CHAPTER 1

SET THEORY

Introduction

In this chapter we will understand the various concepts in relation to set theory from the basic understanding, what is a set? In daily life we often used the word set in various situations, but in mathematical terminology it represents a group or a collection of books, toys or numbers etc., these objects which are the part of the set are called as elements of the set. Some of the key contributors are Georg Cantor (1845-1918) from whose work the modern set theory was largely originated and other prominent contributors are Ernst Zermelo, Abraham Fraenkel and John Von Neumann.

Definition of Set

A set is a well defined collection of objects and these objects are termed as the members or elements of the set. But we need to elaborate on the term "well-defined" it means that each element bears certain characteristics with which it can be identified under a particular head.

For example

(i) The set {a, e, i} belongs to family of vowels

(ii) The set {1, 2, 3, 4} belongs to natural numbers (N).

Set Notation

The two most common way of expressing a set are:

(i) ***Roster, Tabular or Enumeration Form***

In this method all the elements are listed within braces { } or brackets [ ] or parentheses ( ) separated by commas.

For example

All natural numbers less than six can be written as {1, 2, 3, 4, 5}

(ii) ***Set builder Form***

In this method all the elements are listed according to characteristics or properties.

For example

(i) The Set A = {a, e, i, o, u} can be written as

A = {x | x is vowel in English alphabet} or

A = {x : x is vowel in English alphabet}

(ii) The Set B = {1, 3, 5, 7……….} can be written as B = {x | x is a odd number integer}

Note: A Colon (:), a vertical line (|) or a semi colon (;) can be used after x and read as "such that"

Types of Set

(i) ***Finite set***

A set in countable form is a finite set and it means each element can be counted physically.

For example

Set A = {1, 2, 3, 4, 5} is a finite set with five elements.

(ii) ***Infinite set***

A set in uncountable form is an infinite set. The elements in the set cannot be counted.

For example

The set of natural numbers (N)

A = {1, 2, 3,… }

(iii) ***Null, empty or void set***

A set which has no elements is called as null, empty or void set denoted by {Ø} and read as phi in Greek and Latin.

For example

Set A = {Ø} is a null set.

(iv) ***Singleton set***

A set containing only one element is called as singleton set.

For example

Set A = {1}

(v) ***Equality of sets***

Two sets A and B are termed equal if only if every element of Set A is also an element of Set B and also every element of Set B is an element of Set A i.e. A = B if only if {x ∈ A and x ∈ B}. Also called as Axiom of Extension or Axiom of Identity

For example

If Set A = {2, 3, 4} and Set B = {4, 3, 2}. Here Set A = Set B because both sets have common and equal numbers of elements.

(vi) ***Equivalent set***

If the elements of one set can be set to one-one correspondence with the elements of other set, then the sets are called equivalent set denoted by the symbol ~. Here we mean by one–one correspondence is that each element in Set A can be matched with one element in B and vice-versa.

For example

Set A = {a, b, c} and Set B = {x, y, z} , here set A is not equal to set B but the elements of Set A can be put into one–one correspondence with each elements of Set B then Set A ~ Set B.

(vii) ***Subset***

If A and B are two sets such that every element of Set A is also an element of Set B then Set A is said to be a subset of Set B or read as "Set A is contained in Set B" or "Set A is a subset of Set B"

We can write this relationship as A ⊆ B or B ⊇ A, i.e. it means if x ϵ A and A ⊆ B then x ∈ B

For example

Set A = {1, 2, 3} and Set B = {1, 2, 3, 4, 5} then all elements of Set A are also elements of Set B which means A ⊆ B

Properties of Subsets

(i) If Set A is subset of Set B then Set B is called the super set of Set A.

(ii) If Set A ⊆ Set B and Set B ⊆ Set A then Set A = Set B.

(iii) If Set A ⊆ Set B, Set B ⊆ Set C then Set A ⊆ Set C

(viii) ***Proper Subset***

A Set A is called proper subset of a Set B, if each and every element of Set A are contained in Set B and there exists at least one element in Set B such that it is not an element of Set A. Symbolically denoted as $A \subset B$

For example

If Set A = {1, 2, 3, 4, 5} and Set B = {1, 2, 3, 4, 5, 6, 7} here Set A is a proper subset of Set B.

(ix) ***Comparable Sets***

Two set A and B are said to be comparable if one of them is the subset of the other i.e. $A \subseteq B$ or $B \subseteq A$.

(x) ***Non Comparable Sets***

Two set A and B are said to be non-comparable, if there exists and one element in Set A which is not in Set B and one element in Set B which is not in Set A .

Symbolically denoted by $A \not\subset B$ and $B \not\subset A$.

For example

Set A = {1, 2, 3} and Set B = {3, 4, 5} then both the sets are said to be non-comparable.

(xi) ***Disjoint sets***

Set A and Set B are said to be disjoint. If no element of set A is in B and no element of set B is in A.

For example

If Set A = {1, 2, 3} and Set B = {4, 5, 6}, then there is no common elements between Set A and Set B.

(xii) ***Set of sets or family of sets***

If the elements of a set are sets themselves then it is called set of sets or family of sets.

For example

If Set A = {1, 2} then,

The Set = {Ø, {1}, {2}, {1, 2}} is a family of sets whose elements are subset of A.

(xiii) ***Power set***

If for a given Set A, a set consisting of all the subsets of A is called the power of the set. The power set this is denoted by P(A).

For example

If Set A = {1, 2} then P (A) = {Ø}, {1}, {2}, {1, 2}.

***Note**: Suppose a set contains 'n' elements then 2^n subsets can be formed. The set consisting of these 2^n subsets is called power set.*

(xiv) ***Universal Set***

If each set is a subset of some other set. A set which is superset of all the sets under consideration is called as Universal set and is denoted by U.

For example

If Set A = {1, 2, 3, 4} and set B = {5, 6, 7} then both sets A and B are subsets of the universal set of natural number.

Union of Set (Set Operation)

If A and B are two sets then the union of Set A and Set B is the set consisting of either all the elements of Set A or Set B or both and denoted by "A ∪ B" and read as A union B or A cup B

i.e. $A \cup B = \{x : x \in A \text{ or } x \in B \text{ or } x \in \text{ both A and B}\}$

For example

If Set A = {1, 2, 3, 4} and Set B = {4, 5, 6, 7} then

A ∪ B = {1, 2, 3, 4, 5, 6, 7}

And hence the union of two Sets A and B is the logical sum of A and B where each element is written only once.

Properties of Union of Sets

(i) If Set A and Set B are two sets then A ∪ B is also a unique set

For example

If Set A = {1, 2, 3} and Set B = {3, 4, 5}

Therefore, A ∪ B = {1, 2, 3, 4, 5} is also a set.

(ii) ***Commutative Property***

Union of set is commutative i.e. if Set A and Set B are two sets then A ∪ B = B ∪ A

For example

If Set A = {1, 2, 3} and Set B = {3, 4, 5}, then

A ∪ B = {1, 2, 3, 4, 5}.

Similarly

B ∪ A = {3, 4, 5, 1, 2}

$= \{1, 2, 3, 4, 5\}$

$= A \cup B$

Clearly $A \cup B = B \cup A$

Alternative Proof

Union of sets is Communicative i.e. if Set A and Set B are two sets then $A \cup B = B \cup A$.

In order to prove $A \cup B = B \cup A$,

We first prove that $A \cup B \subseteq B \cup A$ and then $B \cup A \subseteq A \cup B$, if both of them holds good then $A \cup B = B \cup A$.

Let x be any element belongs to $A \cup B$.

So $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

So $A \cup B \subseteq B \cup A$ .... (1)

Again let's assume y be an element such that

$y \in B \cup A \Rightarrow y \in B$ or $y \in A$

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in A \cup B$

So $B \cup A \subseteq A \cup B$ .... (2)

From (1) and (2) we get $A \cup B \subseteq B \cup A$ and $B \cup A \subseteq A \cup B$, therefore we get $A \cup B = B \cup A$.

(iii) Union of sets is associative i.e. If Set A, Set B and Set C are three sets then $A \cup (B \cup C) = (A \cup B) \cup C$

For example

If Set A = {1, 2, 3}, Set B = {3, 4, 5} and Set C = {5, 6}, then

$(A \cup B) = \{1, 2, 3, 4, 5\}$

Therefore

$(A \cup B) \cup C = \{1, 2, 3, 4, 5\} \cup \{5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

Similarly,

$(B \cup C) = \{3, 4, 5, 6\}$

Therefore,

$A \cup (B \cup C) = \{1, 2, 3\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

and hence,

$A \cup (B \cup C) = (A \cup B) \cup C$

Alternate Proof

If Set A, Set B and Set C are three sets then

$A \cup (B \cup C) = (A \cup B) \cup C$

Let x be an element which belongs to $A \cup (B \cup C)$

Then $x \in A \cup (B \cup C)$,

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A$ or ($x \in B$ or $x \in C$)

$\Rightarrow$ ($x \in A$ or $x \in B$) or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

Thus, $A \cup (B \cup C) = (A \cup B) \cup C$.

(iv) If Set A is a set, then $A \cup \emptyset = A$ where $\emptyset$ is a null set

For example

If Set A = {1, 2, 3}, then $A \cup \emptyset = \{1, 2, 3\} \cup \{\emptyset\}$

$= \{1, 2, 3\}$

$= A$

Alternate Proof

If Set A is any set then $A \cup \emptyset = A$

Let x be any element such that $x \in A \cup \emptyset$ then $x \in A$ or $x \in \emptyset$ but $\{ \emptyset \}$ being null set therefore $x \notin \{\emptyset\}$, thus $x \in A$.

Hence $A \cup \emptyset = A$

(v) *Union of sets is idempotent*

If Set A is any set then $A \cup A = A$.

For example

If A = {1, 2, 3} then $A \cup A = \{1, 2, 3\} \cup \{1, 2, 3\}$

$= \{1, 2, 3\}$

$= A$

Alternate Proof

If Set A is any set then $A \cup A = A$

Let x be any element so that $x \in A \cup A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A$ thus $A \cup A = A$

(vi) If Set A is a subset of universal Set ∪ then A ∪ ∪ = ∪

For example

If Set ∪ = {1, 2, 3, 4, 5, 6, 7} and Set A = {1, 2, 3}

A ∪ ∪ = {1, 2, 3} ∪ {1, 2, 3, 4, 5, 6, 7}

= {1, 2, 3, 4, 5, 6, 7}

= ∪.

(vii) If Set A and Set B are two sets such that $A \subseteq B$ then $A \cup B = B$ and if $B \subseteq A$ then $A \cup B = A$

For example

If Set A = {1, 2, 3} and Set B = {1, 2, 3, 4, 5}

Then $A \subseteq B$

$A \cup B = \{1, 2, 3\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5\} = B$

Similarly If A = {1, 2, 3, 4, 5}

B = {1, 2, 3}

In this case $B \subseteq A$

$A \cup B = \{1, 2, 3, 4, 5\} \cup \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 4, 5\} = A$

Alternate Proof

If A and B are two sets then $A \subseteq A \cup B$ and $B \subseteq A \cup B$

let x be any element of the Set A such that

$x \in A \Leftrightarrow x \in A$ or $x \in B$

$x \in (A \cup B) \Leftrightarrow A \subseteq A \cup B$

Similarly $B \subseteq A \cup B$ can be proved.

Intersection of Sets

Let A and B are two sets then intersection set of Set A and Set B is the set which consist of common elements which belongs to both A and B denoted by A ∩ B and read as " A Cap B " or "A intersection B". Symbolically represented as

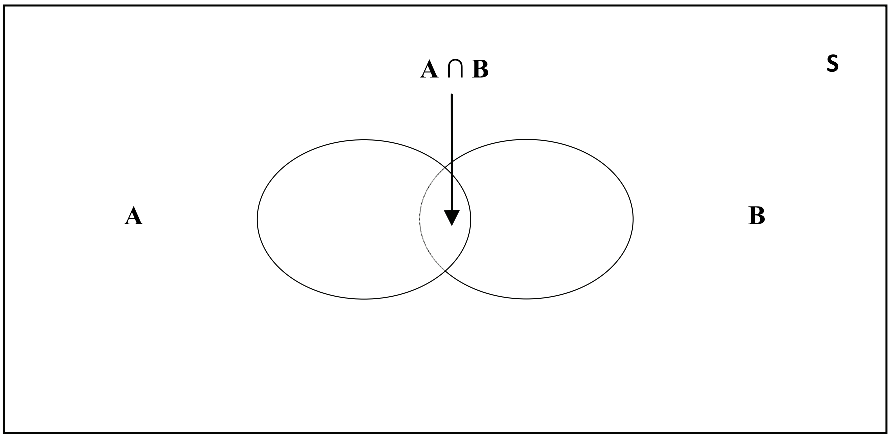
$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Otherwise if $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

For example

If Set A = {1, 2, 3} and Set B = {3, 4, 5} then A ∩ B = {3}

The Venn diagram representation is shown below as:



Properties of Intersection of Sets

The following are the properties which hold with respect to intersection of sets

(i) ***Communicative Property***

Intersection of sets is communicative i.e. if Set A and Set B are two sets then A ∩ B = B ∩ A

For example

If Set A = {1, 2, 3} and Set B = {3, 4, 5}

then A ∩ B = {1, 2, 3} ∩ {3, 4, 5}

= {3}.

Similarly,

B ∩ A = {3, 4, 5} ∩ {1, 2, 3}

= {3}.

Therefore A ∩ B = B ∩ A.

Alternative Proof

Let x ∈ A ∩ B then x ∈ A and x ∈ B ⇔ x ∈ B and x ∈ A i.e. x ∈ B ∩ A.

Hence A ∩ B = B ∩ A.

(ii) ***Associative Property***

The intersection of sets are associative i.e. if Set A, Set B and Set C are three sets then (A ∩ B) ∩ C = A ∩ (B ∩ C).

For example

If Set A = {1, 2, 3, 4}, Set B = {3, 4, 5} and Set C = {4, 5, 6}

Then A ∩ B = {1, 2, 3, 4} ∩ {3, 4, 5}

= {3, 4}.

Hence (A ∩ B) ∩ C = {3, 4} ∩ {4, 5, 6}

= {4}.

Similarly,

B ∩ C = {3, 4, 5} ∩ {4, 5, 6}

= {4, 5}.

Hence,

A ∩ (B ∩ C) = {1, 2, 3, 4} ∩ {4, 5}

= {4}

Therefore (A ∩ B) ∩ C = A ∩ (B ∩ C)

***Alternative Proof*:**

Let x ∈ (A ∩ B) ∩ C ⇔ then x ∈ (A ∩ B) and x ∈ C

⇔ (x ∈ A and x ∈ B) and x ∈ C

⇔ x ∈ A and (x ∈ B and x ∈ C).

⇔ x ∈ A ∩ (B ∩ C).

Hence (A ∩ B) ∩ C = A ∩ (B ∩ C).

(iii) ***Idempotent Property***

The intersection of sets is idempotent i.e. if Set A is any set, then A ∩ A = A

For example

If Set A = {1, 2, 3, 4} then

$A \cap A = \{1, 2, 3, 4\} \cap \{1, 2, 3, 4\}$

$= \{1, 2, 3, 4\}$

$= A$

Alternative Proof

Let x be any element such that $x \in A \cap A$, then $\Leftrightarrow x \in A$ and $x \in A$

$\Leftrightarrow x \in A$

Hence $A \cap A = A$.

(iv) If Set A is any set then $A \cap \emptyset = \emptyset$, $\emptyset$ is the null set

For example

If Set A = {1, 2, 3, 4} and $\emptyset = \{ \}$ then,

$A \cap \emptyset = \{1, 2, 3, 4\} \cap \{ \}$

$= \{ \}$

$= \emptyset$.

(v) If Set A is any set subset of an Universal Set U then $A \cap U = A$

For example

If Set U = {1, 2, 3, 4, 5, 6, 7} and Set A = {1, 2, 3, 4}

then $A \cap U = \{1, 2, 3 ,4, 5, 6, 7\} \cap \{1, 2, 3, 4\}$

$= \{1, 2, 3, 4\}$

$= A$

Alternative Proof

Let x be any element such that $x \in A \Rightarrow x \in A$ and $x \in U$ since $A \subseteq U$

Therefore $x \in A \cap U$,

And hence $A \subseteq A \cap U$ …………………1

But $A \cap U \subseteq A$ … ……………………2

From 1 and 2, we get,

$A = A \cap U$.

(vi) If A and B are disjoint sets then $A \cap B = \varnothing$

For example

if Set A = {1, 2, 3, 4} and Set B = {5, 6, 7}

then $A \cap B = \{1, 2, 3, 4\} \cap \{5, 6, 7\}$

$= \{\varnothing\}$

(vii) If Set A and Set B are two sets then $A \cap B \subseteq A$ and $A \cap B \subseteq B$

As $A \cap B$ contains only those elements which are in common A as well as in B. Therefore $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

For example

If Set A = {1, 2, 3, 4} and Set B = {3, 4, 5, 6}

$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$

$= \{3, 4\}$ which is a subset of Set A and Set B

Alternative Proof

Let x be any element such that $x \in A \cap B$ then $x \in A$ and $x \in B$

$\Leftrightarrow$ $x \in A$ as $A \cap B \subseteq A$. Similarly the other part $A \cap B \subseteq B$ can also be proved in similar manner.

Distributive Laws of Unions and Intersections

Result 1

If Set A, Set B and Set C are three sets, then

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

If Set A = {1, 2, 3, 4}, Set B = {3, 4, 5} and Set C = {4, 5, 6}

Then

$(B \cup C) = \{3, 4, 5\} \cup \{4, 5, 6\}$

$= \{3, 4, 5, 6\}$.

$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$

$= \{3, 4\}$

Similarly,

$(A \cap B) = \{1, 2, 3, 4\} \cap \{3, 4, 5\}$

$= \{3, 4\}$

and

$(A \cap C) = \{1, 2, 3, 4\} \cap \{4, 5, 6\}$

$= \{4\}$

Thus,

$(A \cap B) \cup (A \cap C) = \{3, 4\} \cup \{4\}$

$= \{3, 4\}$

Therefore $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Alternative Proof

Let x be any element belonging to $A \cap (B \cup C)$, then,

$x \in A \cap (B \cup C) \Leftrightarrow x \in A$ and $x \in (B \cup C)$

$\Leftrightarrow x \in A$ and ($x \in B$ or $x \in C$)

$\Leftrightarrow$ ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

$\Leftrightarrow x \in (A \cap B)$ or $x \in (A \cap C)$

$\Leftrightarrow x \in (A \cap B) \cup (A \cap C)$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Result 2

If Set A, Set B and Set C are three sets, then

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

If Set A = {1, 2, 3, 4}, Set B = {3, 4, 5} and Set C = {4, 5, 6}

Then

$(B \cap C) = \{3, 4, 5\} \cap \{4, 5, 6\}$

$= \{4, 5\}$

and

$A \cup (B \cap C) = \{1, 2, 3, 4\} \cup \{4, 5\}$

$= \{1, 2, 3, 4, 5\}$

Similarly

$(A \cup B) = \{1, 2, 3, 4\} \cup \{3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$

$(A \cup C) = \{1, 2, 3, 4\} \cup \{4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

Then $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5\}$

Therefore $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Alternative Proof

Let x be any element belonging to $A \cup (B \cap C)$, then,

$x \in A \cup (B \cap C) \quad \Leftrightarrow x \in A$ or $x \in (B \cap C)$

$\Leftrightarrow x \in A$ or ($x \in B$ and $x \in C$)

$\Leftrightarrow$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$\Leftrightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Leftrightarrow x \in (A \cup B) \cap (A \cup C)$

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Complement of a Set

The complement of a Set A is that set which contains all those elements of the universal set U which are not in A. The complement of set A is the set U – A and is denoted by A^c, A', $\overline{A}$ or ~ A. It can symbolically written as

$A' = U - A = \{ x: x \in U \text{ and } x \notin A \}$

For example

If Set U = {1, 2, 3, 4, 5, 6, 7, 8} and the Set A = {1, 2, 3, 4, 5} then

$A' = U - A = \{6, 7, 8\}$.

Properties of the Complement of Set

(i) The intersection of Set A and its complement A' are disjoint sets i.e. $A \cap A'$ is a null set {Ø}

For example

If the Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and Set A = {1, 2, 3, 4, 5} then

$A' = U - A = \{6, 7, 8, 9\}$

Therefore $A \cap A' = \{1, 2, 3, 4, 5\} \cap \{6, 7, 8, 9\}$

$= \{Ø\}$

Alternative Proof

Let x be an element such that

$x \in A \cap A' \Rightarrow x \in A$ and $x \in A' \Rightarrow x \in \{Ø\}$

(ii) The union of Set A and its complement is the universal set i.e. $A \cup A' = U$, the universal set.

For example

If the Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and Set A = {1, 2, 3, 4, 5} then

$A' = U - A = \{6, 7, 8, 9\}$

Therefore $A \cup A' = \{1, 2, 3, 4, 5\} \cup \{6, 7, 8, 9\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

= Set U.

Alternative Proof

As every set is the subset of the universal set, therefore $A \cup A' \subseteq U$. Let $x \in U$ that implies $x \in A$ or $x \in A' \Rightarrow x \in A \cup A'$. Therefore $U \subseteq A \cup A'$. Hence $A \cup A' = U$.

(iii) Complement of complement a Set A is the set itself i.e. $(A')' = A$

For example

If the Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and Set A = {1, 2, 3, 4, 5} then

$A' = U - A = \{6, 7, 8, 9\}$

Therefore $(A')' = U - A' = \{1, 2, 3, 4, 5\}$ = Set A

Alternative Proof

Let $x \in (A')' \Leftrightarrow x \notin A' \Leftrightarrow x \in A$.

(iv) If the Set A is equal to the universal Set U then $A' = \{\emptyset\}$.

(v) If Set A and Set B are two sets then $A - B = A \cap B'$

For example

If the Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9} , Set A = {1, 2, 3, 4, 5} and Set B = {4, 5, 6, 7} then

$A - B = \{1, 2, 3, 4, 5\} - \{4, 5, 6, 7\}$

$= \{1, 2, 3\}$.

But $B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{4, 5, 6, 7\}$

$= \{1, 2, 3, 8, 9\}$

Thus $A \cap B' = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 8, 9\}$

$= \{1, 2, 3\}$

$= A - B$

Alternate Proof

Let $x \in A - B \Rightarrow x \in A$ and $x \notin B \Leftrightarrow x \in A$ and $x \in B' \Leftrightarrow x \in (A \cap B')$ and hence $A - B = A \cap B'$.

(vi) If $A \subseteq B$ then $A \cup (B - A) = B$

For example

If the Set B = {1, 2, 3, 4, 5, 6, 7, 8, 9} and Set A = {1, 2, 3, 4, 5}

B – A = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {1, 2, 3, 4, 5}

= {6, 7, 8, 9}.

Therefore A ∪ (B – A) = {1, 2, 3, 4, 5} ∪ {6, 7, 8, 9}.

= {1, 2, 3, 4, 5, 6, 7, 8, 9}

= B

Alternate Proof

As given $A \subseteq B$ let $x \in A \cup (B - A) \Leftrightarrow x \in A$ or $x \in (B - A)$

$\Leftrightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in A')$

$\Leftrightarrow x \in (A \cup B)$ and $x \in B$ as $(A \cup A' = B$ as $A \subseteq B)$

$\Leftrightarrow x \in (A \cup B)$

$\Leftrightarrow x \in B$, hence $A \cup (B - A) = B$

Difference of Sets

Let Set A and Set B are two sets then the difference of Set A and B is the set which consist of those elements which belongs to A but does not belong to B denoted by A – B and read as "A difference B "or A minus B" and also denoted by A ~ B. The symbolical representation is

$A - B = (x : x \in A$ and $x \notin B)$ similarly

$B - A = (x : x \in B$ and $x \notin A)$.

For example

If Set A = {1, 2, 3, 4}, Set B = {3, 4, 5} then

A – B = {1, 2, 3, 4} – {3, 4, 5}

= {1, 2}.

Similarly

B – A = {3, 4, 5} – {1, 2, 3, 4}

= {5}

Properties of Difference of Sets

(i) A – A = Ø.

For example

If Set A = {1, 2, 3, 4, 5) then

A – A = {1, 2, 3, 4, 5} – {1, 2, 3, 4, 5}

= Ø

Alternate Proof

Let x ∈ A – A then x ∈ A and x ∉ A but there is no such element satisfying both the conditions and hence, there is no element belonging to A – A i.e A – A = Ø.

(ii) A – Ø = A.

For example

If Set A = {1, 2, 3, 4, 5} and Ø be a null set then

A – Ø = {1, 2, 3, 4, 5} – {Ø}.

= {1, 2, 3, 4, 5} =A, hence A – Ø = A.

Alternate Proof

Let x ∈ A – Ø then x ∈ A and x ∉ Ø which mean x belongs to A, since there is no element belonging to Ø.

Conversely, x ∈ A then x ∈ A – Ø i.e. A – Ø = A.

(iii) A – B, A ∩ B and B – A are mutually disjoint.

For example

If Set A = {1, 2, 3, 4, 5} and Set B = {4, 5, 6, 7} then

A – B = {1, 2, 3, 4, 5} – {4, 5, 6, 7}

= {1, 2, 3}

A ∩ B = {1, 2, 3, 4, 5} ∩ {4, 5, 6, 7}

= {4, 5}

and

B – A = {4, 5, 6, 7} – {1, 2, 3, 4, 5}

= {6, 7}

Alternate Proof

We need to prove that $(A - B) \cap (A \cap B) = \varnothing$

Let $x \in (A - B) \cap (A \cap B)$ then $x \in (A - B)$ and $x \in (A \cap B)$

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) and ($x \in A$ and $x \in B$)

$\Leftrightarrow$ $x \in A$ and $x \in \varnothing$ as there cannot be an element satisfying both the condition i.e. $x \in B$ and $x \notin B$)

$\Leftrightarrow$ $x \in \varnothing$.

Similarly other results can be equally proved in the same lines.

(iv) $(A - B) \cup A = A$

For example

If Set A = {1, 2, 3, 4, 5} and Set B = {4, 5, 6, 7} then

$A - B = \{1, 2, 3, 4, 5\} - \{4, 5, 6, 7\}$

$= \{1, 2, 3\}$.

Therefore,

$(A - B) \cup A = \{1, 2, 3\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$.

Alternate Proof

Let x belongs to $(A - B) \cup A$ then $x \in (A - B)$ or $x \in A$

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) or $x \in A$

$\Leftrightarrow$ $x \in A$ and $x \notin B$

$\Leftrightarrow$ $x \in A$

Hence, $(A - B) \cup A = A$

(v) $(A - B) \cap B = \varnothing$

For example

If Set A = {1, 2, 3, 4, 5} and Set B = {4, 5, 6, 7} then

$A - B = \{1, 2, 3\} = \{1, 2, 3, 4, 5\} - \{4, 5, 6, 7\}$

$= \{1, 2, 3\}$.

Therefore,

$(A - B) \cap B = \{1, 2, 3\} \cap \{4, 5, 6, 7\}$.

$= \varnothing$

Alternate Proof

Let x belongs to $(A - B) \cap B$ then $x \in (A - B)$ and $x \in B$

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) and $x \in B$

Note there cannot be element satisfying both the condition i.e. $x \in B$ and $x \notin B$)

$\Leftrightarrow$ $x \in A$ and $x \in \emptyset$

$\Leftrightarrow$ $x \in \emptyset$, hence $(A - B) \cap B = \emptyset$

De Morgan's Law

1st Law

Let Set A and Set B are two sets then $(A \cup B)' = A' \cap B'$

For example

Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

Set A = {1, 2, 3, 4} and

Set B = {6, 7, 8, 9}

Then

$(A \cup B) = \{1, 2, 3, 4\} \cup \{6, 7, 8, 9\}$

$= \{1, 2, 3, 4, 6, 7, 8, 9\}$

Now,

$(A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 6, 7, 8, 9\}$

$= \{5\}$.

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$

$= \{5, 6, 7, 8, 9\}$

$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{6, 7, 8, 9\}$.

$= \{1, 2, 3, 4, 5\}$

Now,

$A' \cap B' = \{5, 6, 7, 8, 9\} \cap \{1, 2, 3, 4, 5\}$

$= \{5\}$

Therefore $(A \cup B)' = A' \cap B'$

Alternate Proof

$(A \cup B)' = A' \cap B'$

Let x be any element of $(A \cup B)'$,

Then $x \in (A \cup B)' \Leftrightarrow x \in U$ and $x \notin (A \cup B)$

$\Leftrightarrow x \in U$ and $(x \notin A$ or $x \notin B)$

$\Leftrightarrow (x \in U$ but $x \notin A)$ and $(x \in U$ but $x \notin B)$

$\Leftrightarrow x \in A'$ and $x \in B'$

$\Leftrightarrow x \in A' \cap B'$

Hence $(A \cup B)' = A' \cap B'$

2nd Law

Let Set A and Set B are two sets then $(A \cap B)' = A' \cup B'$

For example

Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

Set A = {1, 2, 3, 4} and

Set B = {6, 7, 8, 9}

Then

$(A \cap B) = \{1, 2, 3, 4\} \cap \{6, 7, 8, 9\}$

$= \{\emptyset\}$

Now,

$(A \cap B)' = U - (A \cap B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{\emptyset\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$

$= \{5, 6, 7, 8, 9\}$

$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{6, 7, 8, 9\}$.

$= \{1, 2, 3, 4, 5\}$

Now,

$A' \cup B' = \{5, 6, 7, 8, 9\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Therefore $(A \cap B)' = A' \cup B'$

Alternate Proof

$(A \cap B)' = A' \cup B'$

To prove the above result, let x be any element belonging to $(A \cap B)'$ then

$$x \in (A \cap B)' \Leftrightarrow x \notin (A \cap B)$$
$$\Leftrightarrow x \notin A \text{ or } x \notin B$$
$$\Leftrightarrow x \in A' \text{ or } x \in B'$$
$$\Leftrightarrow x \in A' \cup B'.$$

Hence,$(A \cap B)' = A' \cup B'$

De Morgan's Law on Difference of Sets

$A - (B \cup C) = (A - B) \cap (A - C)$

For example

Set A = {1, 2, 3, 4, 5, 6, 7, 8, 9},

Set B = {1, 2, 3, 4} and

Set C = {6, 7, 8, 9}

Then

$(B \cup C)$ = {1, 2, 3, 4, 6, 7, 8, 9}

And $A - (B \cup C)$ = {5}.

But $(A - B)$ = {5, 6, 7, 8, 9} and $(A - C)$ = {1, 2, 3, 4, 5}

Therefore $(A - B) \cap (A - C)$ = {5}

Hence $A - (B \cup C) = (A - B) \cap (A - C)$

Alternate Proof

Let x be any element such that

$$x \in A - (B \cup C) \Leftrightarrow x \in A \text{ and } x \notin (B \cup C)$$
$$\Leftrightarrow x \in A \text{ and } (x \notin B \text{ or } x \notin C)$$
$$\Leftrightarrow (x \in A \text{ but } x \notin B) \text{ and } (x \in A \text{ but } x \notin C)$$
$$\Leftrightarrow x \in (A - B) \text{ and } x \in (A - C)$$
$$\Leftrightarrow x \in (A - B) \cap (A - C)$$

Hence $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

For Example

Set A = {1, 2, 3, 4, 5, 6, 7, 8, 9},

Set B = {1, 2, 3, 4} and

Set C = {6, 7, 8, 9}

Then

$(B \cap C) = \{\emptyset\}$

And $A - (B \cap C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

But $(A - B) = \{5, 6, 7, 8, 9\}$ and $(A - C) = \{1, 2, 3, 4, 5\}$

Therefore $(A - B) \cup (A - C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Hence $A - (B \cap C) = (A - B) \cup (A - C)$

Alternate Proof

Let x be any element such that

$$x \in A - (B \cap C) \Leftrightarrow x \in A \text{ and } x \notin (B \cap C)$$

$$\Leftrightarrow x \in A \text{ and } (x \notin B \text{ and } x \notin C)$$

$$\Leftrightarrow (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)$$

$$\Leftrightarrow x \in (A - B) \text{ or } x \in (A - C)$$

$$\Leftrightarrow x \in (A - B) \cup (A - C)$$

Hence $A - (B \cap C) = (A - B) \cup (A - C)$.

Some Important results on Difference, Union and Intersection

(i) If Set A and Set B are two sets then $A \cup B = (A - B) \cup B$

For example

Set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} and Set B = {1, 2, 3, 4} then

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

And

$(A - B) = \{5, 6, 7, 8, 9\}$, then,

$(A - B) \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Hence

$A \cup B = (A - B) \cup B$

(ii) If Set A and Set B are two sets then A ∩ (B – A) = {Ø}

For example

Set A = {6, 7, 8, 9} and Set B = {1, 2, 3, 4, 5} then

(B – A) = {5},

Then,

A ∩ (B – A) = {6, 7, 8, 9} ∩ {5}

= {Ø}

Hence proved.

(iii) If Set A and Set B are two sets then (A – B) ∩ B = {Ø}

For example

Set A = {1, 2, 3, 6, 7, 8, 9} and Set B = {1, 2, 3, 4} then

(A – B) = {6, 7, 8, 9}

Therefore

(A – B) ∩ B = {6, 7, 8, 9} ∩ {1, 2, 3, 4}

= {Ø}

Hence proved

(iv) If Set A and Set B are two sets then A – (A – B) = A ∩ B

For example

Set A = {1, 2, 3, 6, 7, 8, 9} and Set B = {1, 2, 3, 4} then

(A – B) = {6, 7, 8, 9}

Therefore

A – (A – B) = {1, 2, 3, 6, 7, 8, 9} – {6, 7, 8, 9}

= {1, 2, 3}

But

A ∩ B = {1, 2, 3, 6, 7, 8, 9} ∩ {1, 2, 3, 4}

= {1, 2, 3}.

Hence, A – (A – B) = A ∩ B

(v) If Set A and Set B are two sets then

(A – B) ∪ (B – A) = (A ∪ B) – (A ∩ B).

For example

Set A = {1, 2, 3, 6, 7, 8, 9} and Set B = {1, 2, 3, 4} then

(A – B) = {6, 7, 8, 9} and (B – A) = {4}

Therefore,

(A – B) ∪ (B – A) = {4, 6, 7, 8, 9}.

Similarly,

(A ∪ B) = {1, 2, 3, 4, 6, 7, 8, 9} and (A ∩ B) = {1, 2, 3}

Therefore

(A ∪ B) – (A ∩ B) = {1, 2, 3, 4, 6, 7, 8, 9} – {1, 2, 3}.

= {4, 6, 7, 8, 9}, hence proved.

(vi) If Set A, Set B and Set C are three sets then

(A ∩ B) – C = (A – C) ∩ (B – C).

For example

Set A = {1, 2, 3, 4, 5, 6, 7, 8, 9},

Set B = {1, 2, 3, 4, 5, 6} and

Set C = {3, 5, 6, 7, 9}, then,

(A ∩ B) = {1, 2, 3, 4, 5, 6}.

Therefore

(A ∩ B) – C = {1, 2, 3, 4, 5, 6} – {3, 5, 6, 7, 9}

= {1, 2, 4}.

Similarly

(A – C) = {1, 2, 4, 8} and (B – C) = {1, 2, 4}.

Therefore

(A – C)∩ (B – C) = {1, 2, 4, 8} ∩ {1, 2, 4}

= {1, 2, 4}.

And hence (A ∩ B) – C = (A – C) ∩ (B – C)

(vii) If Set A, Set B and Set C are three sets then

A ∩ (B – C)=(A ∩ B) – (A ∩ C).

For example

Set A = {1, 2, 3, 4, 5, 6, 7, 8, 9},

Set B = {1, 2, 3, 4, 5, 6} and

Set C = {3, 5, 6, 7, 9}, then,

(B – C) = {1, 2, 4}.

Therefore

A ∩ (B – C) = {1, 2, 3, 4, 5, 6, 7, 8, 9} ∩ {1, 2, 4},

= {1, 2, 4}.

Similarly

(A ∩ B) = {1, 2, 3, 4, 5, 6} and (A ∩ C) = {3, 5, 6, 7, 9}.

Therefore (A ∩ B) – (A ∩ C) = {1, 2, 3, 4, 5, 6} – {3, 5, 6, 7, 9}

= {1, 2, 4}.

And hence A ∩ (B – C) = (A ∩ B) – (A ∩ C)

(viii) If Set A and Set B are two sets then (A – B) = A ∩ B′

For example

If Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Set A = {1, 2, 3, 6, 7, 8, 9} and Set B = { 6, 7, 8, 9} then

(A – B) = {1, 2, 3}

And B′ = U – B = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {6, 7, 8, 9}.

= {1, 2, 3, 4, 5}.

A ∩ B′ = {1, 2, 3, 6, 7, 8, 9} ∩ {1, 2, 3, 4, 5}

= {1, 2, 3}.

Hence (A – B) = A ∩ B′

(ix) If Set A and Set B are two sets then (A – B) = B′ – A′

If Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Set A = {1, 2, 3, 6, 7, 8, 9} and Set B = {6, 7, 8, 9} then

(A – B) ={1, 2, 3, 6, 7, 8, 9} – {6, 7, 8, 9} = {1, 2, 3}

And A′ = U – A = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {1, 2, 3, 6, 7, 8, 9}

= {4, 5}.

B′ = U – B = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {6, 7, 8, 9}.

= {1, 2, 3, 4, 5}.

Therefore $B' - A' = \{1, 2, 3, 4, 5\} - \{4, 5\}$.

$= \{1, 2, 3\}$.

Hence $(A - B) = B' - A'$.

(x) If Set A, Set B and Set C are three sets then $A \cup (B - C) \neq (A \cup B) - (A \cup C)$

For example

If Set A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Set B = {1, 2, 3, 4, 5 ,6} and Set C ={3, 5, 6, 7, 9} , then,

$(B - C) = \{1, 2, 3, 4, 5, 6\} - \{3, 5, 6, 7, 9\} = \{1, 2, 4\}$

Therefore,

$A \cup (B - C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cup \{1, 2, 4\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Similarly

$(A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and

$(A \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Therefore

$(A \cup B) - (A \cup C) = \{\emptyset\}$

Hence $A \cup (B - C) \neq (A \cup B) - (A \cup C)$

Illustration No. 1

If Set A = {1, 2, 3, 4, 5, 6}, Set B = {4, 5, 6, 7}

and Set C = { 3, 5, 6, 7, 9}, then, find

(i) $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

Solution:

$(B \cap C) = \{4, 5, 6, 7\} \cap \{3, 5, 6, 7, 9\}$

$= \{5, 6, 7\}$

Therefore

$A - (B \cap C) = \{1, 2, 3, 4, 5, 6\} - \{5, 6, 7\}$

$= \{1, 2, 3, 4\}$

Similarly

$(A - B) = \{1, 2, 3\}$

and

$(A - C) = \{1, 2, 4\}$

Therefore $(A - B) \cup (A - C) = \{1, 2, 3, 4\}$

Hence, $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $(B \cap C) = \{4, 5, 6, 7\} \cap \{3, 5, 6, 7, 9\}$

$= \{5, 6, 7\}$

Therefore

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

Similarly,

$(A \cup B) = \{1, 2, 3, 4, 5, 6\} \cup \{4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

And

$(A \cup C) = \{1, 2, 3, 4, 5, 6\} \cup \{3, 5, 6, 7, 9\}$,

$= \{1, 2, 3, 4, 5, 6, 7, 9\}$

Therefore,

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 5, 6, 7, 9\}$,

$= \{1, 2, 3, 4, 5, 6, 7\}$.

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(iii) $(B - C) = \{4\}$

Therefore

$A \cap (B - C) = \{4\}$

But $(A \cap B) = \{4, 5, 6\}$

And

$(A \cap C) = \{3, 5, 6\}$

Therefore,

$(A \cap B) - (A \cap C) = \{4\}$

Hence $A \cap (B - C) = (A \cap B) - (A \cap C)$.

Illustration No. 2

If Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Set A = {1, 2, 3, 4, 5}, Set B = {4, 5, 6, 7} and Set C = {5, 6, 7, 8} then find

(i) $(A \cup B) \cap (A \cup C)$

(ii) $(A \cap B) \cup (A \cap C)$

(iii) $(A \cup B \cup C)'$

(iv) $(A \cup B') \cap (A' \cup B)$

Solution:

(i) (A ∪ B) = {1, 2, 3, 4, 5} U {4, 5, 6, 7}

= {1, 2, 3, 4, 5, 6, 7}.

And

(A ∪ C) = {1, 2, 3, 4, 5} ∪ {5, 6, 7, 8}

= {1, 2, 3, 4, 5, 6, 7, 8}.

Therefore,

(A ∪ B) ∩ (A ∪ C) = {1, 2, 3, 4, 5, 6, 7}

(ii) (A ∩ B) = {1, 2, 3, 4, 5} ∩ {4, 5, 6, 7}.

= {4, 5}

And

(A ∩ C) = {1, 2, 3, 4, 5} ∩ {5, 6, 7, 8}

= {5}

Therefore,

(A ∩ B) ∪ (A ∩ C) = {4, 5} ∪ {5}.

= {4, 5}.

(iii) (A ∪ B ∪ C)′ = U – (A ∪ B ∪ C)

But,

(A ∪ B ∪ C) = {1, 2, 3, 4, 5, 6, 7, 8}

Therefore,

(A ∪ B ∪ C)′ = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {1, 2, 3, 4, 5, 6, 7, 8}

= {9}.

(iv) $(A \cup B') = \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, 8, 9\}$,

$= \{1, 2, 3, 4, 5, 8, 9\}$.

And

$(A' \cup B) = \{6, 7, 8, 9\} \cup \{4, 5, 6, 7, 8\}$,

$= \{4, 5, 6, 7, 8, 9\}$.

Therefore,

$(A \cup B') \cap (A' \cup B) = \{1, 2, 3, 4, 5, 8, 9\} \cap \{4, 5, 6, 7, 8, 9\}$

$= \{4, 5, 8, 9\}$.

Illustration No. 3

If Set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, Set $A = \{2, 3, 4\}$, Set $B = \{1, 4, 5, 6\}$ and Set $C = \{1, 2, 4, 5, 7\}$ then find

(i) $(A \cap B)' = A' \cup B'$.

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution:

(i) $(A \cap B) = \{2, 3, 4\} \cap \{1, 4, 5, 6\}$.

$= \{4\}$

Therefore $(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4\}$

$= \{1, 2, 3, 5, 6, 7, 8\}$.

Similarly

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 3, 4\}$,

$= \{1, 5, 6, 7, 8\}$.

And

$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 4, 5, 6\}$,

$= \{2, 3, 7, 8\}$

Therefore,

$A' \cup B' = \{1, 5, 6, 7, 8\} \cup \{2, 3, 7, 8\}$,

$= \{1, 2, 3, 5, 6, 7, 8\}$.

Hence $(A \cap B)' = A' \cup B'$.

(ii) $(B \cap C) = \{1, 4, 5, 6\} \cap \{1, 2, 4, 5, 7\}$
$= \{1, 4, 5\}$.

Therefore,

$A \cup (B \cap C) = \{2, 3, 4\} \cup \{1, 4, 5\}$.
$= \{1, 2, 3, 4, 5\}$.

Similarly,

$A \cup B) = \{2, 3, 4\} \cup \{1, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

And

$(A \cup C) = \{2, 3, 4\} \cup \{1, 2, 4, 5, 7\}$
$= \{1, 2, 3, 4, 5, 7\}$.

Therefore,

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 7\}$.
$= \{1, 2, 3, 4, 5\}$

Hence, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Symmetric Difference of Two Sets

If Set A and Set B are two sets then the difference set is called symmetric difference of two sets if it contains all those elements which belongs to A or to B. Symmetric difference of Set A and Set B is symbolically represented as

$$A \Delta B = (A - B) \cup (B - A)$$

For example

Set A = {1, 2, 3, 5, 6, 7,} and Set B = {1, 3, 4, 8, 9} then

$A \Delta B = \{2, 5, 6, 7\} \cup \{4, 8, 9\}$
$= \{2, 4, 5, 6, 7, 8, 9\}$

Some Properties of Symmetric Difference

(i) Commutative Property $A \Delta B = B \Delta A$.

(ii) Associative Property $(A \Delta B) \Delta C = A \Delta (B \Delta C)$.

(iii) $A \Delta A = \{\emptyset\}$.

(iv) $A \Delta \emptyset = A$

(v) $A \Delta (A \cap B) = A - B$.

(vi) $(A \Delta B) \cup (A \cap B) = A \cup B$.

(vii) $A \Delta B = (A - B) \cup (B - A)$

(viii) $A \Delta B = (A \cup B) - (A \cap B)$.

(ix) $A \Delta B = \{\emptyset\} \Leftrightarrow A = B$.

(x) $(A \Delta B) \cup (A \cap B) = A \cup B$.

Illustration No. 1

If Set A = {1, 2, 3, 5, 6} and Set B = {1, 3, 4, 8, 9} then verify that $A \Delta B = B \Delta A$ and also prove that $A \Delta B = (A \cup B) - (A \cap B)$?

Solution:

$A \Delta B = (A - B) \cup (B - A)$

$= \{2, 5, 6\} \cup \{4, 8, 9\}$

$= \{2, 4, 5, 6, 8, 9\}$

And

$B \Delta A = (B - A) \cup (A - B)$

$= \{4, 8, 9\} \cup \{2, 5, 6\}$.

$= \{2, 4, 5, 6, 8, 9\}$

Similarly, in order to prove that $A \Delta B = (A \cup B) - (A \cap B)$

As $(A \cup B) = \{1, 2, 3, 5, 6\} \cup \{1, 3, 4, 8, 9\}$.

$= \{1, 2, 3, 4, 5, 6, 8, 9\}$.

And

$(A \cap B) = \{1, 3\}$.

$(A \cup B) - (A \cap B) = \{1, 2, 3, 4, 5, 6, 8, 9\} - \{1, 3\}$.

$= \{2, 4, 5, 6, 8, 9\}$.

Hence

$A \Delta B = (A \cup B) - (A \cap B)$

Illustration No. 2

If Set A = {1, 3, 6, 9}, Set B = {3, 4, 7, 9} and Set C = {2, 4, 5, 7} then find

(i) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

(ii) $A \Delta (A \cap B) = A - B$

(iii) $(A \Delta B) \cup (A \cap B) = A \cup B$.

Solution:

$(A \Delta B) = (A - B) \cup (B - A)$

$= \{1, 6\} \cup \{4, 7\}$

$= \{1, 4, 6, 7\}$

Therefore,

$(A \Delta B) \Delta C = \{1, 6\} \cup \{2, 5\}$

$= \{1, 2, 5, 6\}$.

Similarly,

$(B \Delta C) = \{3, 9\} \cup \{2, 5\}$

$= \{2, 3, 5, 9\}$.

Therefore,

$A \Delta (B \Delta C) = \{1, 6\} \cup \{2, 5\}$

$= \{1, 2, 5, 6\}$

Hence,

$(A \Delta B) \Delta C = A \Delta (B \Delta C)$

(ii) $A \Delta (A \cap B) = A - B$

$(A \cap B) = \{3, 9\}$

And

$A \Delta (A \cap B) = \{1, 6\} \cup \{\emptyset\}$

$= \{1, 6\}$.

And

$A - B = \{1, 3, 6, 9\} - \{3, 4, 7, 9\}$.

$= \{1, 6\}$.

And hence,

$A \Delta (A \cap B) = A - B$.

(iii) $(A \Delta B) = (A - B) \cup (B - A)$

$= \{1, 6\} \cup \{4, 7\}$.

$= \{1, 4, 6, 7\}$.

And $(A \cap B) = \{3, 9\}$

Therefore,

$(A \Delta B) \cup (A \cap B) = \{1, 4, 6, 7\} \cup \{3, 9\}$

$= \{1, 3, 4, 6, 7, 9\}$.

And

$A \cup B = \{1, 3, 4, 6, 7, 9\}$.

And hence $(A \Delta B) \cup (A \cap B) = A \cup B$.

Cardinal Number of a Set

The numbers of elements in a finite set say Set A is called as cardinal number of A and symbolically represented as n (A).

For example

If Set A = {1, 2, 3, 4, 5, 6, 7} then n(A) = 7 as A contains only seven elements.

Some Important Results

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Note: If Set A and Set B are disjoint sets then $(A \cap B) = \{\emptyset\}$.

Therefore $n(A \cup B) = n(A) + n(B)$

(ii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(A \cap C) + n(A \cap B \cap C)$.

(iii) $n(A') = n(U) - n(A)$.

(iv) $n(A) = n[(A \cap B') \cup (A \cap B)]$
$= n[(A - B) \cup (A \cap B)] = n(A - B) + n(A \cap B)$.

(v) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$.

Illustration No. 1

In a class of 30 students, 14 has taken mathematics, 10 has taken mathematics but not economics. Find the number of students who had taken mathematics and economics also find the number of students who had taken economics but not mathematics?

Solution:

Let

A = Set of students who have taken mathematics as subject.

B = Set of students who have taken economics as subject.

Then given are $n(A \cup B) = 30$, $n(A) = 14$ and $n(A \cap B') = 10$.

Now we need to find the students who have taken both the subjects i.e. $n(A \cap B)$ and the number of students who have taken economics as subject but not mathematics i.e. $n(B \cap A')$

But

$n(A) = n(A \cap B') + n(A \cap B)$

$14 = 10 + n(A \cap B)$.

$n(A \cap B) = 14 - 10$.

$= 4$

But

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

$30 = 14 + n(B) - 4$.

$n(B) = 30 - 10 = 20$.

Therefore,

$n(B) = n(B \cap A') + n(A \cap B)$

$20 = n(B \cap A') + 4$.

$n(B \cap A') = 20 - 4 = 16$.

Illustration No. 2

Out of 200 students in a management school 120 students read Indian Economic Review and 100 read Harvard Business Review if the number of students who read neither of the journals is 40 then find the number of students who read both them?

Solution:

Let

A = Set of students who read Indian Economic Review.

B = Set of students who read Harvard Business Review.

Then given are $n(U) = 200$, $n(A) = 120$, $n(B) = 100$ and

$n(A' \cap B') = 40$.

We need to find the number of students who read both the journals i.e. Indian Economic Review and Harvard Business Review i.e. $n(A \cap B)$

But,

$A' \cap B' = (A \cup B)'$ therefore $n(A \cup B)' = 40$.

Since $n(A \cup B)' = n(U) - n(A \cup B)$ or $n(A \cup B) = 200 - 40 = 160$

But as we know,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

$$160 = 120 + 100 - n(A \cap B).$$

$$n(A \cap B) = 220 - 160 = 60.$$

Thus there are 60 students who read both the journals.

Illustration No. 3

In an examination 100 students secured 80% and more marks in Economics or Accounts. Out of these 70 obtained 80% and more marks in Economics and 20 in both Economics and Accounts. Then find how many of them have secured 80% and more marks in Accounts only?

Solution:

Let

A = Set of students who scored over 80% in Economics.

B = Set of students who scored over 80% in Accounts.

Then given are $n(A \cup B) = 100$, $n(A) = 70$, and $n(A \cap B) = 20$.

We need to find the number of students who scored over 80% in Accounts.

But we know,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$100 = 70 + n(B) - 20.$$

$$n(B) = 100 - 50 = 50.$$

As we need to find the students who scored over 80% in Accounts are

$$N(A' \cap B) = n(B) - n(A \cap B)$$

$$= 50 - 20 = 30.$$

Illustration No. 4

In a pollution study of 200 cities has revealed the following report, 70 were polluted by Sulphur compounds, 60 were polluted by Lead and 80 were polluted by Fly Ash, 40 were polluted by Sulphur and Fly Ash, 30 were polluted by Sulphur and Lead , 35 were polluted by Lead and Fly ash and 10 cities were polluted by all three compounds. Then find

(i) How many were polluted by one of three impurities?

(ii) How many cities are only polluted by Sulphur, Lead and Fly Ash?

(iii) How many of the cities are not polluted at all?

Solution:

Let

S = City polluted by Sulphur compounds.

L = City polluted by Lead.

F = City polluted by Fly Ash.

(i) Since we need to find the number of cities polluted by at least one of the pollutants is given by

$n(S \cup L \cup F) = n(S) + n(L) + n(F) - n(S \cap L) - n(L \cap F)$
$\qquad - n(S \cap F) + n(S \cap L \cap F)$

$= 70 + 60 + 80 - 40 - 30 - 35 + 10$

$= 115.$

Thus 115 cities are polluted by atleast one of the pollutants.

(ii) Cities polluted by only one pollutant are

Case I: By Sulphur Compounds,

$n(S \cap L' \cap F') = n[S \cap (L \cup F)']$

$= n(S) - n(S \cap L) - n(S \cap F) + n(S \cap L \cap F)$

$= 70 - 30 - 40 + 10.$

$= 10$

Case II By Lead,

$n(L \cap S' \cap F') = n[L \cap (S \cup F)']$

$= n(L) - n(L \cap S) - n(L \cap F) + n(S \cap L \cap F)$

$= 60 - 30 - 35 + 10.$

$= 5$

Case III : By Fly Ash,

$n(F \cap S' \cap L') = n[F \cap (S \cup L)']$

$= n(F) - n(F \cap S) - n(F \cap L) + n(S \cap L \cap F)$

$= 80 - 35 - 40 + 10$

$= 15$

(iii) To find the number of cities which are not polluted were

$= n(U) - n((S \cup L \cup F)$

$= 200 - 115 = 85$

Ordered Pairs

An ordered pair consists of two elements, say a and b represented within parenthesis as (a, b) where a is the first member and b the second member.

For example: the odd numbers and their squares can be represented in the form of ordered pairs as

(1, 1); (3, 9); (5, 25); (7, 49)..............................

Note

Two ordered pair (a, b) and (c, d) are said to be equal if and only if a = c and b = d. In other words $(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$.

Cartesian Products

If Set A and Set B are two sets then the set of all ordered pairs whose first member belongs to Set A and the second member belongs to Set B is the Cartesian product of A and B in that order and read as A cross B and symbolically represented as

$A \times B = \{ (x , y) ; x \in A \text{ and } y \in B\}$

Note

(i) If $A \times B$ and $B \times A$ have same number of elements then $A \times B \neq B \times A$ unless and until A = B . Therefore the Cartesian product of two sets is commutative if the two sets are equal.

(ii) If Set A has m elements and Set B has n elements then $A \times B$ has mn elements.

(iii) If Set A and Set B are disjoint then the Cartesian product $A \times B$ is also disjoint.

(iv) If either of Set A or Set B is a null set then the set $A \times B$ is also a null set.

(v) If either of Set A or Set B is a infinite set and the other is a non empty set then the set $A \times B$ is also an infinite set.

(vi) If Set A or Set B are finite sets then $n(A \times B) = n(A) \times n(B)$.

Some Important Properties

(i) If Set A, Set B and Set C are three sets and $A \subseteq B$, then $(A \times C) \subseteq (B \times C)$

For example

Let $\{1, 2\} \in A \times C$ then $1 \in A$ and $2 \in C$

$\Leftrightarrow$ 1 $\in$ B and 2 $\in$ C (Since $A \subseteq B$)

$\Leftrightarrow$ {1, 2} $\in$ (B × C)

Hence $(A \times C) \subseteq (B \times C)$

Alternate Proof

Let { x, y } $\in$ A × C then x $\in$ A and y $\in$ C

$\Leftrightarrow$ x $\in$ B and y $\in$ C (Since $A \subseteq B$)

$\Leftrightarrow$ { x, y} $\in$ (B × C)

Hence $(A \times C) \subseteq (B \times C)$

(ii) If Set A, Set B and Set C are three sets and $A \subseteq B$, and $C \subseteq D$ then $(A \times C) \subseteq (B \times D)$

For example

let {1, 2} $\in$ A × C then 1 $\in$ A and 2 $\in$ C

$\Leftrightarrow$ 1 $\in$ B and 2 $\in$ D (Since $A \subseteq B$ and $C \subseteq D$)

$\Leftrightarrow$ {1, 2} $\in$ (B × D)

Hence $(A \times C) \subseteq (B \times D)$

Alternate Proof

let {x, y} $\in$ A × C then x $\in$ A and y $\in$ C

$\Leftrightarrow$ x $\in$ B and y $\in$ D (Since $A \subseteq B$ and $C \subseteq D$)

$\Leftrightarrow$ {x, y} $\in$ (B × D)

Hence $(A \times C) \subseteq (B \times D)$

(iii) If Set A and Set B are two sets and $A \subseteq B$, then

$(A \times A) = (A \times B) \cap (B \times A)$.

For example

Let {1, 2} $\in$ A × A then 1 $\in$ A and 2 $\in$ A

$\Leftrightarrow$ (1 $\in$ A and 2 $\in$ A) and (1 $\in$ A and 2 $\in$ A)

$\Leftrightarrow$ (1 $\in$ A and 2 $\in$ B) and (1 $\in$ B and 2 $\in$ A) (Since $A \subseteq B$)

$\Leftrightarrow$ {1, 2} $\in$ (A × B) and {1, 2} $\in$ (B × A)

$\Leftrightarrow$ {1, 2} $\in$ $(A \times B) \cap (B \times A)$

Hence $(A \times A) = (A \times B) \cap (B \times A)$

Alternate Proof:

Let $\{x, y\} \in A \times A$ then $x \in A$ and $y \in A$

$\Leftrightarrow$ ($x \in A$ and $y \in A$) and ($x \in A$ and $y \in A$)

$\Leftrightarrow$ ($x \in A$ and $y \in B$) and ($x \in B$ and $y \in A$) { Since $A \subseteq B$ }

$\Leftrightarrow$ $\{x, y\} \in (A \times B)$ and $\{x, y\} \in (B \times A)$

$\Leftrightarrow$ $\{x, y\} \in (A \times B) \cap (B \times A)$

Hence $(A \times A) = (A \times B) \cap (B \times A)$.

(iv) If Set A, Set B and Set C are three sets then

$A \times (B - C) = (A \times B) - (A \times C)$

For example

Let $(1, 2) \in A \times (B - C)$ then $1 \in A$ and $2 \in (B - C)$

$\Leftrightarrow$ $1 \in A$ and $2 \in B$ and $2 \notin C$

$\Leftrightarrow$ ($1 \in A$ and $2 \in B$) and ($1 \in A$ and $2 \notin C$)

$\Leftrightarrow$ $\{1, 2\} \in (A \times B)$ and $\{1, 2\} \notin (A \times C)$

$\Leftrightarrow$ $\{1, 2\} \in [(A \times B) - (A \times C)]$

Hence $A \times (B - C) = (A \times B) - (A \times C)$.

Alternate Proof

Let $(x, y) \in A \times (B - C)$ then $x \in A$ and $y \in (B - C)$

$\Leftrightarrow$ $x \in A$ and $y \in B$ and $y \notin C$

$\Leftrightarrow$ ($x \in A$ and $y \in B$) and ($x \in A$ and $y \notin C$)

$\Leftrightarrow$ $\{x, y\} \in (A \times B)$ and $\{x, y\} \notin (A \times C)$

$\Leftrightarrow$ $\{x, y\} \in [(A \times B) - (A \times C)]$

Hence $A \times (B - C) = (A \times B) - (A \times C)$.

Illustration No. 1

If Set A = {1, 2, 3} and Set B = {1, 2} then prove that $A \times B \neq B \times A$

Solution:

The Cartesian product $A \times B$ = {(1, 1) , (1, 2) , (2, 1), (2, 2) ,(3, 1) (3, 2)}

Similarly $B \times A$ = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)}

Thus (3, 1) and (3, 2) are elements of A × B are not elements B × A. Similarly (2,3) is a element in B × A but not A × B, therefore A ×B ≠ B × A

Illustration No. 2

If Set A = {1, 2} , Set B = {2, 3} and Set C {3, 4} then find

(i) A × (B ∪ C)

(ii) (A × B) ∩ (A × C).

(iii) A × (B ∩ C).

Solution:

(i) (B ∪ C) = {2, 3} ∪ {3, 4}

= {2, 3, 4}.

Therefore,

A × (B ∪ C) = {1, 2} × {2, 3, 4}.

= { (1, 2), (1, 3), (1, 4), (2, 2), (2, 3) ,(2, 4) }.

(ii) (A × B) = { 1, 2} × { 2, 3}

= { (1,2) , (1,3), (2,2), (2,3) }.

And (A × C) = { 1, 2} × { 3, 4}

= { (1, 3) , (1, 4), (2, 3), (2, 4) }

Therefore

(A × B) ∩ (A × C) = { (1, 3), (2, 3) }.

(iii) (B ∩ C) = { 2, 3 } ∩ {3, 4}

= { 3 }.

Therefore A × (B ∩ C) = { 1, 2} × { 3 }.

= { (1, 3) , (2, 3) }.

Illustration No. 3

If Set A = {1, 2} , Set B = {1, 2, 3} and Set C {1, 2, 3, 4} then verify whether A × (B ∩ C) = (A × B) ∩ (A × C) ?

Solution:

(B ∩ C) = {1, 2, 3} ∩ {1, 2, 3, 4}

= {1, 2, 3}

Therefore

$A \times (B \cap C) = \{1, 2\} \times \{1, 2, 3\}.$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$

But,

$(A \times B) = \{1, 2\} \times \{1, 2, 3\}$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$

And

$(A \times C) = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}.$

Therefore,

$(A \times B) \cap (A \times C) = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}.$

Hence $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Summary

A set is a well defined collection of objects and these objects are tended as the members or elements of the set.

A set which has no elements is called as null, empty or void set denoted by {Ø} and read as phi in Greek and Latin.

If for a given set A, a set consisting of all the subsets of A is called the power of the set. The power set this is denoted by P (A).

If A and B are two sets then the union of Set A and Set B is the set consisting of either all the elements of Set A or Set B or both and denoted by "A ∪ B" and read as A union B or A cup B

i. e. A ∪ B = {x : x €A or x € B or x € both A and B}

The complement of a set A is that set which contains all those elements of the universal set U which are not in A. The complement of set A is the set U – A and is denoted by A^c, A′ , A or ~ A. It can symbolically written as

$A' = U - A = \{x : x \in U \text{ and } x \notin A\}$

The number of elements in a finite set say Set A is called as cardinal number of A and symbolically represented as n (A).

If Set A and Set B are two sets then the set of all ordered pairs whose first member belongs to Set A and the second member belongs to Set B is the Cartesian product of A and B in that order and read as A cross B and symbolically represented as

$A \times B = \{(x, y) ; x \in A \text{ and } y \in B\}$

Self Practice Exercises

1. If Set A = {1, 2, 3, 4} and Set B = {3, 4, 5, 6} then find
   (i) $A \cup B$ (ii) $A \cap B$
2. If Set A = {1, 2, 3, 4, 5}, Set B = {4, 5, 6, 7} and
   Set C = {2, 3, 5, 6, 7} then find
   (i) $A \cap (B \cup C)$. (ii) $A \cup (B \cap C)$.
3. If Set A ={ 1, 2, 3 } , Set B = { 2, 3, 4, 5} and Set C = {3, 5, 6} then find
   (i) $(A \cup B) \cap (B \cup C)$ (ii) $(A \cap B) \cup (B \cap C)$
4. If Set A ={1, 2, 3, 4} , Set B = {2, 3, 4, 5} and Set C = {3, 4, 5, 6} then find
   (i) $(A - B) \cup C$ (ii) $A - (B \cap C)$
   (iii) $(A - B) \cup (B - C)$ (iv) $(A \cap B) - (B \cap C)$
5. If Set A ={ 1, 2, 3, 4, 5 } and Set B = { 3, 4, 5, 6, 7, 8 } then find
   (i) $A - (A - B)$ (ii) $(A - B) \cap (B - A)$ (iii) $(A - B) \cup (B - A)$
6. If Set U = {0, 1, 2, 3, 4, 5} , Set A = {0, 1, 2, 3} and
   Set B = {2, 3, 4} then find,
   (i) $(A' \cup B) \cap (B' \cup A)$ (ii) $(A' \cap B) \cup (B' \cap A)$.
7. If Set A = {1, 2, 3, 4, 5}, Set B = {3, 4, 5, 6, 7} and Set C = {5, 6, 7, 8} then find
   (i) $A \cup (B \cap C)$ (ii) $A \cap (B \cup C)$ (iii) $A \cap (B \cap C)$
8. If Set A = {1, 2, 3}, Set B = {2, 3, 4, 5} and Set C ={2, 4, 5, 6, 8} then prove that
   (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$
   (ii) $(A \cap B) - C = (A - C) \cap (B - C)$
9. If Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, Set A = {4, 5, 7} , Set B = {1, 2, 3, 4} and Set C = {3, 4, 5, 7} then find
   (i) $A' \cup B'$ (ii) $A' \cap B'$. (iii) $(A \cup B \cup C)'$.
10. if Set U = {1, 2, 3, 4, 5, 6, 7} , Set A = {1, 2, 3}, Set B = {3, 4, 5} and Set C = {4, 5, 7} then find
    (i) $A' \cap (B \cup C)$ (ii) $(A' \cap B) \cup (A \cap B')$
11. If Set A = {1, 2, 3, 4, 5} and Set B = {3, 4, 5, 6, 7} then find
    (i) $(A - B) \cup (B - A)$ (ii) $(A - B) \cap (B - A)$

12. If Set A = {1, 2, 3, 4, 5}, Set B = {3, 4, 5, 6} and Set C = {3, 5, 7, 8} then find
    (i) $(A \cap B) \cup (B - C)$ (ii) $(A \cup B) \cap (B - C)$

13. In a survey conducted in a town of 2000 population to see the readership of newspaper, it has revealed that 1000 read Hindustan Times, 800 read Times of India and 700 read Indian Express. If there was 400 who read both Hindustan Times and Times of India, 300 who read both Times of India and Indian Express, 500 read only Hindustan times and 100 read all the three papers. Then find,
    (i) How many read both Hindustan Times and Indian Express?
    (ii) How of them read only one paper?
    (iii) How many of them read none of the newspaper?

14. In a class of management course of 120 students appeared for their tri semester examinations of Business Mathematics, Business Statistics and Business Economics. After the results were declared it was found that 50 failed in Business Mathematics, 48 failed in Business Statistics and 64 failed in Business Economics. Moreover 18 failed in Business Mathematics alone, 12 failed Business Statistics alone and 10 in Business Statistics and Business Economics then find
    (i) How many failed in all the subjects?
    (ii) How many passed in all the subjects?
    (iii) How many passed in Business Economics alone?

15. In an examination 86 passed in marketing paper, 96 passed in managerial economics paper and 104 has passed in quantitative techniques paper. Only 16 students could manage to pass all the three, 28 passed in marketing and managerial economics paper and 42 passed in marketing and quantitative techniques paper and 40 passed in managerial economics paper and quantitative techniques paper then find,
    (i) How many students passed in only marketing paper?
    (ii) Find the ratio of students passing in managerial economics and quantitative techniques paper?
    (iii) If a student is declared pass in the examination provided he or she clears at least two subjects. Find how many were declared pass?

16. In a picnic party it was found that 20 people like all three beverages tea, coffee and cold drink. It was found that 40 people like coffee and tea, 50 like coffee and cold drink and 50 like tea and cold drink. It was noticed that 110 liked coffee, 100 tea and 100 cold drink. From the above mentioned information find
    (i) How many picnic goers preferred only coffee, only tea and only cold drink?
    (ii) How many of the picnic goers liked both tea and cold drink but not coffee?
    (iii) How many of the picnic goers liked at least two of the beverages?
17. The combined membership of Accounts Society and Mathematical society is 100. Find the number of members in Mathematical society knowing that 60 members are the members of accounts society and 25 are the members of both the institutions?
18. If Set A = { 1, 3 } , Set B = { 2, 4 } and Set C = { 3, 4} then find
    (i) $A \times (B \cup C)$ (ii) $A \times (B \cap C)$ (iii) $(A \times B) \cap (A \times C)$
19. If Set A ={ a, b} , Set B = { c, d } and Set C = { e, f } show that $A \times B \neq B \times A$ and also find $(A \times B) \cup (A \times C)$?
20. If Set A = { 1, 2, 3 } , Set B = { 1, 2 } and Set C = { 2, 4} then find
    (i) $A \times (B \cap C)$ (ii) $(A \times B) \cap (A \times C)$

Answers

1. (i) {1, 2, 3, 4, 5, 6}
   (ii) {3, 4}
2. (i) {2, 3, 4, 5}
   (ii) {1, 2, 3, 4, 5, 6, 7}
3. (i) {2, 3, 4, 5}
   (ii) {2, 3, 5}
4. (i) {1, 3, 4, 5, 6}
   (ii) {1, 2}
   (iii) {1, 2}
   (iv) { 2 }
5. (i) {3, 4, 5} , (ii) { Ø }, (iii) {1, 2, 6, 7, 8}

6. (i) {2, 3, 5} (ii) {0, 1, 4}
7. (i) {1, 2, 3, 4, 5, 6, 7}, (ii) {3, 4, 5} and (iii) { 5 }
9. (i) {1, 2, 3, 5, 6, 7, 8, 9}, (ii) {6, 8, 9} and (iii) {6, 8, 9}
10. (i) {4, 5, 7} and (ii) {1, 2, 4, 5}
11. (i) {1, 2, 6, 7} and (ii) { Ø }
12. (i) {3, 4, 5, 6} and (ii) {4, 6}
13. (i) 100, (ii) 1000 and (iii) 300
14 (i) 20, (ii) 20 and (iii) 28.
15. (i) 32, (ii) 22: 19 and (iii) 78
16. (i) 90, (ii) 30 and (iii) 100
17. 65.
18. (i) { (1, 2), (1, 3), (1, 4), (3, 2), (3, 2), (3, 4) }
    (ii) { (1, 4), (3, 4)}
    (iii) { (3, 4) }.
19. { (a, c), (a, d), (a, e), (a, f), (b, c), (b, d), (b, e), (b, f)}.
20. (i) { (1, 2),(2, 2), (3, 2) }
    (ii) { (1, 2),(2, 2), (3, 2) }