free Space or Vacuum.
4. What are the values of Relative Permitivity for Air and Mica?
5. Define the terms

   (a) Electric Field Intensity

   (b) Electric Flux Density

   (c) Potential

   (d) Potential Difference

   and mention their units.
6. Define the term Current and mention its unit.
7. What is Steady State Current?
8. What is the relationship between the Current and the Charge?
9. Define the unit of Current.
10. What is meant by Resistance Parameter? What are its Units?
11. How are Mega-Ohms, Kilo-Ohms, Milli-Ohms and Micro-Ohms denoted?
12. Explain the Color Code for Resistors used in Electronic Circuits.
13. What are the factors that affect a Resistance?
14. Given an expression for the Resistance?
15. Explain the different ways of connecting a Rheostat.
16. What will be the output in the case of a Potential Divider connection?
17. What is meant by Temperature Coefficient of Resistance?
18. Discuss the effect of Temperature on Resistance. Mention materials having negative temperature coefficients. What is meant by Negative Temperature Coefficient?

19. What is meant by Electrical Conductance? What are its Units?
20. Explain the differences between Ideal and Practical Sources of Voltage and Current.
21. What should be the nature of the Internal Resistance for
    (a) Voltage Source?
    (b) Current Source?
22. Distinguish between D.C. and A.C. Sources giving their waveforms with respect to time.
23. State and Explain Ohm's Law.
24. Explain the difference between Potential Rise and Potential Drop.
25. What is the difference between Linear and Non-Linear Resistances? Draw their *v*-*i* Characteristics. Give examples.
26. What is meant by Electrical Power? Give different forms of expression for Electrical Power. Give its Units.
27. Give expressions for Instantaneous Value of Power Loss and Power Supplied.
28. State and Explain Kirchoff's Voltage Law (KVL).
29. State and Explain Kirchoff's Current Law (KCL).
30. Give KVL and KCL in Matrix Form.
31. Explain the different methods of connecting Resistances.
32. Derive the Equivalent Series Resistance for a number of Resistances which are connected in Series.
33. Derive the Equivalent Parallel Resistance for a number of Resistances which are connected in Parallel.
34. Explain the nature of the Division of Currents in Parallel Circuits.
35. What are the Salient Features of Series and Parallel Connections of Resistances?
36. How do you reduce Series-Parallel Resistances into a Single Equivalent Resistance?
37. How are the Voltages and Currents measured in a circuit?
38. What should be the Resistance of a Voltmeter and an Ammeter? Why? How should they be connected in a circuit?
39. In D.C. circuits, What should be the nature & Polarities of the Voltmeter and Ammeter while connecting?

40. Explain Star and Delta Connection of Resistances.
41. Give the connection diagram for Mesh or Star connection using three and more than three resistances. How are their equivalent resistances obtained?
42. Give expressions for the Equivalent Star Resistances for a given Delta connected Resistances.
43. Give expressions for the Equivalent Delta Resistances for a given Star connected Resistances.
44. Explain the features of resistances.
45. Explain the uses of resistances.
46. Explain the differences between Electrical and Electronic Circuits.
47. What is meant by Energy Storing Elements? Name them.
48. What is the salient difference between the Resistance and Capacitance or Inductance Parameters?
49. Compare the Resistance Parameter and Capacitance or Inductance Parameter.
50. What is a Capacitance? What is its other name? What is its Unit?
51. Derive an expression for the value of a Parallel Plate Capacitance.
52. Obtain the relationship between the charge Q, the voltage V and the capacitance C.
53. Can a Capacitor pass DC current through it? Why?
54. What are the expressions for the current flowing through a Capacitance and the voltage?
55. Give an expression for the current through a capacitance if both voltage across the capacitance and the value of the capacitance change with respect to time.
56. Obtain the expression for the voltage across a capcitance including initial voltage.
57. What is meant by Elastance? What is its Unit?
58. Obtain different forms of expressions for the energy stored in a capacitance. What will be its nature?

59. Why is it that the voltage across a capacitance does not change suddenly?
60. What is the Electric Stress?
61. Why should a capacitor be discharged prior to its use?
62. Define a Farad.
63. What will be the nature of the capacitance of Electrical Transmission Lines?
64. What is a Guard Ring? Why should a Guard Ring be provided for a parallel plate capacitor?
65. Obtain an expression for the Effective Capacitance for capacitances connected in Series.
66. If a capacitance consists of a multiple dielectric media placed between the plates, what will be nature of the capacitance?
67. Obtain an expression for the energy stored in a capacitance.
68. Obtain an expression for the equivalent capacitance when the capacitors are connected in parallel.
69. Obtain an expression for the total capacitance in the case of series-parallel connection of capacitors.
70. Obtain an expression for the force on a dielectric slab placed between the plates of a capacitor.
71. What is a Gang Condensor? Where is it used?
72. Give a brief note on Dielectric Strength and Breakdown.
73. What is meant by electrostatic induction?
74. What is the principle of lightning arrestor used to prevent damage due to lightning stroke?
75. Enumerate the features of a Capacitance.
76. Enumerate the uses of a Capacitance.
77. What is meant by an Inductance? What is its Units?
78. What is the nature of the energy stored in an inductor?

79. What is the average power in an inductance?
80. State and Explain Maxwell's Right Hand Cork-Screw Rule. When is it used?
81. Draw the magnetic field lines in the case of a current carrying conductor with current flowing into the plane of the paper, with current flowing away from the plane of the paper indicating both the direction of current and the magnetic flux.
82. Draw the magnetic fields set up by a coil or a solenoid.
83. State and Explain Right Hand Grip Rule.
84. What is meant by Self Induced EMF in a coil?
85. Give expressions for the voltage across an inductance and the current through the inductance.
86. Obtain an expression for the self inductance of a coil.
87. Give an expression for the magnetisation force H.
88. Derive different forms of expressions for the self inductance of a coil.
89. Obtain different forms of expression for the energy stored in an inductance in terms of the current or the flux density B or magnetisation force H or both B and H.
90. Obtain an expression for the total inductance when two or more inductances are connected in series.
91. Obtain an expression for the total inductance when two or more inductances are connected in parallel.
92. Obtain an expression for the total inductance when many inductances are connected in series-parallel.
93. What is meant by Mutual Induction and Coefficient of Mutual Induction?
94. Explain the dot conventions for Mutual Inductance.
95. Obtain expressions for mutual inductance between two coils - M_{12} and M_{21}.
96. Obtain an expression for the total inductance when two inductances with mutual induction are connected in series.

97. Obtain an expression for the total inductance when two inductances with mutual induction are connected in parallel.
98. Explain the dot convention when more than two coils with mutual induction are present.
99. Obtain an expression for the total inductance when many inductances with mutual induction are connected in series-parallel.
100. Obtain an expression for mutual inductance in terms of the dimensions of two coils.
101. Derive an expression for the Coefficient of Magnetic Coupling between two coils.
102. State the Principle of Constant Flux Linkages.
103. Why is it that the current through an inductance does not change suddenly?
104. What is a Variable Inductance? Where is it used?
105. What is meant by Magnetic Hum? How do you prevent it?
106. Enumerate the features and uses of a Inductance.

* * * * * *

EXERCISE - 1

Note : By answering all the questions given in this exercise one can answer subjective type of questions easily including problems. The worked examples in the chapter may help in solving these questions.

1. (a) Distinguish between potential difference between two points and the potential gradient.

   (b) Find the potential difference and the potential gradient between two points A and B when the potential of point A is 30V and the potential of point B is –20V if they are separated by a distance of 2mts.

   [Ans: V_{AB} = 50Volts; E = 25V/mts]

2. (a) What is meant by electric current? What is its units?

   (b) If a charge of 25C passes a given point in a circuit in a time of 125ms, determine the current in the circuit.

   [Ans: I = 200A]

3. (a) What is meant by resistance? Give an expression for resistance in terms of its dimensions.

   (b) An aluminium wire 400mts long has a resistance of 0.25Ω. Find its area of cross-section. Find the area of cross-section required if the wire is of copper (specific resistances of copper and aluminium are 1.73 ´ 10^{-8} and 2.83 ´ 10^{-8}Ω-m)

   [Ans: a = 0.4528Sq-cm; 0.2768Sq-cm]

4. (a) What are the factors that effect a resistance and How?

   (b) Give the color code for the resistors used in electronic circuits.

   (c) A conductor of 10mts long and with 2mm^2 cross-sectional area has resistance of 0.4Ω. Calculate the specific resistance of the conductor material. What is its conductance?

   [Ans: $\rho = 0.8 \times 10^{-7}$ Ω-m; G = 2.5Siemens]

5. (a) What is the effect of temperature on resistance?

   (b) Define temperature coefficient of resistance.

   (c) The resistance of the armature of a D.C. generator is 0.92Ω at 25^{o}C. After working for 6Hrs on full load, the resistance increases to 1Ω. Calculate the temperature rise of the armature, if the temperature coefficient of resistance is 0.0040 at 0^{o}C.

   [Ans: 23.9^{o}C]

6. (a) How does the temperature coefficient of a resistance vary with respect to temperature?

   (b) What is meant by Negative Temperature Coefficient of Resistance? Mention some materials having Negative Temperature Coefficient of Resistance.

   (c) A coil has resistance of 20Ω when its mean temperature is 30°C and 24Ω when its mean temperature is 50°C. Find the temperature coefficient of resistance at 0°C. Also find the mean temperature rise when its resistance is 26Ω and the surrounding temperature is 20°C.

   [Ans: α_0 = 0.0143/°C; 40°C]

7. (a) What is meant by EMF of a source?

   (b) Distinguish between ideal and practical voltage sources drawing the V-I characteristics.

   (c) Distinguish between ideal and practical current sources drawing the V-I characteristics.

   (d) The internal resistance of a 12V battery is 0.9Ω. What will be its terminal voltage when the current drawn from the battery is 2A?

   [Ans: V = 10.2Volts]

8. (a) Distinguish between DC and AC sources of voltage and current drawing the waveforms with respect to time.

   (b) What is meant by Instantaneous Voltage and Instantaneous Current? How are they represented?

   (c) What is meant by Polarity of a source? When do you generally use it? Can you use the polarities for AC sources?

9. (a) State and explain Ohm's Law.

   (b) State Ohm's Law in Point Form.

   (c) A lamp takes 2.5A at 230V when switched on at room temperature (25°C). At the normal working temperature the current drops to 0.2A. What is the cold resistance and hot resistance of the filament at the operating temperature?

   [Ans: 92Ω;1150Ω]

10. (a) What is meant by electrical power and electrical energy?

    (b) Twenty lamps of 60W are used each for 4Hrs in a building. Calculate the current drawn when all the lamps are working and the monthly electricity charge at Rs. 1.20 per unit. Assume supply at 240V.

    [Ans: 0.25A; Rs. 172.8]

11. A factory has a 220V supply and has the following loads:

Lighting:	Five Hundred 100W and Five Hundred 60W lamps: 5Hrs per day.
Heating:	120KW: 10Hrs per day.
Motors:	48KW with average efficiency of 80%: 4Hrs per day.
Others:	15KW: 4Hrs per day

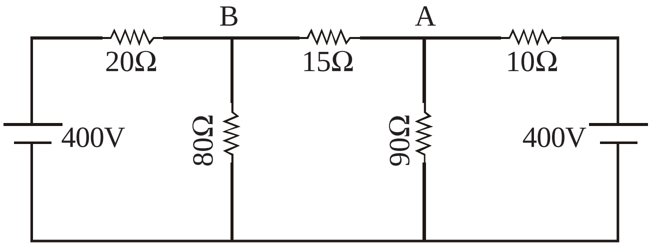
Assume 6 day week, calculate the daily and weekly consumption of the factory. Calculate the current drawn with only the lighting load, lighting and heating.

[Ans: 1900KWH; 11400KWH; 363.6A; 909A]

12. (a) Explain Kirchoff's Voltage Law (giving an example of a circuit with atleast two sources of different polarities and two or three resistances in series)

(b) Explain Kirchoff's Current Law (for a junction with two or three source currents of opposite directions and two or three load currents)

(c) Using Kirchoff's Laws, find the magnitude and direction of current flowing through the 15Ω resistance in the given circuit.



[Ans: 1A from A to B]

13. (a) Explain how the total resistance of a circuit having more than one resistance in series is calculated.

(b) A 60W, 240V lamp is connected in series with a 40W, 200V lamp across 250V supply. Calculate the current taken, voltage across each lamp and power given by each lamp. Assume that the resistance of the lamps remain constant.

[Ans: 0.12755A; 127.45V; 15.62W, 16.27W]

(c) A voltmeter has a resistance of 20,000Ω. When connected in series with an external resistance across a 230V supply, the instrument reads 160V. What is the value of external resistance?

[Ans: R = 8750Ω]

14. (a) Explain how the parallel equivalent resistance of a circuit having more than one resistance in parallel is calculated. Also explain in terms of conductances.

    (b) A resistor R is connected in series with a parallel circuit comprising of two resistances 12 and 8Ω respectively. The total power dissipated in the circuit is 96W when the applied voltage is 24V. Draw the circuit diagram and calculate the value of R.

[Ans: R = 1.2Ω]

15. (a) Explain the division of current in the parallel branches.

    (b) A circuit consists of three resistances of 12Ω, 18Ω and 36Ω respectively joined in parallel, and the parallel combination is connected in series with a fourth resistance R_4. The whole circuit is supplied at 60V and it is found that power dissipated in 12Ω resistance is 36W. Draw the circuit diagram and determine the value of fourth resistance R_4 and the total power dissipated in the circuit.

[Ans: R_4 = 11.32Ω; 207.84W]

16. Three loads A, B and C are connected in parallel to a 250V source. Load A takes 50A. Load B is a resistance of 10W and load C takes 6.25KW. Calculate

    (i) R_A and R_C (ii) I_B and I_C

    (iii) Power in Loads A and B (iv) Total Current

    (v) Total Power

    (vi) Total Effective Resistance

[Ans: R_A = 5Ω; R_C = 10Ω; I_B = I_C = 25A;

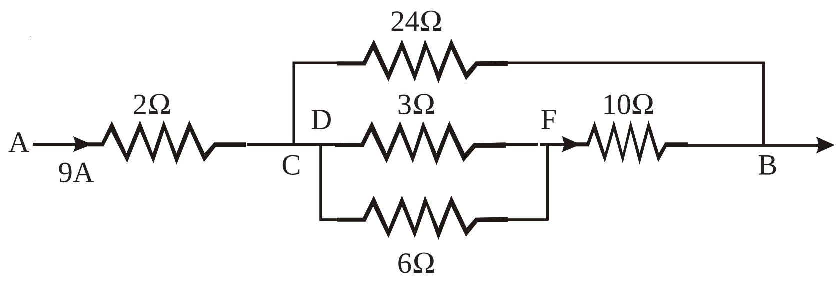
P = 25KW; R_{Eq} = 2.5Ω]

17. (a) How do you reduce the circuit and obtain the total resistance in the case of a circuit with series and parallel resistances?

    (b) A 10Ω resistor is connected in series with a group of two resistances of 15Ω each connected in parallel. Draw the circuit diagram and find out what resistance R must be shunted across this parallel combination so that the total current taken from 20V supply is 1.5A.

[Ans: R = 6Ω]

18. Determine the effective resistance between terminals A and B in the circuit of the given figure. If the current drawn at A is 9A, find the current in and the voltage drop across each element.



[Ans: 9A, 18V; 3A, 72V; 4A, 12V; 2A, 12V; 6A, 60V]

19. (a) Explain the Potential Divider connection of a Rheostat.

(b) A direct voltage of 240V is applied to a uniform 200Ω resistor connected between A and C. A load 40Ω resistance is connected between the terminal A and the tapping B (between the terminals A and C) Draw the circuit diagram and find the resistance AB in order that 1A flows in the 40Ω resistor.

[Ans: R = 71.65Ω]

20. (a) Explain how three resistances can be connected in Star and Delta.

(b) How do you extend Star-Mesh connection to more than three resistances?

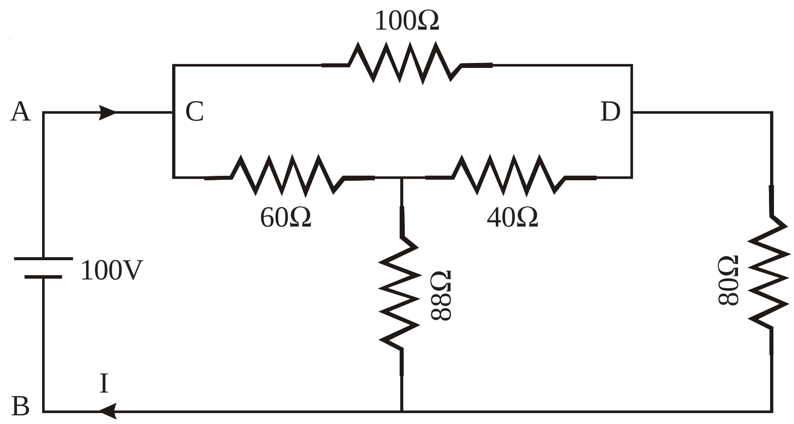
(c) Derive expressions for equivalent star resistances for given delta and equivalent delta resistances for given star.

(d) How do you extend the finding out the equivalent resistances in the case of Star-Mesh connections having more than three resistances?

21. (a) Enumerate the features and uses of resistance.

(b) Enumerate the differences between Electrical Circuits and Electronic Circuits.

22. In the given figure, 160V are applied to the terminals AB. Determine the resistance between the terminals A & B and the current.



[Ans: R_{AB} = 80Ω; I = 2A]

23. (a) What is a capacitor?

(b) A capacitor having a capacitance of 80μF is connected across a 500V DC supply. Calculate the charge.

[Ans: 40mC]

(c) A capacitor consists of two similar square aluminium plates, each 10cm × 10cm mounted parallel and opposite each other. What is their capacitance in μμF (pF or 10^{-12} Farads) when the distance between them is 1cm and the dielectric air? If the capacitor is given a charge of 500 μμC, What will be the difference of potential between the plates? How will be the difference of potential between plates? How will this be affected if the space between the plates is filled with wax which has a relative permitivity of 4?

[Ans: 8.854pF; 56.5V; 14.1V]

24. (a) A potential difference of 400V is maintained across a capacitor of value 25μF. Calculate

(i) the Charge

(ii) the Electric Field Strength

(iii) the Electric Flux Density

in the dielectric, if the distance between plates of capacitor is 0.5mm and area of the plates is 1.2m^2.

Given $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

[Ans: (i) 0.01C; (ii) 0.8 × 10^6V/m;
(iii) 0.0083C/m^2]

(b) Capacitance of a capacitor formed by two parallel metal plates each $100cm^2$ in area seperated by a dielectric 5mm thick is 20pF. A potential difference of 200V is applied. Calculate

(i) Total Charge on the plates
(ii) Potential Gradient in the dielectric medium
(iii) Electric Flux Density
(iv) Relative Permitivity of the dielectric.

[Ans: (i) 0.4μC; (ii) 4×10^4V/m; (iii) $4 \times 10^{-6}C/m^2$; (iv) 11.29]

25. (a) Derive an expression for the energy stored in a capacitance in terms of Capacitance and Voltage, Charge and Capacitance, Electric Field Intensity and Electric Flux Density

(b) Calculate the capacitance and energy stored in a parallel plate capacitor which consists of two metal plates each $60cm^2$ seperated by a dielectric of 1.5mm thickness and of $e_r = 3.5$ if a potential difference of 1000V is applied across it.

[Ans: 124pF; 62μJ]

26. (a) Find an expression for the effective capacitance when two or more capacitances are connected in series.

(b) In a parallel plate capacitor, the plates are spaced 6mm apart in air. Now two slabs of materials having dielectric constants 3 and 6 and thickness 2 and 4mm respectively are inserted between the plates. By what distance should one plate be further moved so as to restore the capacitance of the capacitor to its original value.

[Ans: 4.666mm]

27. (a) If two capacitors having capacitances of 6 and 10μF respectively are connected in series across a 200V supply. Find

(i) The Potential Difference across each capacitor
(ii) The charge on each capacitor

[Ans: (i) 125V, 75V; (ii) 750μC]

(b) Each plate in a parallel plate capacitor has area of $15cm^2$ and are seperated by 4mm thck sheet of mica of relative permitivity $\varepsilon_r = 6$. Calculate

(i) the capacitance of this capacitor
(ii) Next one plate is moved away permitting on air gap of 1mm between it and the mica sheet. Calculate the value of this capacitor.

(iii) If potential difference of 10V is maintaine across the plates, find the charge on the plates in two cases.

[Ans: (i) 19.92pF; (ii) 7.97pF; (iii) 79.7pC, 79.7pC]

28. (a) Deduce an expression for the equivalent capacitance when the capacitances are connected in parallel.
(b) Three capacitors have capacitances of 2, 4 and 8μF respectively are connected in
(i) Parallel
(ii) Series
Find the total capacitance in each case.

[Ans: (i) 14μF; (ii) 1.413μF]

29. Determine the capacitance of a capacitor consisting of two parallel metal plates 30cm × 30cm surface area, seperated by 5mm in air. What is the total energy stored by the capacitor if the capacitor is charged to a Potential Difference of 500V? What is the Energy Density?

[Ans: 159.2pF; 19.90 × 10^{-6}J; 4.427 × 10^{-6}J/m^3]

30. Deduce an expression for the joint capacitance of two capaictors C_1 and C_2
(i) in series
(ii) in parallel
If C_1 = 100μF, C_2 = 50μF, Calculate the joint capacitance and the total energy stored with a steady applied potential difference of 1000V in each case.

[Ans: C = 33.3μF; 150μF; 16.66J; 75J]

31. (a) Derive an expression for the self inductance of a coil in terms of its dimensions.
(b) A solenoid consists of 1500 turns of wire wound on a length of 60cm. A search coil of 500 turns enclosing a mean area of 20cm^2 is placed centrally in the solenoid. Find the Mutual Inductance.

[Ans: 3.14mH]

32. (a) Derive an expression for the energy stored in an inductance.
(b) A magnetic field is produced by a coil of 300 turns which is wound on a closed iron ring. The ring has a cross-section of 20cm^2 and mean length of 120cm. The permeability of the iron is 800. If the current in the coil is 10A. Find the energy stored in the magnetic field.

[Ans: 7.55J]

33. (a) Obtain an expression for the voltage drop across an inductance.
(b) An air cored toroidal coil has 450 turns and a mean diameter of 30cm and a cross-sectional area of 5cm^2. Calculate the inductance of the coil and the average induced EMF if a current of 4A is reversed in 60 milli-seconds.

[Ans: 135μH; 18mV]

34. (a) If the current through a coil having an inductance of 0.5H is reduced from 5A to 2A in 0.05sec, calculate the mean value of the EMF induced in the coil

[Ans: –30V]

(b) A coil of 300 turns, wound on a core of non-magnetic material, has an inductance of 10mH. Calculate
(i) the flux produced by current of 5A
(ii) the average value of the EMF induced when a current of 5A is reversed in 8milli-sec.

[Ans: (i) 167μWb; (ii) 12.5V]

35. An iron rod 2cm in diameter and 20cm long is bent into a closed ring and is wound with 3000 turns of wire. It is found that when a current of 0.5A is passed through this coil, the flux density in the coil is 0.5Wb/m^2. Assuming that all the flux links with every turn of the coil, what is the B-H ratio for the iron and what is the inductance of the coil. What voltage would be developed across the coil if the current through the coil is interrupted and the flux in the iron falls to 10% of its former value in 0.001second.

[Ans: 6.67×10^{-5}H/m; 0.94H; 424V]

36. (a) Define Mutual Inductance.
(b) Explain dot convention for mutual inductance between two coils and also for three coils.
(c) Derive an expression for coefficient of mutual coupling.
(d) Two identical coils X and Y of 1000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5A flowing in X produces a flux of 0.05mWb in it, find the mutual inductance between X and Y.

[Ans: 8×10^{-3}H]

37. (a) Obtain expressions for the effective inductance of two mutually coupled inductances connected in
(i) series (ii) parallel

(b) When two coils are connected in series, their effective inductance is found to 10.0H. However, when the connections to one coil are reversed, the effective inductance is 6.0H. If the coefficient of coupling is 0.6, calculate the self-inductance of each coil and the mutual inductance.

[Ans: L_1 = 7.63H or 0.37H; L_2 = 0.37H or 7.63H; M = 1.0H]

38. Two coils A of 5000 turns and B of 3000 turns lie in parallel planes. A current of 6A in coil A produces a flux of 0.1mWb. If 60% of the flux produced by coil A links with the turns of coil B, calculate the EMF induced in coil B when the current in coil A changes from 5A to –5A in 0.01second.

[Ans: 30V]

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