1

Electrical Circuit Concepts

1.1   INTRODUCTION

Since the first discovery of electricity in the year 600 BC by Thales of Miletus, the knowledge and use of electricity has multiplied by enormous counts. All the modern day conveniences are dependent to the most extent on electricity. Without electricity the life of the modern man will come to a standstill.

Network theory is a basic discipline of electric science. It describes how the energy is transferred from one device to another. Thorough understanding of this subject is essential for every type of engineer and physicist. The objective of this text is to provide a thorough understanding and proficiency in the subject of engineering circuit analysis.

1.2   NETWORK MODEL

Just as in every branch of science and engineering, where the behaviour of a complex physical device is studied by constructing its mathematical model, in network theory also, an idealised model for the device is constructed and its behaviour is analysed. Network theory deals with the determination of response of this network model to a stimulus or excitation as shown in Fig. 1.1. In electrical engineering it is often possible to separate the excitation and response.
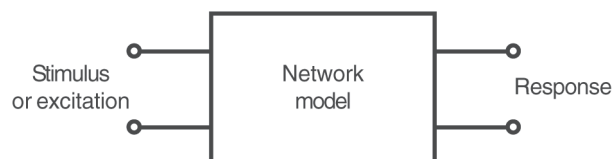

Fig. 1.1 Excitation and response

The network model itself is an interconnection of various idealised models of physical devices. A basic characteristic of many physical networks encountered in engineering practice is that the response is proportional to the excitation. If the input is doubled, the output is also doubled. This property of the network is called *linearity*. In this text we will be concerned with the study of networks which obey this property.

We are often concerned with the flow of electrical energy from one device to another and not in the internal distribution of the energy in any particular device. Thus we will treat the basic building blocks of our network models to be '*lumped*' elements rather than distributed elements. Thus their behaviour can be analysed by the effects produced at the terminals of the device, through which the energy leaves or enters the device. A simple model of an electric device is represented in Fig. 1.2.
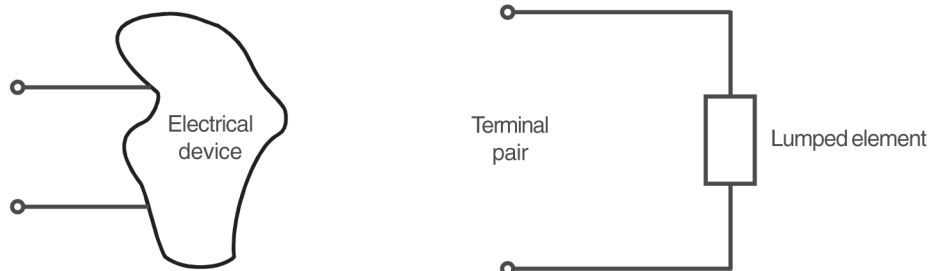

Fig. 1.2 An electrical device and its lumped model

The networks are constructed with these ideal elements interconnected by ideal connectors which absorb zero energy.

1.3   NETWORK VARIABLES

As we are concerned with the flow of energy from one device to another in a network, we must know what are the variables that are used in the study of the flow of energy. Energy by itself is not a measurable quantity and is therefore measured by association with other measurable quantities. For example, the mechanical energy is measured in terms of force and distance.

The electrical energy can be measured by defining a measurable quantity called *electric charge*. Charges were first observed by rubbing dry substances together. Suppose a light material such as pith ball is suspended by a string and a hard rubber comb, rubbed with a woollen cloth, is brought near it, it is observed that the pith ball tends to swing away from it. If the pith ball is approached with the woollen cloth now, the pith ball is found to be attracted towards it. This phenomenon is explained by saying that there are forces acting due to electric charges present on the pith ball, the comb and the woollen cloth. Since these forces can be either attraction or   repulsion as seen in the above experiment, it is postulated that there are two kinds of electrical charges : positive and negative and like charges repel and unlike charges attract. Benjamin Franklin first named the charges that are present on the comb to be negative and that on the woollen cloth, positive.

According to atomic physicists, the entire matter in the universe is made up of fundamental building blocks called *atoms* and these atoms are composed of different kinds of fundamental particles. The three most important particles are the electron, proton and the neutron. The ultimate unit of electric charge is the electron, with a mass of 9.107×10^{-31} kg. The fundamental unit of charge, called *the Coulomb*, named after Charles Coulomb, is 6.24×10^{18} electrons. These electrons move in orbits around the nucleus of

an atom and many of them break loose and move at random in the interatomic spaces. An excess of electrons in a body represents negative charge and deficit of electrons represents a positive charge.

Hans Christian Oersted, in the year 1819, observed that a flow of electrical charge produced a force on a nearby magnetic compass needle and he found that the force was proportional to the rate of flow of charge. Since the measurement of this force was much easier than the measurement of forces between static charges, a new variable, current, was defined as the rate of flow of electric charge and its unit, ampere = 1 Coulomb/sec. Thus the current, rather than the charge, has come to be the basic variable of circuit analysis. Charges are represented by the symbols q or Q and the currents are represented by i or I depending on whether they are varying with respect to time or constant respectively. Thus

$$i = \frac{dq}{dt} \qquad \qquad .....(1.1)$$

We observe two important aspects of the flow of current. First, the current has direction. The number of coulombs per second has a direction and the current is said to be flowing in a circuit in a particular direction. Secondly, the current flow may be due to the flow of electrons in a particular direction or due to motion of positive charge, or due to motion of positive charge in one direction and negative charge in the other direction. Conventionally, the direction of current is taken to be the flow of positive charge in the direction of a reference arrow placed for convenience.

In Fig. 1.3, two equivalent methods are shown to represent the current in a conductor. 1A of current in Fig. 1.3(a) means a net positive charge of 1 C/sec is flowing in the direction of arrow, whereas, in Fig. 1.3(b) a net negative charge of –1 C/sec is flowing in the direction of arrow. Since the magnetic effect produced by a flow of positive charge in one direction and that due to a flow of negative charge in the opposite direction is identical, the currents are equivalent in both the cases. Hence to avoid ambiguity, the current is always considered to be an equivalent flow of positive charge. Thus a current of positive charge flowing in the direction of an arbitrary reference direction is taken to be positive. Again coming back to our lumped model of the elements, the current is measured at the terminals and no consideration is given to the finite speed of propogation of electricity through the material. Thus the current entering one terminal of the element is taken to be equal to the current leaving the other terminal and
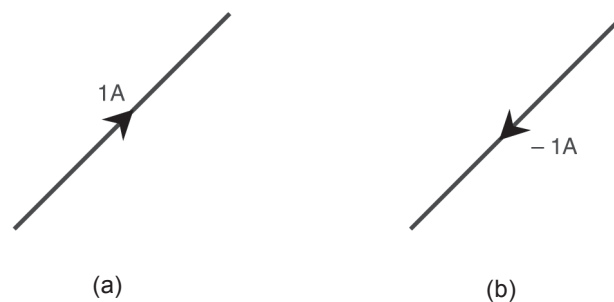

(a)   (b)

Fig. 1.3 Refrence direction for currents

currents are measured and defined only at the terminals of the lumped element as shown in Fig. 1.4. Coming back to the measurement of electrical energy, in addition to the concepts of charge and current, we need to define an additional variable to measure the amount of energy lost or gained by each charge as it passes through the element. This variable is called *voltage* or *potential* and is denoted by the letters e, E or v, V.
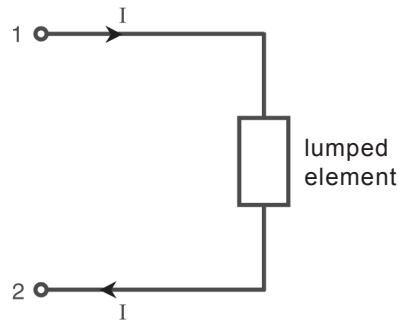

Fig. 1.4 Current in a lumped element

The voltage across a terminal pair of a device is a measure of the work required to move the electric charge through the device. Thus

$$v = \frac{dW}{dQ} \qquad\qquad .....(1.2)$$

where   v = voltage in Volts
             W = energy in Joules
             Q = charge in Coulombs

From the definitions of current and voltage given in eqs. (1.1) and (1.2), it is clear that the product of voltage and current is the power associated with the device. Power is measured in Joules/sec or watts and is represented by the symbols p or P. Thus

$$p = v \cdot i \qquad\qquad .....(1.3)$$

Just as a current has direction, voltage has a polarity depending on whether energy is supplied to the device or received from it. A plus sign at the terminal where the current enters the device indicates that the energy is absorbed by the device and negative sign at that terminal indicates that the device is supplying energy. We therefore use a pair of signs, plus and minus, at the two terminals of an electric device to give a reference polarity for the voltage. Thus, placing a +ve sign at the terminal 1 and –ve sign at the terminal 2 indicates that the terminal 1 is at a higher potential than the terminal 2. As for the current, the reference marks for voltage are for algebraic reference only. If the actual voltage has the same polarity as the reference marks, the voltage is positive and it is negative if it has opposite polarity. Thus the two figures 1.5(a) and 1.5(b) are equivalent.
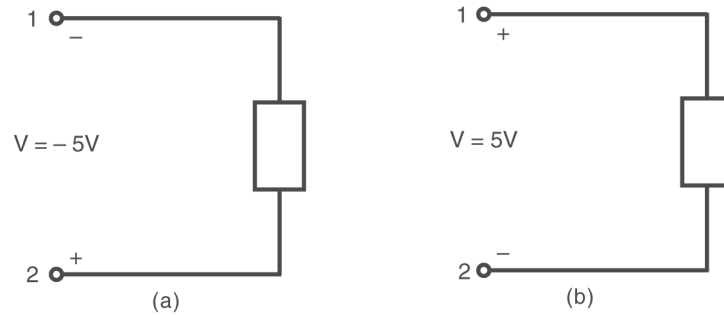
Fig. 1.5 Reference directions for voltage

Consistent signs must be assigned to the current and voltage in a device if the power is absorbed by the device or supplied by the device. First an arbitrary direction is assigned for the current and the sign of voltage is made to agree. A plus sign is assigned to the terminal at which the current enters the device. With this convention, a positive sign for power indicates that the device is absorbing power or it is a sink, whereas a negative sign for the power indicates that the device is supplying power and hence it is a source.
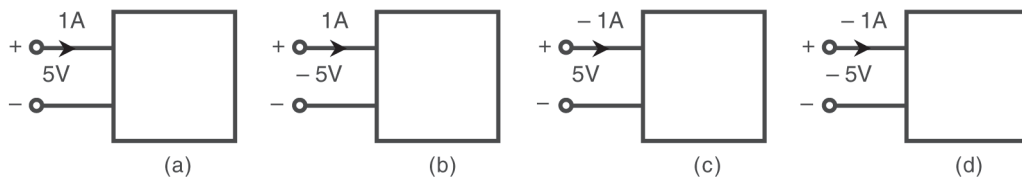
Fig. 1.6 Reference directions for voltage and current

In Fig. 1.6 four different combinations of the polarities are shown for an electric device. In Fig. 1.6(a) the power is $P = 5 \times 1 = 5$ watts and hence it is a sink. In Fig. 1.6(b) the power is $P = -5 \times 1 = -5$ W and hence it is a source. In a similar manner Fig. 1.6(c) is a source and Fig. 1.6(d) is a sink.

1.4 NETWORK ELEMENTS

The network elements are the mathematical models of two terminal electric devices which can be characterised by their voltage and current relationship at the terminals. These network elements are interconnected to form a network and can be divided into two major groups as shown in Fig. 1.7.
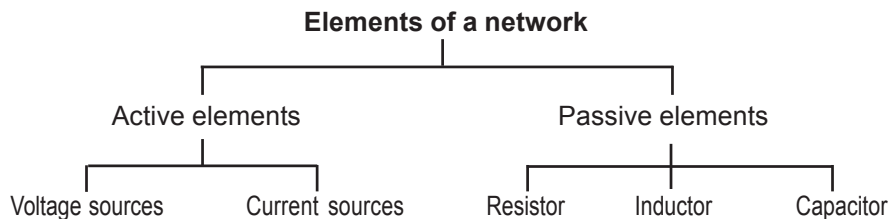

Fig. 1.7 Classification of elements of a network

1.4.1   Active Elements : Voltage and Current Sources

Active elements are those elements which are capable of delivering energy to the networks or devices which are connected across them. There are essentially two kinds of active elements in a network.

Independent voltage source :

An independent voltage source is characterised by the property that the voltage across its terminals is independent of current passing through it. The symbolic representation and the v – i relationship of the element is given in Fig. 1.8.

(a)   (b)
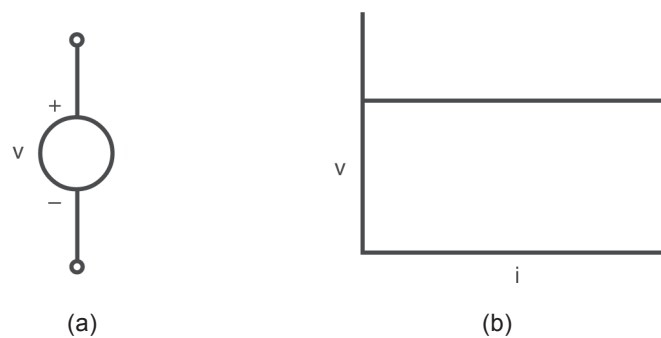
Fig. 1.8 Independent voltage source (a) its circuit representation and (b) v – i characteristic

If the voltage source has a constant voltage, it is termed as a d.c voltage source and is represented by either of the symbols shown in Fig. 1.9(a) or (b).

(a)   (b)
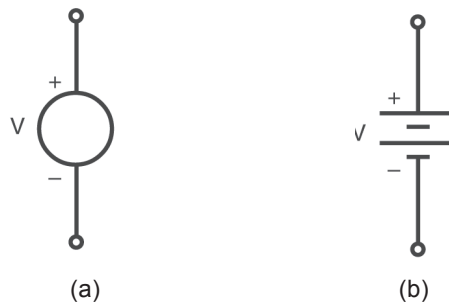
Fig. 1.9. Symbolic representation of d.c. sources

The sources described above are '*ideal*' sources and do not represent exactly any practical device encountered in physical world, because, theoretically it can deliver infinite amount of power to the circuit to which it is connected. This device is supposed to deliver unlimited number of coulombs per second, each receiving 'v' Joules as it passes through the source. This, clearly, is impossible, but there are several practical devices which can be modelled very closely with these elements. The v – i characteristic of a practical voltage source is characterised by a drooping characteristic as given in Fig. 1.10.
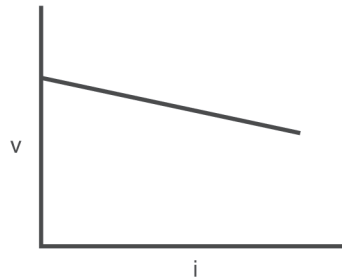
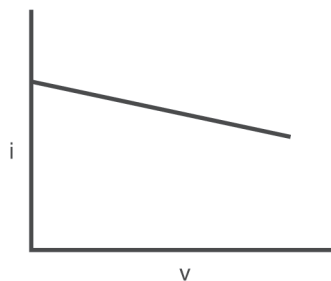Fig. 1.10 v – i characteristic of a practical voltage source

One point worth emphasising here is about the polarity signs placed in Fig. 1.8(a). The '*plus*' sign placed at the upper terminal does not mean that it is always positive with respect to the lower terminal. In fact, it only means that the upper terminal is 'v' volts positive with respect to the other terminal at a particular instant of time. If 'v' is negative at any instant the upper terminal is actually 'v' volts negative with respect to the lower terminal at that instant.

Independent current source :

The second ideal source is an independent current source which delivers a constant current independent of the voltage across its terminals. The symbol and the v – i characteristic are given in Fig. 1.11.
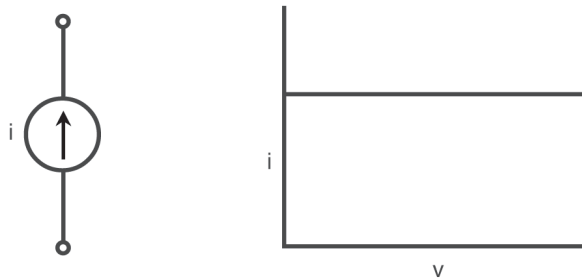
Fig. 1.11 The symbol and the v – i characteristic of an independent current source

This, again, can be considered as a close approximation to a practical device. This too, theoretically, is capable of supplying infinite power from its terminals since it will be able to deliver the same current even if the voltage across it is infinite ! However, this element is very useful in representing the behaviour of some electronic devices. The v – i characteristic of a practical device is given in Fig. 1.12 which is a drooping characteristic.

Fig. 1.12 v – i characteristic of a practical current source

The two active elements described above are called *independent sources* because their values are not affected by the currents or voltages in other elements of the network to which these elements are connected. There are other types of sources which are very useful in describing many electronic devices that are represented by their equivalent circuits. These are known as *dependent* or *controlled sources* in which the voltage or the current associated with the element is dependent on either the current or voltage in some other element in the network. In order to distinguish these sources from the independent sources, a diamond shaped symbol is used to represent them as shown in Fig. 1.13.

(a) voltage source   (b) current source

Fig. 1.13 Symbols for dependent sources

There are essentially four types of controlled sources depending on whether it is a current source or a voltage source and whether it is controlled by a current or voltage in some other element of the network. Thus we have

   (a) Voltage controlled voltage source
   (b) Current controlled voltage source
   (c) Voltage controlled current source   and
   (d) Current controlled current source.

1.4.2 Passive Elements

The passive elements are those elements which are capable of only receiving power. They cannot deliver power. However there are some passive elements which can store finite energy and then return this energy to external elements. Since these elements cannot deliver unlimited energy over an infinite time interval, they are treated as passive elements only. There are essentially three kinds of passive elements called *the resistor*, *the inductor* and *the capacitor* as shown in Fig. 1.7.

As described earlier, a network is an interconnection of two or more elements. If this interconnection involves atleast one closed circuit, the network is also called as *an electrical circuit*. Thus every circuit is a network but every network need not be a circuit. Also, if the network contains at least one independent source, the network is called *an active network*. If the network does not contain any active element it is called *a passive network*.

Example 1.1

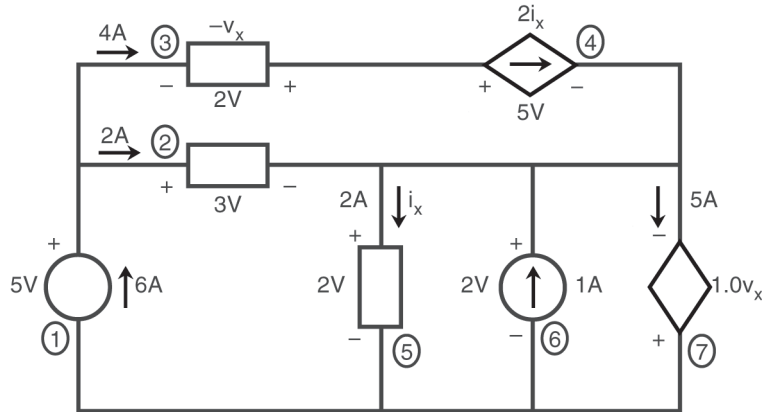Find the power absorbed by each element in the circuit of Fig. 1.14.

Fig. 1.14 Network for example 1.1

Solution :

The elements are identified by the encircled numbers. For any element if the current is entering the +ve terminal, the current is taken as +ve and hence the power absorbed will be positive.

Thus for element (1)

Power absorbed is

$$P_1 = 5 \times (-6) = -30 \text{ W}$$

and for the other elements

$$P_2 = 3 \times 2 \times = 6 \text{ W}$$
$$P_3 = 2 \times (-4) = -8 \text{ W}$$
$$P_4 = 5 \times 4 = 20 \text{ W}$$
$$P_5 = 2 \times 2 = 4 \text{ W}$$
$$P_6 = 2 \times (-1) = -2 \text{ W}$$
$$P_7 = (1.0 \times -2) \times (-5) = +10 \text{ W}$$

1.5   R-L-C ELEMENTS

1.5.1 The Resistor

The passive elements introduced in section 1.4 will now be described in detail. First, the idealised passive element called *linear resistor*, will be considered. The mathematical model for a linear resistor is described by the famous Ohm's law for most of the conducting materials. Ohm's law states that :

"*The voltage across any conducting material is directly proportional to the current flowing through the conductor*" or

$$v = R\,i \qquad\qquad\qquad ......(1.3)$$

Where 'R', the constant of proportionality, is called *the resistance of the material*. The resistance is measured in ohms. The v – i characteristic of a resistor is shown in Fig. 1.15.
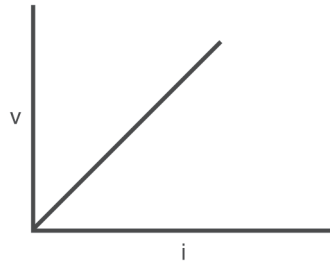
Fig. 1.15 v – i characteristic of a resistor

Since the characteristic is a linear relationship, the resistor is a linear element. The network symbol and the polarities of voltage and current for absorbing power in a resistor are shown in Fig. 1.16.

Fig. 1.16 Network model of a resistor and sign convention for voltage and current

The product v . i gives the power absorbed by the resistor and this power manifests itself as heat in the resistor. As resistor is a passive element it cannot deliver or store power. The expressions for power are

$$P = vi = i^2R = \frac{v^2}{R} \qquad\qquad .....(1.4)$$

It is pertinent to emphasise that the linear resistor is an idealised model of a physical device and the relationship of eq. (1.3) holds good for a certain range of currents only. If it exceeds a particular value, excessive heat is produced and the value of the resistance is found to change with temperature. It is no longer linear and hence the eq. (1.3) cannot be used. However, in this text, we will be concerned with the linear resistors only.   The resistance of a wire of length '*l*' meters, and a cross sectional area of A Sq. m. is given by

$$R = \frac{\rho l}{A}$$

where is ρ the resistivity of the wire in ohm – meters

The reciprocal of resistance is defined as conductance. Thus, conductance is the ratio of current to voltage.

$$G = \frac{1}{R} = \frac{i}{v} \qquad\qquad .....(1.5)$$

The unit of conductance is Siemens (S).

Example 1.2

Consider the resistor in Fig. 1.17(a). A voltage v(t) of waveform given in Fig. 1.17(b) is applied at its terminals. Obtain the waveform of current through it.
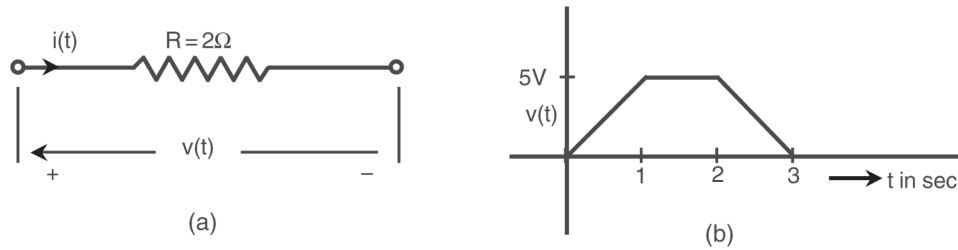

Fig. 1.17 The resistance and voltage waveform for example 1.2

Solution :

The v – i relationship for a resistor is

$$v = i\,R$$

or　　　　　$$i = \frac{v}{R} = vG$$

$$G = \frac{1}{R} = \frac{1}{2} = 0.5S$$

For $0 \le t \le 1$ sec

$$v(t) = 5\,t$$
$$i(t) = 5t(0.5) = 2.5t\ A$$

For $1 \le t \le 2$ sec

$$v(t) = 5\ V$$
$\therefore$　　　　$$i(t) = 5(0.5) = 2.5\ A$$

For $2 \le t \le 3$ sec

$$v(t) = -\,5t + 15$$
$$i(t) = (-\,5t + 15)0.5$$
$$= -\,2.5t + 7.5\ A$$
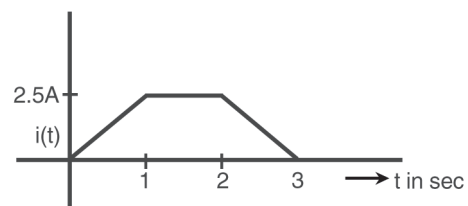
Fig. 1.17(c) Current waveform for example 1.2

The current waveform is given in Fig. 1.17(c).

Example 1.3

Through a resistor of value 2Ω a current of i(t) = 2 sin50t A is passed. What is the voltage across its terminals and what is the power consumed by it ?

Solution :

$$R = 2 \ \Omega$$

$$i(t) = 2 \ \text{Sin} \ 50t \ \ A$$

$$v(t) = \ i(t) \ . \ R = (2\text{Sin} \ 50t) \times 2$$
$$= \ 4\text{Sin} \ 50t \ \ V$$

$$p(t) = v(t) \ . \ i(t)$$
$$= 4 \ \text{Sin} \ 50t \ . \ 2 \ \text{Sin} \ 50t$$
$$= 8 \ \text{Sin}^2 \ 50t \ \ \text{Watts}$$

1.5.2 The Inductor

This is another important passive element which does not consume energy but is capable of storing energy. Once it stores energy, it is then capable of supplying this energy to external devices. But unlike active source, it is not capable of supplying unlimiteed energy or a finite average power over an infinite interval of time.

The mathematical model for an inductor will be given purely in a network point of view as a v – i relationship at the terminals. For an inductance

$$v = L\frac{di}{dt} \qquad\qquad\qquad .....(1.7)$$

From eq. (1.7) it is clear that if the current is a constant, the voltage across the inductor is zero, which means that the two terminals of the inductor are connected by a wire of zero resistance. This is called a short circuit between the two terminals. Hence we can say that the inductance behaves like a short circuit to d.c. Another characteristic we observe from eq. (1.7) is that if there is an

Fig. 1.18 Network symbol for inductor

instantaneous or sudden change in the current, the rate of change of current is infinite and the voltage across the inductor is also infinite. Thus a sudden change in the current in an inductor can occur only if the voltage across it is infinite. Hence the inductor opposes abrupt changes in the current. Further if an inductor carrying current is open circuited, for example, by opening a switch, an arc appears across the switch due to the same reason.

An alternate form of eq. (1.7) describes the relation between i and v.

Rewriting eq. (1.7) as

$$di = \frac{1}{L} \ v \ dt \qquad\qquad\qquad .....(1.8)$$

Integrating on both sides, and assuming that the current in the inductor was zero at the time $t = - \ \infty$. (The time $t = - \ \infty$ is a conceptual time to ensure that the current was zero at $t = - \ \infty$. This time could be the time at which the inductance coil was wound and obviously the current at that time was zero !)
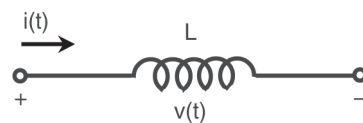
$$\int_0^i di = \frac{1}{L}\int_{-\infty}^t vdt$$

$$i = \frac{1}{L}\int_{-\infty}^t vdt \qquad\qquad ....(1.9)$$

We observe that both the equations, eq. (1.7) and eq. (1.9), are linear and hence the inductor is a linear element.

The power entering the inductor at any instant is given by

$$p = v \cdot i = Li\,\frac{di}{dt} \qquad\qquad .....(1.10)$$

When the current is constant, $\dfrac{di}{dt} = 0$ and $p = 0$ and no additional energy is stored in the inductor. The energy associated with the lines of flux will be fixed. If the current increases, the derivative of the current is positive and the power is positive. Thus additional energy is stored in the inductor. The total energy stored in the inductor at any time is given by

$$w_L = \int_{-\infty}^t vidt = \int_{-\infty}^t iL\frac{di}{dt}\,dt$$

$$= \int_0^i Li\,di = \frac{1}{2}\,Li^2 \qquad\qquad .....(1.11)$$

Observe that the limits of integration are chosen appropriately in eq. (1.11) to suit the variable of integration. Note also that the energy stored in the inductor at any instant depends on the value of the current at that instant only and not on its past history.

Example 1.4

An inductor shown in Fig. 1.19(a) is supplied with a current waveform given in Fig. 1.19(b). Draw the waveforms for the voltage and energy in the inductor.

(a) (b)
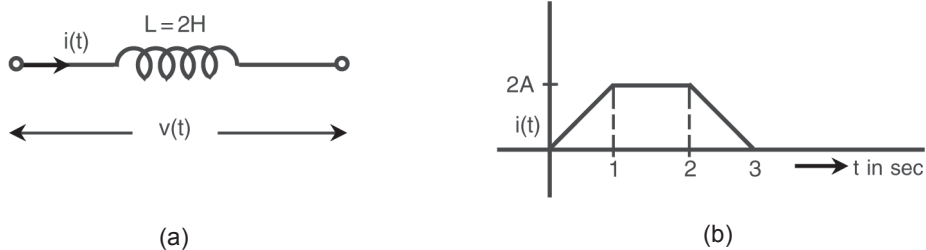
Fig. 1.19 An inductor and its associated current waveform for example 1.4

Solution :

For $0 \leq t \leq 1$ sec

$\qquad$ i(t) = 2t A

$\therefore \qquad$ v(t) = L $\dfrac{di}{dt}$ = 2 × 2 = 4V $\qquad\qquad$ w_L (t) = $\dfrac{1}{2}$ L i^2 = $\dfrac{1}{2}$ × 2 × (2t)^2 = 4 t^2 J

For $1 \leq t \leq 2$ sec

$\qquad$ i(t) = 2 A $\qquad\qquad\qquad\qquad\qquad$ w_L (t) = $\dfrac{1}{2}$ × 2 × 2^2 = 4 J

$\qquad$ v(t) = 0

For 2

$\qquad$ i(t) = − 2t + 6   A $\qquad\qquad\qquad$ w_L(t) = $\dfrac{1}{2}$ × 2 × (− 2t + 6)^2 = 4t^2 − 24t + 36   J

$\qquad$ v(t) = 2 × − 2 = − 4V

The resulting waveform of voltage and energy are given in Fig. 1.19(c).
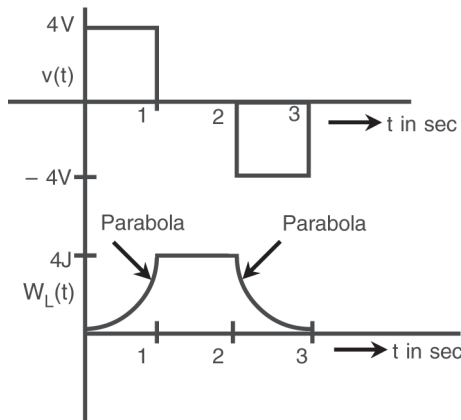
Fig. 1.19(c) Solution of example 1.4

Example 1.5

A waveform of current shown in the figure is applied to an inductor of value 0.5 H. Obtain the waveform of voltage across it.

Solution :

At t = 0 and t = 1s there is an abrupt change in the current. Here $\dfrac{di}{dt} = \infty$.
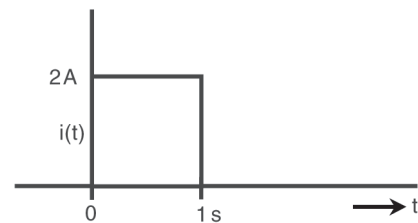
Current wave for example 1.5

Hence at t = 0 and t = 1s we have infinite voltage spikes appearing across the inductor terminals. As discussed earlier, these infinite spikes are required to allow the current to change suddenly. These spikes are called *impulses*, which are mathematically defined later. These spikes are physically not possible as, a finite time, however small it may be, is required for the current to rise to a given value. However these spikes can be of very large magnitude.

For 0   (observe that t $\neq$ 0 or 1s)

$$i(t) = 2A \quad \text{and} \quad v(t) = L \frac{di}{di} = 0v$$

Thus the waveform of voltage across inductor consists of two infinite spikes called *impulses* at t = 0 and 1 sec and is zero for 0 < t < 1s. The waveform is shown in the figure.

Example 1.6

Consider a waveform of voltage given in Fig. 1.20(a) applied to an inductor of value 2 mH. Obtain the waveforms of current and energy in the inductor. Assume that at t = 0 the energy and thus current in it to be zero.
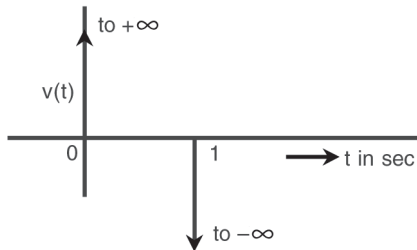
Waveform of v(t) for example 1.5
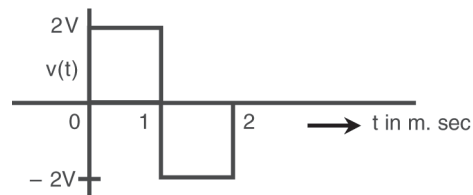
Fig. 1.20(a) waveform of voltage for example 1.6

Solution :

For 0 $\leq$

v(t) = 2 V

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(t)dt = \frac{1}{L} \int_{-\infty}^{o} v(t)dt + \frac{1}{L} \int_{o}^{t} v(t)dt \qquad .....(1.12)$$

The first term in eq. (1.12) represents the current in the inductor at t = 0. Thus

$$i(t) = i(0) + \frac{1}{L} \int_{0}^{t} 2dt = 0 + \frac{1}{2 \times 10^{-3}} 2t \Big|_{0}^{t} = 10^{3}t \quad A \qquad .....(1.13)$$

$$w_L(t) = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times 10^{-3} \times 10^6 t^2 = 10^3 t^2 \quad J$$

For $1 \leq t \leq 2$ ms

$$v(t) = -2V$$

Again, $i(t) = \quad = \quad +$

$$= i(1 \times 10^{-3}) + \frac{1}{2 \times 10^{-3}}[-2t]_{1 \times 10^{-3}}^{t}$$

From eq. (1.13) at $t = 1$ m sec.

$$i(1 \times 10^{-3}) = 10^3 \times 1 \times 10^{-3} = 1A$$

$$i(t) = 1 + 10^3[-t + 10^{-3}] = -10^3 t + 2 \quad A$$

$$w_L(t) = \frac{1}{2} \times 2 \times 10^{-3}(-10^3 t + 2)^2 = 10^3 t^2 - 4t + 4 \times 10^{-3} \quad J$$

The waveform of current and energy are shown in Fig. 1.20(b).
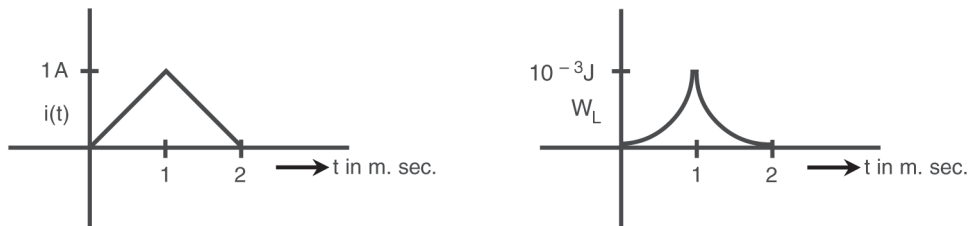
Fig. 1.20(b) Waveform of current and energy for example 1.6

1.5.3 The Capacitor

The capacitor is the third network element which is a passive element, but, like inductor, it can also store energy. Once it stores energy, it is capable of supplying this energy to external devices. It cannot provide unlimited energy or a finite average power over an infinite time interval. Hence it is not an active element like an ideal voltage or current source.

The inductor stores energy because of the current carried by it. In a similar manner, the capacitor is a device which stores energy by virtue of the voltage across it. Historically the capacitor was the first element to be discovered. In 1745 Van Mussenbrock devised an experiment to store static electricity by placing on insulator between two metal sheets, and then charged it by rubbing. Cunaeus, one of his friends, touched this device and received a violent shock ! Thus a device to store electricity was discovered.

Later in the year 1812 Simeon Poisson, gave a mathematical explanation for the energy storage on a capacitor. He compared the forces between charge on the plates to the gravitational force between a mass and earth and the energy associated with these stored charges to the potential energy of a mass at rest above the earth. He also showed that the energy is proportional to the area of the plates and inversely proportional to the spacing between them. Thus, capacitance, which is the property of a capacitor by which it can store energy, is given by

$$C = \frac{\in A}{d}$$

where   A is the area of the plates in sq. m.

                d is the distance between the plates in m.

and $\in$ is the permittivity of the insulating

                material between the two conductors

For air or vacuum,

           $\in = \in_0 = 8.854$ pF/m.

Coming back to the circuit model of a capacitor, it is described by the v – i relationship

$$i = C \frac{dv}{dt} \qquad\qquad .....(1.14)$$

The two commonly used symbols and the polarities of voltage and current for passivity requirement are shown in Fig. 1.21.
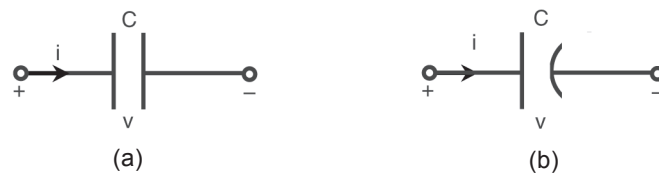
Fig. 1.21 circuit models of capacitor

An alternate form of eq. (1.14) can be derived as follows :

$$dv = \frac{1}{C} \, i \, dt \qquad\qquad .....(1.15)$$

Integrating on both sides of eq. (1.15) and assuming that the voltage across the capacitor was zero at t = – ∞, a conceptual time when the capacitor was manufactured and hence the voltage across it was zero, we get

$$\int_o^v dv = \int_{-\infty}^t \frac{1}{C} i \, dt$$

$$v = \frac{1}{C} \int_{-\infty}^t i \, dt \qquad\qquad .....(1.16)$$

From eq. (1.14), it is clear that, if the voltage across the capacitor is a constant, the current through it would be zero. But if the voltage changes suddenly, the current would by infinite. Thus a capacitor opposes any sudden change in voltage across it. An infinite current is required to be passed through it if the voltage has to change suddenly by a finite value. If the two terminals of a charged capacitor are shorted, a spark is produced because of this infinite current.

The two eqs. (1.14) and (1.16) describing the behaviour of a capacitor, are linear and hence the capacitor is a linear element. The power entering the capacitor at any instant is given by

$$P = v \cdot i$$

$$= v \cdot c \frac{dv}{dt} \qquad .....(1.17)$$

When $\dfrac{dv}{dt} = 0$ or $v(t)$ is a constant, $p = 0$ and no additional energy is stored in the capacitor. If the voltage is increasing, the power is positive and additional energy is stored in the capacitor. The total energy in the capacitor is given by

$$w_C = \int_{-\infty}^{t} vidt = \int_{-\infty}^{t} C v \frac{dv}{dt} dt$$

$$= C \int_{o}^{v} v\, dv = \frac{1}{2} Cv^2 \qquad .....(1.18)$$

Thus the energy stored in the capacitor at any instant depends on the value of the voltage at that instant only and not on the past history.

Example 1.8

The capacitor in Fig 1.22(a) is supplied with a voltage waveform shown in Fig. 1.22(b). Obtain the current and energy waveforms in the capacitor.

Solution :

For $0 \leq t \leq 1$

$$v(t) = 2t \ \ V$$

$$i = C \frac{dv}{dt} = 1.2 = 2 \ \ A$$

$$w_C = \frac{1}{2} Cv^2 = \frac{1}{2} \cdot 1 \times 4t^2 = 2t^2 \ \ J$$

Thus the current is a constant during the interval $0 \leq t \leq 1$.
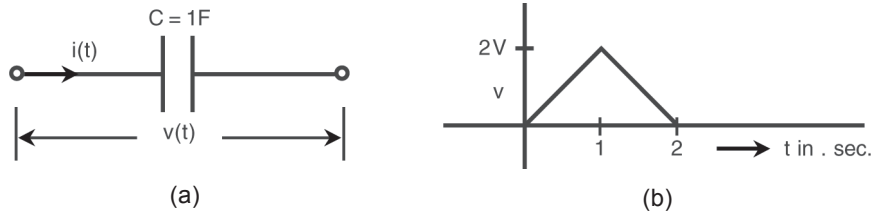
Fig. 1.22 The capacitor and the voltage waveform for example 1.8

For $1 \leq t \leq 2$

$$v(t) = -2t + 4 \quad V$$

and $\quad i(t) = 1 \cdot (-2) = -2 \quad A$

The energy stored in the capacitor is

$$w_C = \frac{1}{2} Cv^2 = \frac{1}{2} \times 1 \times (-2t + 4)^2 = 2t^2 - 8t + 8 \quad J$$

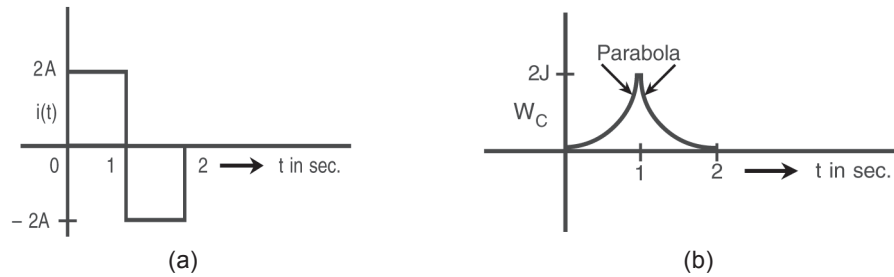The waveforms of current and energy are given in Fig. (1.22).

Fig. 1.23 Waveforms of i and W_C for example 1.8

Example 1.9

Consider the capacitor in Fig. 1.24(a) and a waveform of voltage shown in Fig. 1.24(b) applied across it. Find the waveform of current and energy in the capacitor.

Solution :

For $0 \leq t < 1$ m.sec.

$$v(t) = \frac{2}{1 \times 10^{-3}} \, t = 2 \times 10^3 t$$

$$i(t) = C \frac{dv}{dt} = 1 \times 10^{-6} \times 2 \times 10^3 = 2 \text{ mA}$$

$$w_C = \frac{1}{2} Cv^2 = \frac{1}{2} \times 1 \times 10^{-6} \times 4 \times 10^6 t^2 = 2t^2 \text{ J}$$

(a)   (b)
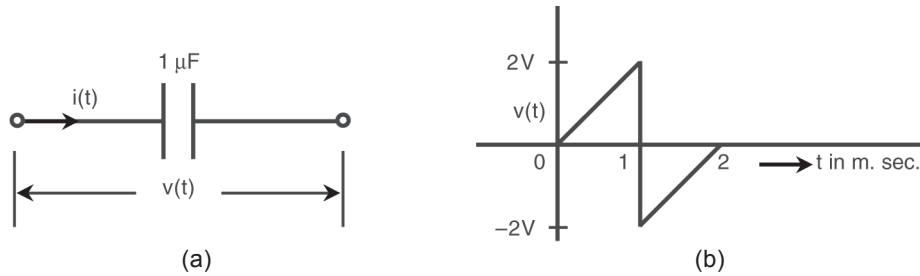
Fig. 1.24 The capacitor and the voltage waveform for example 1.9

At t = 1 m.sec the voltage across the capacitor instantaneously changes from +2V to –2V thus making $\dfrac{dv}{dt}$ infinite. Hence an impulse of current will be produced at t = 1 m.sec.

For $1 < t \leq 2$ m. sec.

$$v(t) = 2 \times 10^3 \, t - 4$$

$$i(t) = C\frac{dv}{dt} = 1 \times 10^{-6} \times 2 \times 10^3 = 2 \text{ mA.}$$

The energy during this interval is

$$W_C(t) = \frac{1}{2} \, C \, v^2 = \frac{1}{2} \times 1 \times 10^{-6} \times$$

$$(2 \times 10^3 t - 4)^2$$
$$= 2 \, t^2 - 8 \times 10^{-3} \, t + 8 \times 10^{-6} \text{ J}$$

The waveform of current and energy are given in Fig. 1.25.
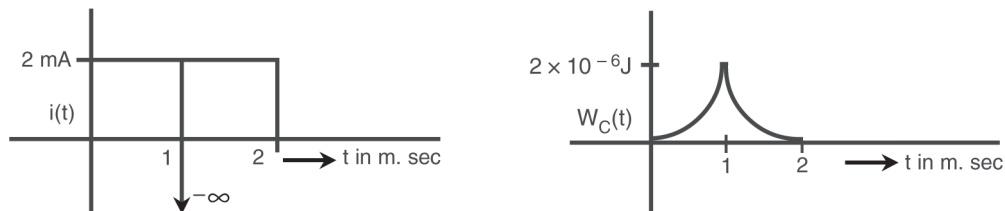
Fig. 1.25 The current and energy waveforms for example 1.8

Example 1.10

A current of waveform shown in Fig. 1.26(a) is applied to a capacitor of value 2 μ F. Find the voltage waveform.

Solution :

For $0 \leq t \leq 1$ m sec.

Fig. 1.26(a) Current waveform for example 1.10

$$i(t) = \frac{5 \times 10^{-3}}{1 \times 10^{-3}} t = 5t \text{ A}$$

$$v(t) = \frac{1}{C} \int_{-\infty}^{o} i(t) dt$$

Assuming no voltage to be present across the capacitor at t = 0

$$v(t) = \frac{1}{C} \int_{-\infty}^{o} i(t) dt + \frac{1}{C} \int_{o}^{t} i(t) dt$$

$$= v(0) + \frac{1}{2 \times 10^{-6}} \int_{o}^{t} 5t \, dt$$

$$= 0 + \frac{5}{2 \times 10^{-6}} \frac{t^2}{2} \Big|_0^t$$

$$= 1.25 \times 10^6 t^2 \text{ V} \qquad \qquad .....(1.19)$$

For $1 \leq t \leq 2$ m sec

$$i(t) = 5 \times 10^{-3} \text{ A}$$

$$v(t) = \frac{1}{C} \int_{-\infty}^{t} i(t) \, dt$$

$$= \frac{1}{C} \int_{-\infty}^{1 \times 10^{-3}} i(t) \, dt + \frac{1}{C} \int_{1 \times 10^{-3}}^{t} 5 \times 10^{-3} dt$$

$$\qquad \qquad .....(1.20)$$

The first integral in the above expression is the voltage across the capacitor at t = 1 m sec. From eq. (1.34).

$$v(1 \times 10^{-3}) = 1.25 \times 10^6 \times 1 \times 10^{-6} = 1.25 \text{ V}$$

Eq. (1.35) becomes

$$v(t) = 1.25 + \frac{1}{2 \times 10^{-6}} \int_{1 \times 10^{-3}}^{t} 5 \times 10^{-3} dt$$

$$= 1.25 + 2.5 \times 10^3 t \Big|_{1 \times 10^{-3}}^{t}$$

$$= 2.5 \times 10^3 t - 1.25 \qquad \qquad .....(1.21)$$

For $2 \leq t \leq 3$

$$i(t) = - 5\,t + 15 \times 10^{-3}\text{A}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{2\times10^{-3}} i(t)dt \; + \; \frac{1}{2\times10^{-6}}\int_{2\times10^{-3}}^{t}\left(-5t+15\times10^{-3}\right)dt$$

$$= v(2\times10^{-3}) + \frac{1}{2\times10^{-6}}\left[\frac{-5t^2}{2}+15\times10^{-3}\,t\right]_{2\times10^{-3}}^{t}$$

From eq. (1.36),

$$v\,(2 \times 10^{-3}) = 3.75 \text{ V}$$

$$v(t) = 3.75 - 1.25 \times 10^6\,t^2 + 7.5 \times 10^3 t - 15 + 5$$

$$= - 1.25 \times 10^6 t^2 + 7.5 \times 10^3 t - 6.25 \text{ V}$$

The waveform of voltage is shown in Fig. 1.26(b).
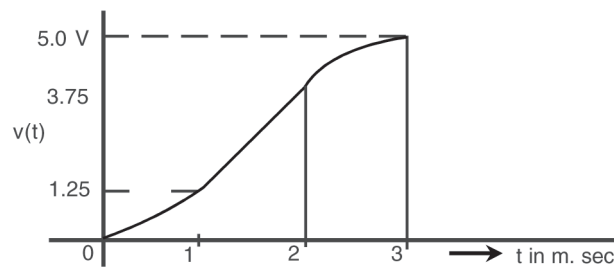
Fig. 1.26(b) Waveform of voltage across the capacitor for example 1.10

1.6   ELECTRICAL NETWORK

An electrical network is an interconnection of passive and active devices which are represented by their network models. As discussed in chapter 1, the network models are two terminal lumped elements and we are concerned with the current through these elements and voltages across them. The elements are interconnected with wires having zero resistance and absorbing zero power. These wires may have different lengths and shapes and the elements may be oriented in different positions. The same network may be drawn in different ways to look completely dissimilar and an engineer must be able to recognise the equivalence of these networks.

The four networks shown in Fig. 1.27 appear to be different but they are equivalent. One must be able to redraw the network in its simplest form.

The junction points where two or more elements are connected together are known as *nodes*. One single element with two nodes at its each end is called as a *branch*. Thus in Fig. 1.27(a) there are four nodes and six branches. Suppose we start at a node, say node 1, and travel along the branches to the other nodes and come back to the starting node without going through any node twice, we call this path *a circuit*.
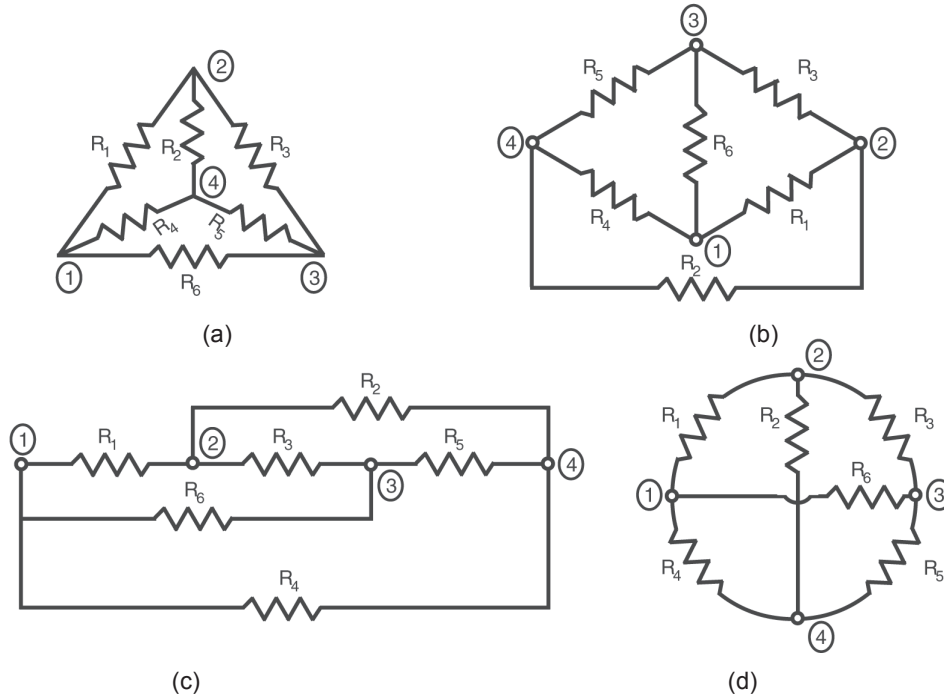
Fig. 1.27 A network represented in 4 different configurations

For example, in Fig. 1.27(a) the branches with elements R_1, R_2, R_4 constitute a circuit. In a similar manner we can recognise more circuits in Fig. 1.27 (a). Elements R_1, R_3, R_5, R_2, R_1 do not constitute a circuit because the node 2 is encountered twice in travelling from node 1 to itself. Having defined a node and a circuit we are now ready to consider two basic laws, which form the foundation for the study of network theory. These are the two laws enunciated by a German Professor, Gustav Robert Kirchhoff, in 1848.

1.7   KIRCHHOFF'S LAWS

Kirchhoff 's laws are the consequence of the law of conservation of energy and conservation of charge.

1.7.1 Kirchhoff 's Current Law

The first law, which is known as Kirchhoff's current law, or simply KCL, states that

" The algebraic sum of the currents at a node is equal to zero ".

Thus at a node

$$\Sigma\, i = 0 \qquad\qquad\qquad \text{.....(1.22)}$$

This law is based on the fact that there cannot be accumulation of charge at a node and thus store energy, since node is not a circuit element. Any charge which enters a node at any instant must therefore leave the node immediately. In terms of current, the total current entering the node must be equal to the total current leaving the node. If currents leaving the node are taken to be positive and currents entering are taken to be negative, this choice being arbitrary, the algebraic sum of the currents at the node will be zero. This is illustrated in Fig. 1.28.
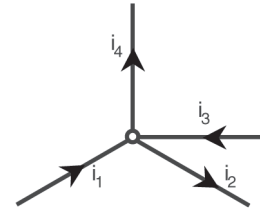
Fig. 1.28 An example of KCL

$$-i_1 + i_2 - i_3 + i_4 = 0 \qquad ....(1.23)$$
$$\text{or} \qquad i_1 + i_3 = i_2 + i_4 \qquad .....(1.24)$$

The directions of currents shown in Fig. 1.28 are the assumed reference   directions. The actual currents may have negative values. For example all the reference directions of currents at a junction may be pointing away form the node as in Fig. 1.29 and according to KCL.
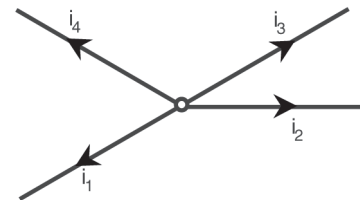
$$i_1 + i_2 + i_3 + i_4 = 0$$

.....(1.25) Fig. 1.29 All currents leaving the node

This may appear to be absurd, but recognising the fact that these are reference directions only and some of the currents may have negative values, eq. (1.25) is quite appropriate.

1.7.2 Kirchhoff 's Voltage Law

The second law of Kirchhoff, known as Kirchhoff 's voltage law, or in short   KVL, is stated as

" The algebraic sum of voltages around any closed path in a network is equal to zero ".

This, again, is based on law of conservation of energy. This law can be best understood in terms of its gravitational analogy. The energy level of a point A with respect to another point B can be found by moving a unit mass from point A to point B and finding the energy given up by the mass in this process. If a mass is moved from point A to B and then from point B to A, the net work done on the mass must be equal to zero. In a similar manner, the energy level of a point A with respect to a point B can be obtained by moving 1 coulomb of charge from A to B and noting the energy gained or lost by the charge. The energy gained or lost by the charge is independent of the path taken in travelling form A to B, as otherwise, a charge can be made to gain energy continuously by taking the charge repeatedly form A to B along the high energy path and back to A along the low energy path. In Fig. 1.30, if 1 coulomb of charge moves form A to B along the element 1, it loses v_1 Joules of energy. If the same charge is moved through elements 2 and 3 to reach the point B, it loses $(v_2 - v_3)$ Joules of   energy. Since these

two energies must be the same, we have.

$$v_1 = v_2 - v_3 \qquad\qquad .....(1.26)$$

or $\qquad\qquad v_2 - v_3 - v_1 = 0 \qquad\qquad .....(1.27)$

Thus if we move 1 coulomb of charge, starting from any node of a network, along any closed path or a circuit, the net energy gained or lost must be equal to zero.

In applying the KVL, one may adopt the following procedure to minimise errors in writing the equations. Starting from any node in the closed path, move along a clockwise direction (if you chose, you may follow anti – clockwise direction also !) and assign positive sign for the voltage across the element, if you are entering it at positive reference terminal and assign negative sign for the voltage, if you are entering a negative, terminal. The algebraic sum of all these voltages must be zero. Following this procedure, starting from node A, the KVL for the closed path of network in Fig. 1.30, will be,

Fig. 1.30 Illustration of Kirchhoff 's Voltage Law

$$v_2 - v_3 - v_1 = 0 \qquad\qquad .....(1.28)$$

Eq. (1.28) is same as eq. (1.27). It is to be emphasised here that the polarities of voltages shown in Fig. 1.30 are only reference directions and the voltages themselves may have positive or negative values.

The Ohm's law, combined with the two laws of Kirchhoff, enables us to analyse a network consisting of resistors and sources. By the analysis of a network, we mean, determining the currents and voltages in all the elements forming the network.

1.8   RESISTIVE NETWORKS

A resistive network consists of only resistors and sources. In this chapter we will consider sources which have a constant value, i.e., d.c sources only. The method of solution we would follow, is to define certain unknown variables, usually some currents or voltages, write equations in terms of these variables and then solve these equations using rules of algebra. The equations are based on the volt ampere relations of the elements and the two Kirchhoff 's laws. This procedure is a mechanical process and can be applied to solve any network easily. First, we will consider simple networks consisting of several resistors connected in series or parallel, and then analysis more complicated networks.

1.8.1   Resistors in series

By definition, all the elements that carry the same current are said to be in series. It is to be noted here that the elements must carry 'same' current and not equal current to be considered as connected in series. In a complex network two or more resistor in different parts of the network may carry equal currents but they are not connected in series. The elements must form a single chain between two terminals. It is often convenient to replace this chain of resistors connected in series with a single equivalent resistor between the two terminals. The two networks have same volt ampere relation at the two terminals, as shown in Fig. 1.31.
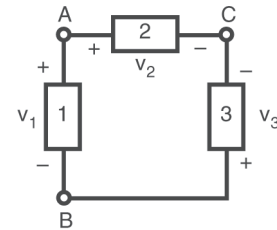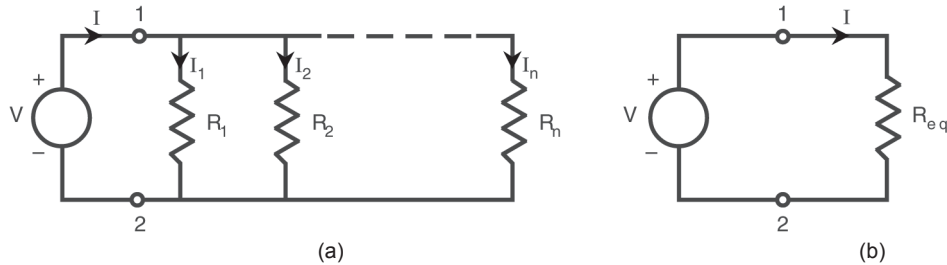
Fig. 1.31 Resistors connected in series and its equivalent

From a trivial application of KCL it can be seen that the same current 'I' flows through each of the resistors in the circuit of Fig. 1.31(a). Assuming a reference direction for the current to be directed out from the positive terminal of the source and using passivity condition the voltages across the resistors are taken to have positive signs at the terminal where the current enters the resistor. Using Kirchhoff's law around the circuit travelling in a clockwise direction starting from terminal 1,

$$V_1 + V_2 + ... + V_n - V = 0 \qquad \qquad .....(1.29)$$

But $\qquad V_1 = IR_1, \ V_2 = IR_2 \ ..... V_n = IR_n \qquad \qquad .....(1.30)$

Therefore $\qquad IR_1 + IR_2 + ... + IR_n - V = 0$

or $\qquad I(R_1 + R_2 + ... + R_n) = V \qquad \qquad .....(1.31)$

$$I = \frac{V}{R_1 + R_2 + ... + R_n} \qquad \qquad .....(1.32)$$

For equivalent network in Fig. 1.31(b), if we assume that the same current is produced when the applied source is same, we have

$$I = \frac{V}{R_{eq}} \qquad \qquad .....(1.33)$$

From eq. (1.32) and (1.32)

$$R_{eq} = R_1 + R_2 + - - - + R_n \qquad \qquad ....(1.34)$$

Thus the two networks of Fig. 1.31(a) and (b) are equivalent at the two terminals 1 and 2 if they satisfy the relation given by eq. (1.34). If the two networks to the right of the terminals 1 and 2 in Fig. 1.31(a) and (b) are enclosed in a 'black box', it is impossible to tell one from the other.

Example 1.11

Solve for the current I in the circuit of Fig. 1.32(a).
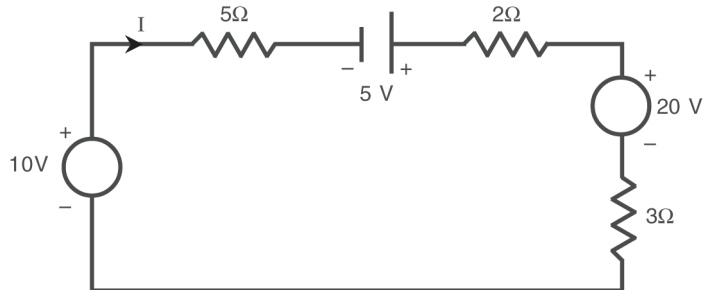
Fig. 1.32(a) Circuit for example 2.1

Solution :

As the order in which the elements are connected is immaterial in a series circuit, the network may be redrawn as shown in Fig. 1.32(b).
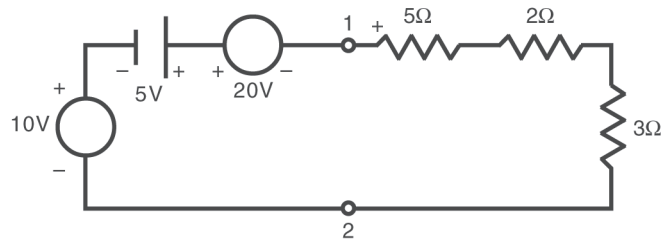
Fig. 1.32(b) Network of Fig. 1.32(a) redrawn

Now the sources connected between the terminals 1 and 2 can be replaced by a single voltage source V_{eq} and the resistors connected in series between the same terminals can be replaced with a single resistor R_{eq}. as shown in Fig. 1.32(c).

$$V_{eq} = 10 + 5 - 20 = - 5 \text{ V}$$
$$R_{eq} = 5 + 2 + 3 = 10 \ \Omega$$

Observe the polarity for V_{eq}. Since the reference polarity for voltage source is taken such that terminal 1 is at a higher potential compared to the terminal 2, the actual voltage is – 5 V.

Thus $\qquad I = \dfrac{V_{eq}}{R_{eq}} = \dfrac{-5}{10} = - 0.5 \ \text{A}$

or equivalently a current of 0.5 A is flowing in the resistor form terminal 2 to 1. If the reference polarity is reversed the circuit may be written as shown in Fig. 1.32(d).
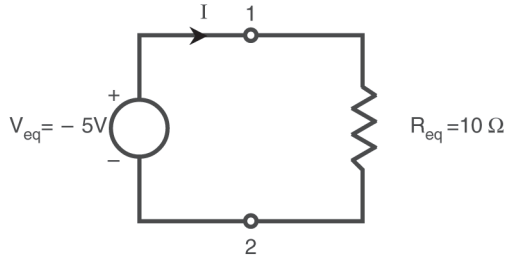
Fig. 1.32(c) Equivalent network for circuit in
Fig. 1.32(a)
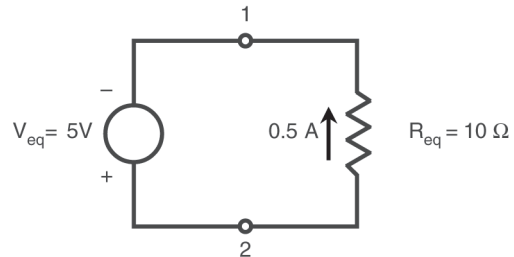
Fig. 1.32(d) Alternate form for circuit in
Fig. 1.32(c)

The example 1.11 illustrates the following important points :

1.   The order in which the elements are connected in series can be changed as desired.

2.   The reference polarities for voltages at any set of terminals is arbitrary.

3.   Once the reference polarities are chosen, the actual voltages are either positive or negative.

4.   All the voltage sources connected in series between two terminals can be replaced by a single equivalent voltage source.

5.   All the resistors connected in series between two terminals can also be replaced by a single equivalent resistor.

1.8.2 Resistors in Parallel

A set of resistors are in parallel if the same voltage is present across each of the resistors. For the network in Fig. 1.33(a).
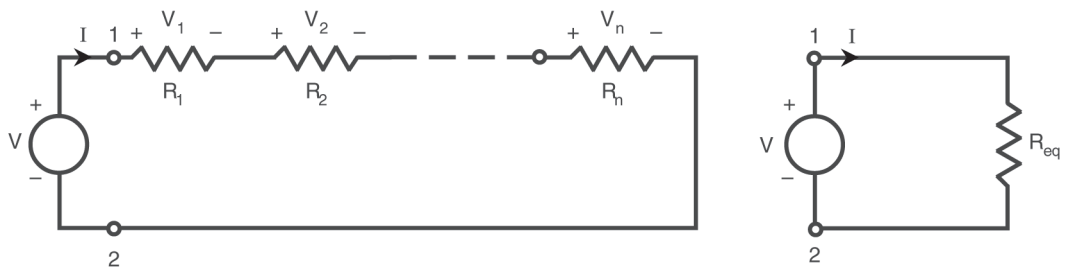
Fig. 1.33(a) Resistors in parallel

By applying KCL at node 1

$$I = I_1 + I_2 + ... + I_n \qquad .....(1.35)$$

But

$$I_1 = \frac{V}{R_1} , \quad I_2 = \frac{V}{R_2} ..... I_n = \frac{V}{R_n} \qquad ....(1.36)$$

Therefore $\qquad I = \dfrac{V}{R_1} + \dfrac{V}{R_2} + ... + \dfrac{V}{R_n}$

$$= V \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + ... + \dfrac{1}{R_n} \right) \qquad .....(1.37)$$

Similarly for network in Fig. 1.33(b), for the same voltage and current

$$I = \dfrac{V}{R_{eq}} \qquad .....(1.38)$$

Since both the networks have same V – I equations, we have from eq. (1.37) and eq. (1.38).

$$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + ... + \dfrac{1}{R_n} \qquad .....(1.39)$$

From the definition of conductances of the resistors, eq (1.39) may be written as

$$G_{eq} = G_1 + G_2 + ... + G_n \qquad .....(1.40)$$

As in the case of series connected resistors, the order in which these resistors are connected in parallel between two terminals is not important. Any number of independent current sources connected across these terminals may also be combined into a single equivalent current source with a value of current equivalent to the algebraic sum of the individual currents.

Example 1.12

Consider the network in Fig. 1.34(a). Reduce it to a single current source and single resistor network at the terminals 1 and 2 and find the voltage across them.
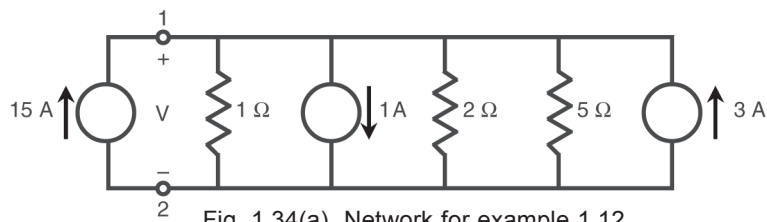
Fig. 1.34(a) Network for example 1.12

Solution :

The network is redrawn as shown in Fig. 1.34(b).
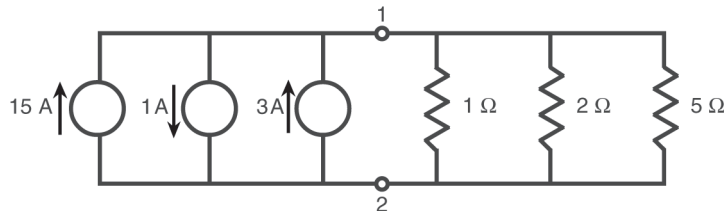
Fig. 1.34(b) Network of Fig. 1.34(a) redrawn

The equivalent network between terminals 1 and 2 is given in Fig. 1.34(c) with

$$I_{eq} = 15 - 1 + 3 = 17 \text{ A}$$

$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{5} = 1.7$$

or $$R_{eq} = \frac{1}{1.7} \ \Omega$$

$$V = I_{eq} \times R_{eq}$$

$$= 17 \times \frac{1}{1.7}$$

$$= 10 \text{ V}$$
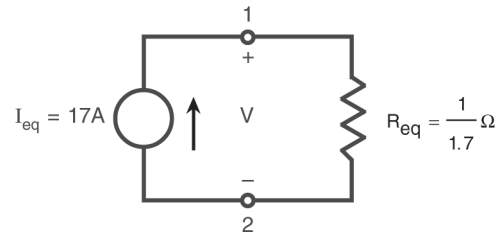
Fig. 1.34(c) Equivalent network of circuit in Fig. 1.34(a)

In many networks, two resistors connected in parallel is very common and thus it merits special attention. For two resistors in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \qquad ...(1.41)$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad \qquad .....(1.42)$$

It is easier to use relation (1.42) rather than eq. (1.41) to find R_{eq} when two resistors are connected in parallel.

1.8.3 The Voltage Divider

A practical application of resistors connected in series across a voltage source is the voltage divider. When a large voltage source is available and we desire to have a smaller voltage, we use a voltage divider circuit shown in Fig. 1.35.

In the circuit of Fig. (1.35)

$$R_{eq} = R_1 + R_2 + R_3$$

and $$I = \frac{V}{R_1 + R_2 + R_3}$$

Fig. 1.35 A voltage divider circuit

The voltage across the resistor R_3 is

$$V_0 = I R_3$$

$$= \frac{R_3}{R_1 + R_2 + R_3} \cdot V \qquad \qquad .....(1.43)$$

Eq. (1.43) implies that the voltage across any resistor in a voltage divider circuit will have the same proportion to the total voltage as that resistance to the total resistance. Thus for a set of series resistors, voltages across the resistors will be in proportion to the corresponding resistances.

1.8.4 The Current Divider

When several resistors are connected in parallel across a current source, we have a current divider problem as shown in Fig. 1.36.

The equivalent resistance is given by

$$R_{eq} = \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3}}$$

and $\qquad V = I\, R_{eq}$
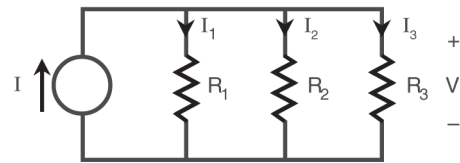
Fig. 1.36. A Current divider circuit

$$= I\, \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3}}$$

Once the voltage is known across the resistor, the individual current can be obtained as

$$I_1 = \frac{V}{R_1} = \cfrac{\cfrac{1}{R_1}}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3}}\; I \qquad\qquad .....(1.44)$$

Eq. (1.44) shows that the current divides as the inverse of the resistance or directly to the conductance.

The case of two resistors connected in parallel occurs frequently and the current in individual resistors can be calculated easily. In Fig. 1.37.

the currents I_1 and I_2 are given by

$$I_1 = \cfrac{\cfrac{1}{R_1}}{\cfrac{1}{R_1} + \cfrac{1}{R_2}}\; I$$
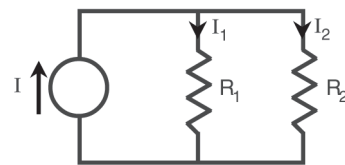
Fig. 1.37 Two resistors is parallel

$$= \frac{1}{R_1} \times \frac{R_1 R_2}{R_1 + R_2}\, I = \frac{R_2 I}{R_1 + R_2} \qquad\qquad .....(1.45)$$

Similarly $\qquad I_2 = \dfrac{R_1 I}{R_1 + R_2} \qquad\qquad\qquad .....(1.46)$

Eqs. (1.45) and (1.46) show that the current in any branch is equal to the total current multiplied by the resistance in the other branch divided by the sum of the resistances.

Example 1.13

Find the current in the 2 Ω resistor in Fig. 1.38

From current division principle

$$I = \frac{1}{1+2} \cdot 3$$

$$= 1\,A$$
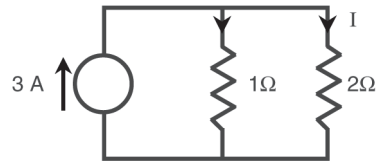
Fig. 1.38 Network for example 2.3

1.8.5 Ladder Network

A set of resistors connected alternately in series and parallel is known as a *ladder network*. By combining the resistors in series and in parallel, a ladder network between two terminals can be replaced by an equivalent resistor at these terminals.

Example 1.14

Find the currents in all the branches of the network in Fig. 1.39.
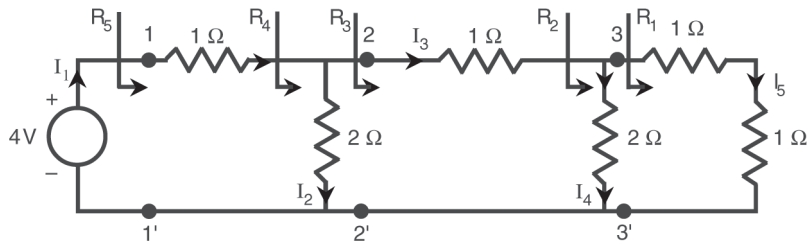
Fig. 1.39 (a) Ladder network for example 1.14

Solution :

Reducing the network successively starting from right at terminals 3 and 3', the resulting networks at different stages of combination are shown in Fig. 1.39.
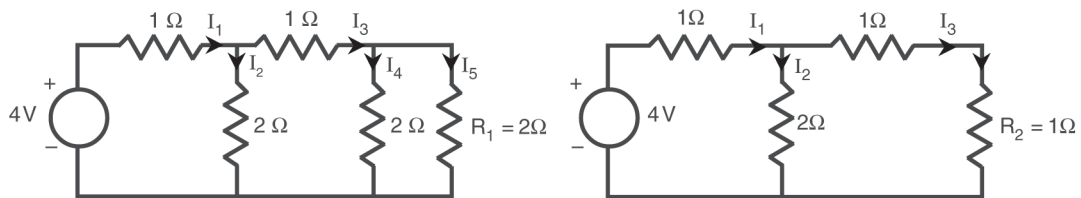
(b)   (c)
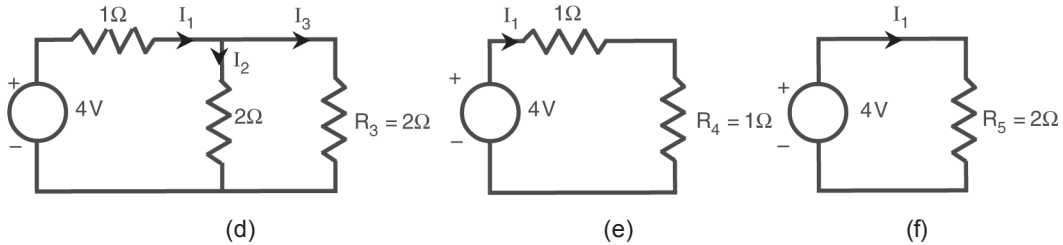
Fig. 1.39 Various intermediate networks obtained by series parallel combination of resistors

In Fig. 1.39(b), R_1 is the series combination of the two right most resistances.

$$R_1 = 1 + 1 = 2 \ \Omega$$

In Fig. 1.39(c) R_2 is the parallel combination of 2 Ω resistor and R_1 i.e.,

$$R_2 = \frac{2 \times 2}{2 + 2} = 1 \ \Omega$$

Proceeding in a similar manner

$$R_3 = 2 \ \Omega \qquad R_4 = 1 \ \Omega \quad \text{and finally} \ R_5 = 2 \ \Omega$$

Thus the current supplied by the source is, from Fig. 1.39(f)

$$I_1 = \frac{4}{2} = 2 \ A$$

To find the currents in all the branches, we proceed backwards and apply the principle of current division. Thus the current in 1 Ω resistor and R_4 is same as current I_1 in Fig. 1.39(f). The currents I_2 and I_3 in Fig. 1.39(d) are given by

$$I_2 = \frac{I_1 \times 2}{2 + 2} = \frac{2 \times 2}{4} = 1 \ A$$

$$I_3 = \frac{I_1 \times 2}{2 + 2} = \frac{2 \times 2}{4} = 1 \ A$$

The currents I_1, I_2, I_3 are shown in Fig. 1.39(c). Again currents I_4, I_5 are given by current division as

$$I_4 = I_3 \times \frac{2}{2 + 2} = 1 \times \frac{2}{4} = 0.5 \ A$$

$$I_5 = I_3 \times \frac{2}{2 + 2} = 1 \times \frac{2}{4} = 0.5 \ A$$

Thus the currents in all the resistors are found.

Fig. 1.39(g) Equivalent network

If the network to the right of terminals 1 and 1' is to be replaced by its equivalent resistance, the network appears as in Fig. 1.39(g).

1.8.6 Source transformations

We had defined ideal voltage and current sources in Chapter 1. But real world sources are seldom ideal. The practical sources have a drooping v – i characteristic rather than a constant value as shown in Fig. 1.40(a) and (b).
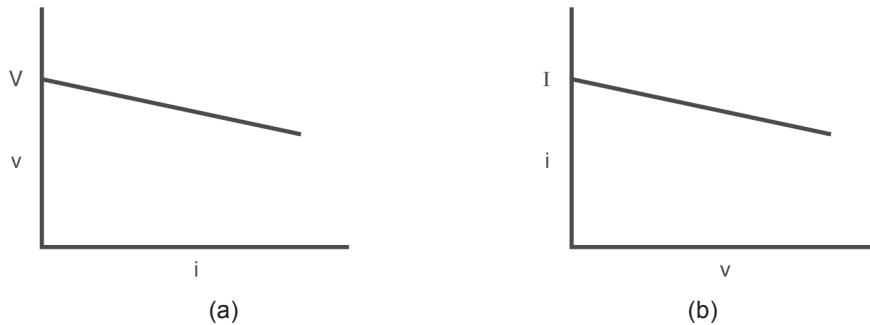
Fig. 1.40 v – i characteristics of
(a) Practical voltage source (b) Practical current source

These real physical sources can be represented by an ideal voltage source in series with a resistance or an ideal current source in parallel with a resistance as shown in Fig. 1.42(a) and (b).
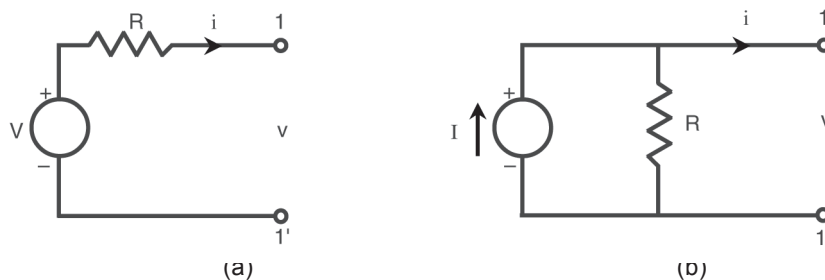
Fig. 1.42. Representation of (a) practical voltage source (b) practical current source

The volt – ampere relation for the circuit in Fig. 1.42(a) is

$$v = V - iR \qquad\qquad .....(1.47)$$

This relationship gives rise to the characteristic shown in Fig. 1.41(a)

Similarly the volt – ampere equation for Fig. 1.42(b) is given by

$$i = I - \frac{v}{R} \qquad\qquad .....(1.48)$$

This relationship gives rise to the characteristic shown in Fig. 1.41(b).

It is often convenient to transform a branch containing an ideal voltage source in series with a resistor to a branch containing an ideal current source in parallel with a resistor or vice versa.

Towards this end, let us solve eq. (1.48) for v to get,
$$v = IR - iR \qquad \qquad .....(1.49)$$

Comparing eq. (1.49) with eq. (1.47), we get
$$V = IR \qquad \qquad .....(1.50)$$

Similarly solving for i in eq. (1.47)
$$i = \frac{V}{R} - \frac{v}{R} \qquad \qquad .....(1.51)$$

Comparing with eq. (1.48), we get
$$I = \frac{V}{R} \qquad \qquad .....(1.52)$$

From eq. (1.47), eq. (1.48), eq. (1.49) and eq. (1.52) we can develop the   equivalent circuits shown in Fig. 1.43(a) and (b).
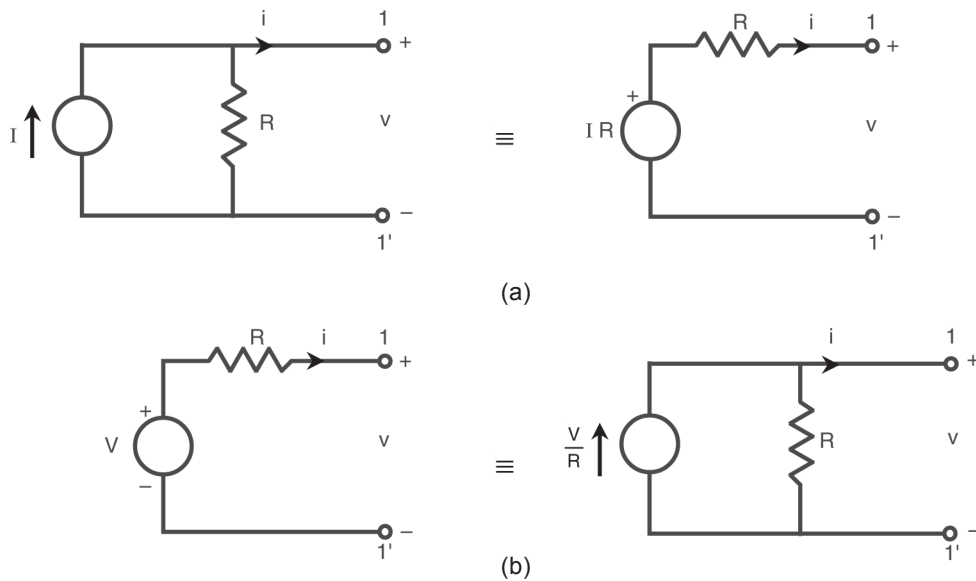
Fig. 1.43 Source transformations

1.8.7 Voltage source with no series resistance

Practical voltage sources will always have a series resistance along with an ideal voltage source. But sometimes these resistances are so small that they can be neglected. But such branches with only ideal voltage sources in a network cause problems. It is not possible to transform this branch into an equivalent current source branch. To avoid this problem, a technique by which the ideal voltage source with no series resistance is removed and replaced with a real voltage source with a series resistance is developed.

Consider the network in Fig. 1.44(a).

(a)   (b)   (c)
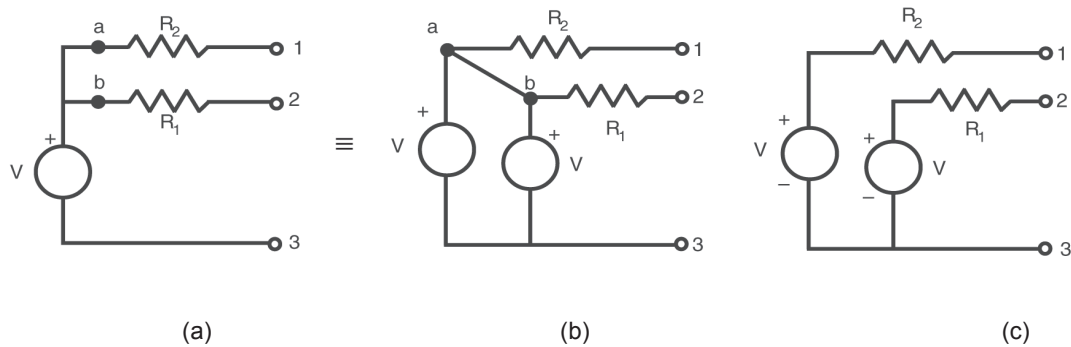
Fig. 1.44 A transformation for voltage sources without resistance in series

The voltage source V is replaced by two voltage sources of value V in parallel as shown in Fig. 1.44 (b). The points a and b, are at the same potential V, whether the short circuit between a and b is present or not. Hence removing the short circuit we get the equivalent circuit shown in Fig. 1.44(c).

1.8.8 Current source with no parallel resistor

A real current source can be represented by an ideal current source in parallel with a resistance. But sometimes this resistance is so large in comparison with other resistances in the network, that it can be neglected. Hence we are left with an ideal current source with no resistance in parallel. This also poses problem in that, it cannot be converted to a branch with a voltage source. This problem can be overcome by considering the following circuit in Fig. 1.45(a) and replacing the current source between terminals 1 and 2 by three current sources of same value as shown in Fig. 1.45(b).

(a)   (b)
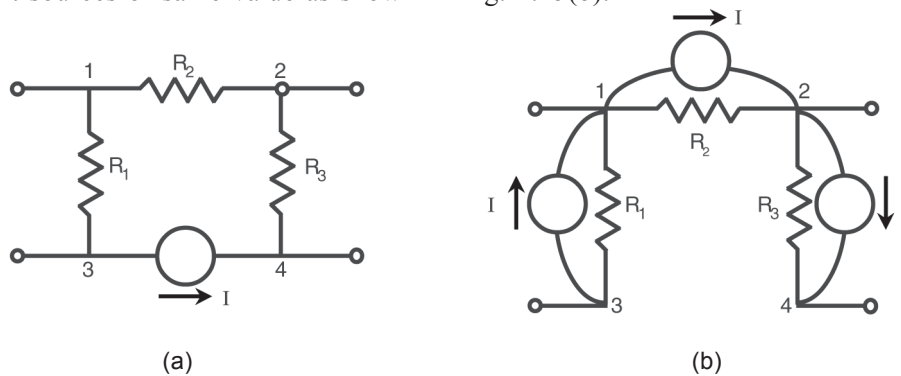
Fig. 1.45 A transformation for current source with no parallel resistance

The effect of current source is to deliver 1A from node 3 to node 4. The same effect is produced by the 3 sources of 1A connected across the three resistors without affecting the

currents or voltages at the other nodes. 1 Amp of current enters the node 1 from node 3 and the same current immediately leaves the node 1 and enters the node 2. Again the same current leaves the node 2 and enters the node 4. This in effect sends a current from node 3 to 4 via nodes 1 and 2 instead of directly, without affecting the currents at other nodes.

1.8.9   Voltage sources with parallel resistors or current sources

When a voltage source is connected in parallel with a set of resistors or current sources at two terminals in a network, the combination can be replaced by the voltage source only. Two examples are given in Fig. 1.46.

(a)

(b)
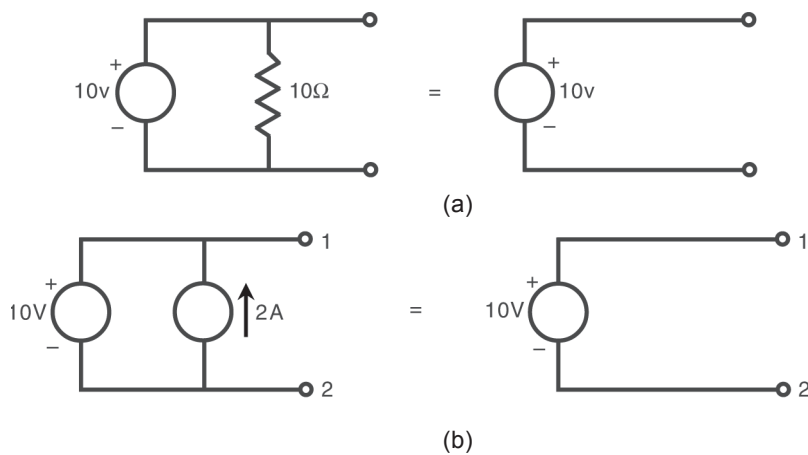
Fig. 1.46 Example of voltage source in parallel with (a) resistor (b) current source

More complex example is given in Fig. 1.47.
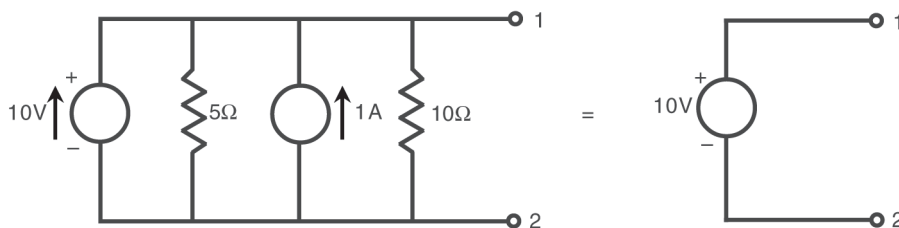
Fig. 1.47 Another example of voltage source in parallel with other elements

1.8.10   Current sources in series with resistors or voltage sources

When a current source is in series with resistors or voltage sources or any combination of these at a set of two terminals, the combination can be replaced by a single current source at these terminals. Three examples are given in Fig. 1.48 illustrating this equivalence.

(a)

(b)

(c)
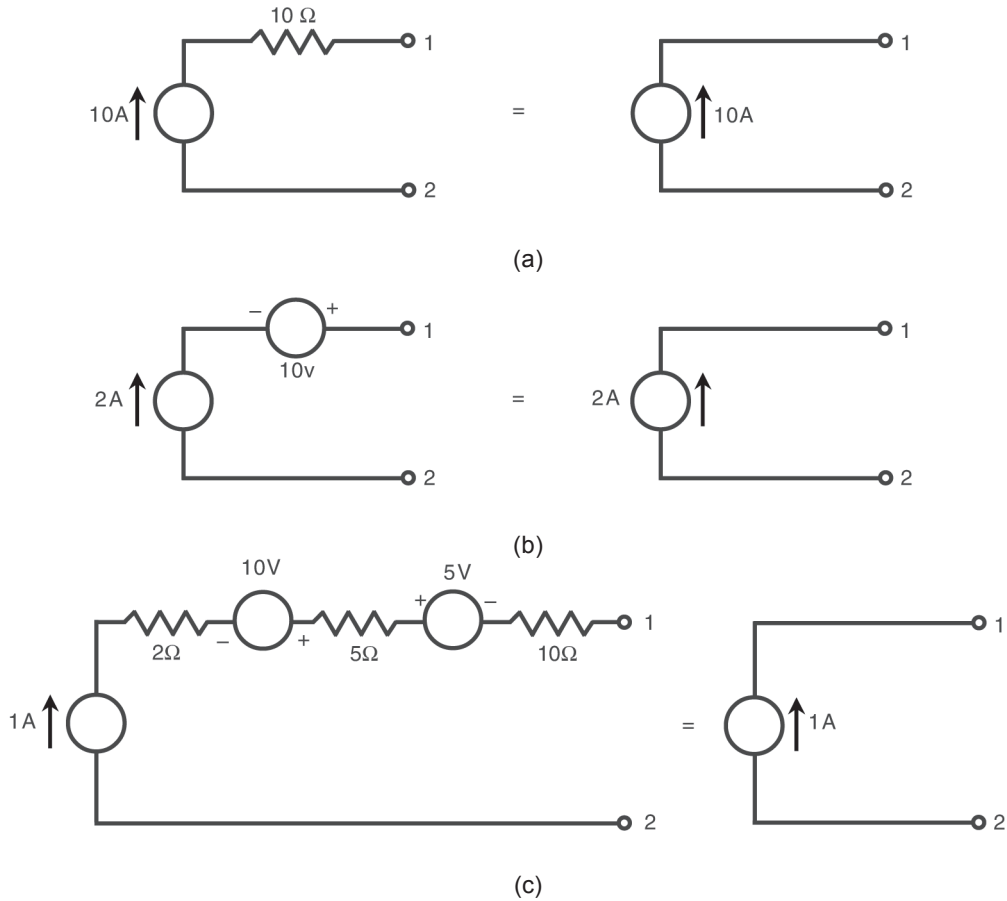
Fig. 1.48 Examples of current source in series with other elements

Example 1.15

Use source transformation to simplify the network in Fig. 1.49(a) and find the equivalent network containing only one voltage source and a resistance at the terminals 1,2.
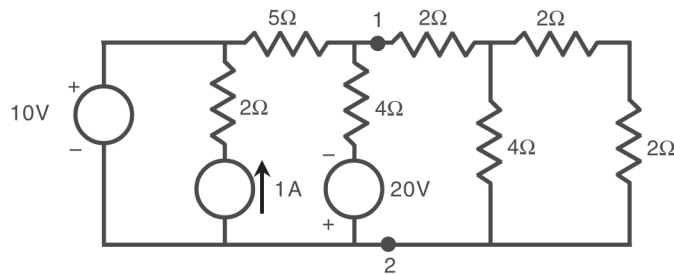
Fig. 1.49(a) Network for example 1.15

Solution :

The network to the right of terminals 1 and 2 is a ladder network and can be simplified to get its equivalent resistance

$$R_{eq} = 2 + \frac{4(2+2)}{4+2+2} = 4 \ \Omega$$

Ignoring the branch containing the current source and a series resistance which is connected in parallel with a 10 V voltage source, the network can be written in simplified form as :
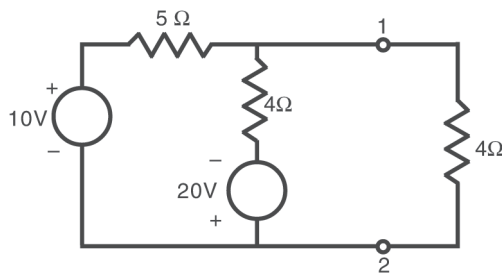
(b)

Converting the two branches containing voltage source and series resistance to branches containing current source in parallel with a resistance, using source transformation, we have
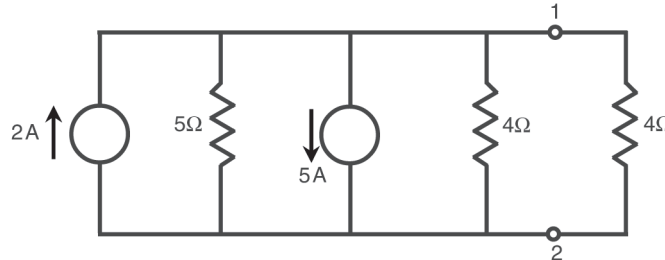
(c)

Combining the current sources which are in parallel, and resistances in parallel, across terminals 1 and 2, we get
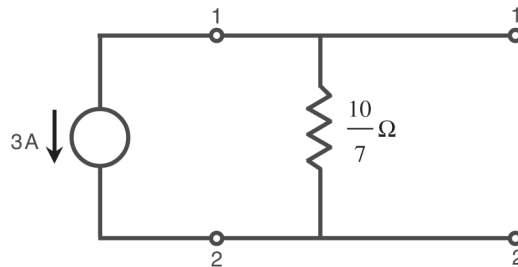
(d)

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{4} + \frac{1}{4} = 0.7$$

$$R = \frac{1}{0.7} = \frac{10}{7} \ \Omega$$

Finally applying source transformation to the current source in parallel with the resistance, we get the equivalent network shown in Fig. 1.49(e) which is the desired result.
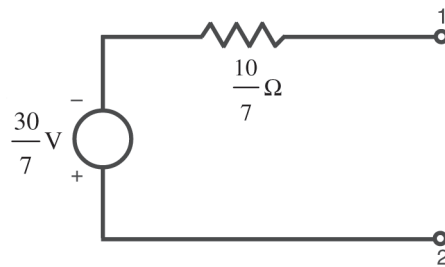
(e)

Example 1.16

Find the power delivered by the 5 V voltage source in the network of Fig. 1.50
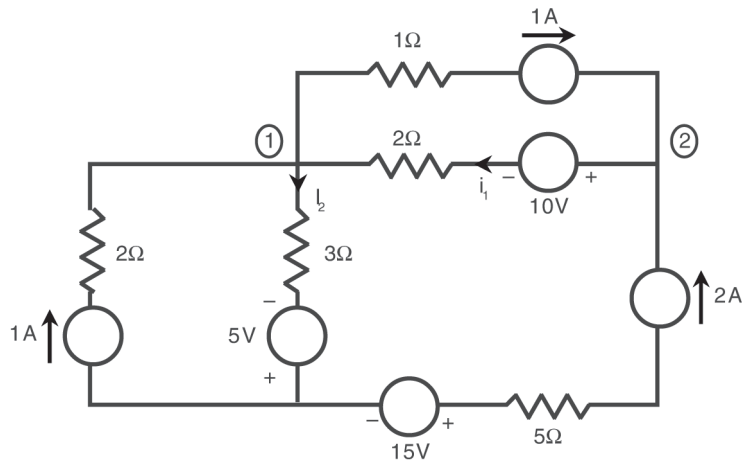
Fig. 1.50(a) Network for example 1.16

Solution :

Applying KCL at node 2, we have

$$I_1 = 2 + 1 = 3 \text{ A}$$

Applying KCL at node 1, we have

$$I_2 = 1 - 1 + 3 = 3 \text{ A}$$

Since the current in the 5 V source branch is known and the direction of current is directed out of positive terminal of the 5 V voltage source, the power delivered by the source is given by

$$P_{5V} = 5 \times 3 = 15 \text{ W}$$

If desired, the power delivered by each of the other sources in the network can also be calculated.

Power delivered by the 15 V voltage source is

$$P_{15V} = 15 \times 2 = 30 \text{ W}$$

Similarly

$$P_{10V} = 10 \times (-3) = -30 \text{ W}$$

To calculate the power delivered by the current sources, we need to know the voltages across them. These voltages can be calculated by applying KVL to a closed loop containing these sources.
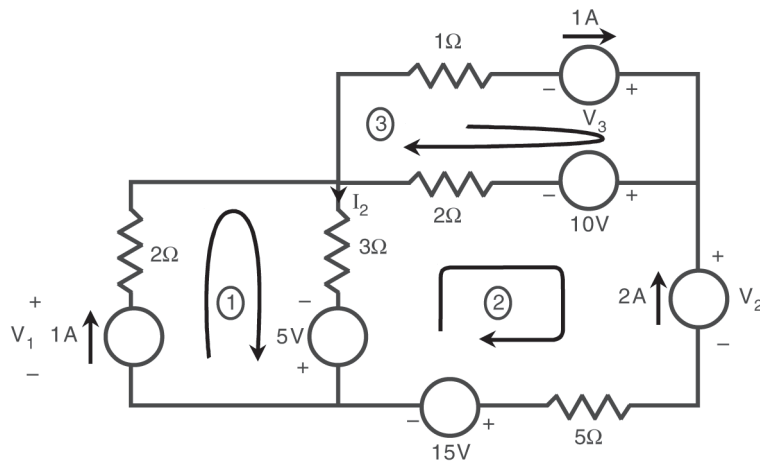
Fig. 1.50(b) Network of example 1.16 with three closed paths identified

For closed path 1, applying KVL

$$-V_1 + 1 \times 2 + 3 \times 3 - 5 = 0$$

which gives,

$$V_1 = 6 \text{ V}$$

$$\therefore \qquad P_{1A} = 6 \times 1 = 6 \text{ W}$$

For closed path 2

$$5 - 3 \times 3 - 3 \times 2 - 10 + V_2 - 5 \times 2 + 15 = 0$$

$$V_2 = +15V$$

$$P_{2A} = 15 \times 2 = 30 \text{ W}$$

For closed path 3

$$1 \times 1 - V_3 + 10 + 3 \times 2 = 0$$

$$V_3 = 17 \text{ V}$$

$$P_{1A} = 17 \times 1 = 17 \text{ W}$$

The power delivered to the resistors can be calculated by summing the power consumed by each resistor.

Power delivered to the resistors

$$P_R = 1^2 \times 2 + 3^2 \times 3 + 2^2 \times 5 + 3^2 \times 2 + 1^2 \times 1$$

$$= 2 + 27 + 20 + 18 + 1$$

$$= 68 \text{ W}$$

Total power delivered by the sources is the algebraic sum of powers delivered by the sources,

$$P_S = 15 + 30 - 30 + 6 + 30 + 17$$

$$= 68 \text{ W}$$

It is obvious that the total power delivered by the sources is consumed by the resistors in the network.

1.8.11 Star – Delta or Y – Δ Transformation

In many of the networks, the resistances are neither connected in series nor in parallel and hence the network cannot be simplified using the series or parallel combination of resistors. In such cases star – delta or wye – delta (Y – Δ) transformation, which will be developed in this section, is useful.

Consider a 3 – terminal network in which 3 resistors are connected at a common point O as shown in Fig. 1.51(a). Similarly consider another 3 – terminal network with 3 resistors connected to form a triangle as shown in Fig. 1.51(b).

(a)   (b)
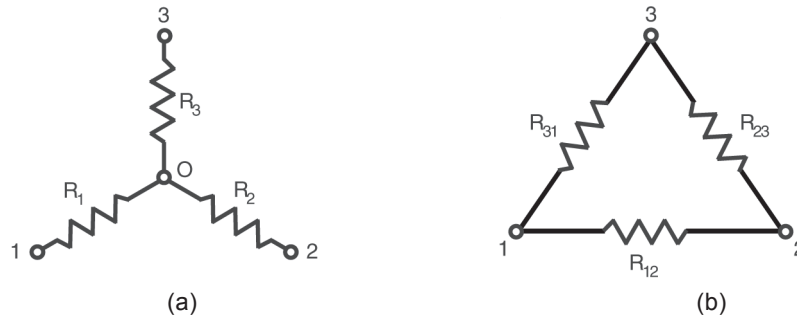
Fig. 1.51 Star – Δ or Y – Δ networks

If we are interested only in the effects produced by these networks when they are connected to other networks at these three terminals or when they form a part of another network, we can establish equivalence of these two networks and replace one with the other at the three terminals.

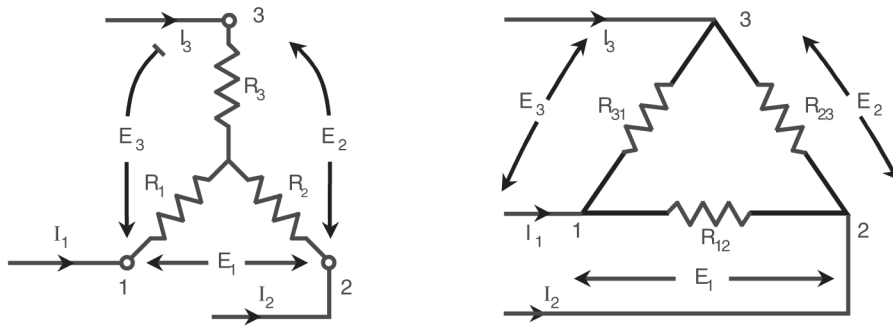The conditions for the equivalence are shown in Fig. 1.51(c).

Fig. 1.51(c) Conditions for equivalence of the two networks

For similar conditions existing at the three terminals of these two networks, i.e. if the currents entering the three terminals I_1, I_2 and I_3 and the three voltages across terminal pairs, E_1, E_2 and E_3 are equal to the corresponding currents and voltages of the other network, then these two networks are equivalent.

Finding the equivalent resistance between the terminals 1, 2 with terminal 3 open, for both the networks in Fig. 1.51(c), and equating them, we get

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad .....(1.53)$$

($\therefore R_{31}$ and R_{23} are in series and together they are in parallel with R_{12} in the Δ network)

Similarly, repeating this with each of the other pair of terminals, we have

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \qquad .....(1.54)$$

and $$R_3 + R_1 = \frac{R_{31}(R_{23} + R_{12})}{R_{12} + R_{23} + R_{31}} \qquad .....(1.55)$$

Solving the set of equations (2.32), (2.33) and (2.34)

we get $$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \qquad .....(1.56)$$

$$R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{31}} \qquad .....(1.57)$$

and $\quad R_3 = \dfrac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$ $\hspace{4cm}$ .....(1.58)

A general expression for a resistance in Y − network can be written in terms of the resistances of the Δ − network with reference to Fig. 1.51(a), we have Resistance in

$$Y = \dfrac{\text{Product of the resistances of adjacent branches of } \Delta}{\text{Sum of the } \Delta \text{ resistances}} \hspace{1cm} ....(1.59)$$

For example, in Fig. 1.51(d) R_1 can be obtained as the product of the two adjacent resistances R_{12} and R_{31} divided by the sum of the Δ − resistances.

Fig. 1.51(d) Δ to star conversion

In a similar manner, the equivalent Δ resistances can be obtained in terms of resistances in star, by shorting a pair of terminals, say, 1 and 2 and finding the equivalent resistance between 1 and 3. Using conductances rather than resistances, the equivalent conductances of the two networks can be equated. Thus

$$G_{31} + G_{32} = \dfrac{G_3(G_1 + G_2)}{G_1 + G_2 + G_3}$$

$$....(1.60)$$

Similarly for other pairs of terminals, we have

$$G_{12} + G_{31} = \dfrac{G_1(G_2 + G_3)}{G_1 + G_2 + G_3} \hspace{2cm} .....(1.61)$$

and $\hspace{2cm} G_{12} + G_{32} = \dfrac{G_2(G_1 + G_3)}{G_1 + G_2 + G_3} \hspace{2cm} .....(1.62)$

Solving the set of equation (1.60) to (1.61), we get

$$G_{12} = \dfrac{G_1 G_2}{G_1 + G_2 + G_3} \hspace{3cm} .....(1.63)$$

$$G_{23} = \dfrac{G_2 G_3}{G_1 + G_2 + G_3} \hspace{3cm} .....(1.64)$$

$$G_{31} = \dfrac{G_3 G_1}{G_1 + G_2 + G_3} \hspace{3cm} .....(1.65)$$

A general expression for the conductances of Δ network in terms of the Y − conductances is given by

$$\text{Conductance of } \Delta \text{ branch} = \dfrac{\text{Product of conductances of adjacent branches of Y}}{\text{Sum of Y conductances}}$$

$$.....(1.66)$$
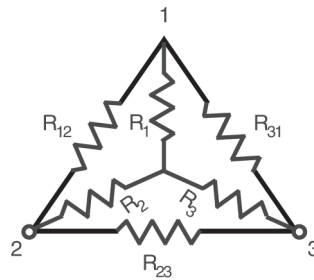
For example in Fig. 1.51(e) G_{12} can be obtained as the product of conductances the two adjacent branches G_1 at G_2, divided by sum of the Y conductances.

It is often more convenient to deal with the resistances than conductances. Therefore, by taking the reciprocals of the respective conductances in equations (1.63), (2.43) and (2.44) and simplifying, we get

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad\qquad .....(1.67)$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad\qquad .....(1.68)$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad\qquad .....(1.69)$$

A general expression for Δ – resistances can be written as

$$\text{Resistances in } \Delta = \frac{\text{sum of products of resistances of Y taken two at a time}}{\text{The resistance of Y not connected to this } \Delta - \text{resistance}}$$

For example in Fig. 1.51(e), R_{12} is obtained as the sum of product of resistances of Y – network taken two at a time and divided by the resistances R_3 which is not connected to R_{12}.

Using the Δ – Y or Y to Δ transformation and series parallel combinations of resistances any passive resistive network can be reduced to its equivalent resistance at a given pair of terminals.

Example 1.17

Find the equivalent resistance between the terminals 1 and 2 of the network shown in Fig. 1.52.

Solution :

We observe that, in this network, no two resistances are either in series or in parallel. Hence no simplification is possible by series or parallel combinations. However, one can recognise, 3 resistances connected in Δ or Y and convert them to Y or Δ. There are 3 resistor R_1, R_3, and R_4 connected in Δ between terminals 1, 3 and 4.

Fig. 1.52(a) Network for example 1.17

Similarly there are 3 resistor R_2, R_3, and R_5 connected in Δ between terminals 3, 4 and 2. Any one of them can be converted into an equivalent Y . Alternatively there is a set of Y connected resistors R_1, R_2, and R_3 between terminals 1, 4 and 2 and another set of star connected resistor R_4, R_3 and R_5 between terminals 1, 3 and 2. Any one of these Y's can also be converted to a corresponding Δ. Converting the Δ – between the terminals 1, 3 and 4, we get the network in, Fig. 1.52(c).
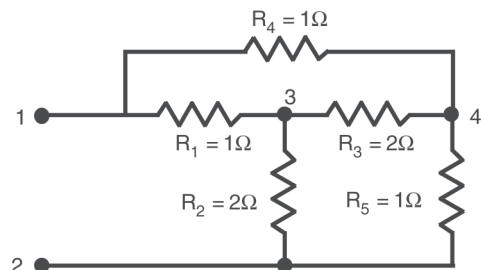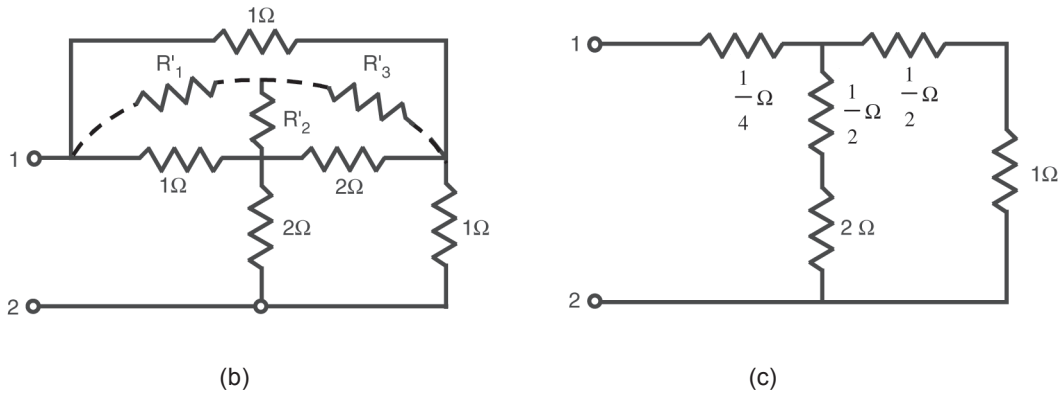
(b)   (c)

Fig. 1.52 Δ of network 1.52(b) is converted to Y and redrawn in (c)

$$R'_1 = \frac{1 \times 1}{1+1+2} = \frac{1}{4} \ \Omega$$

$$R'_2 = \frac{1 \times 2}{1+1+2} = \frac{2}{4} = \frac{1}{2} \ \Omega$$

$$R'_3 = \frac{2 \times 1}{1+1+2} = \frac{2}{4} = \frac{1}{2} \ \Omega$$

Now the resistance in Fig. 1.52(c) can be combined using series parallel combinations. The resulting networks are shown in Fig. 1.52(d), (e) and (f).
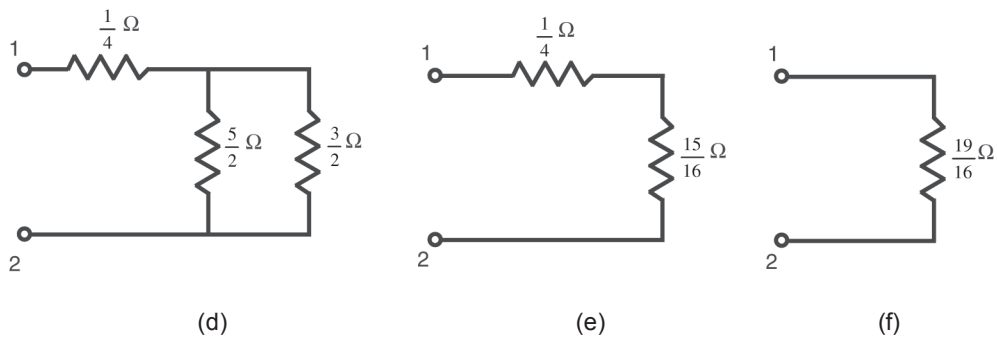

(d)   (e)   (f)

Fig. 1.52 Reduction of network in Fig. 1.52 to its equivalent resistance at terminals 1 and 2

Thus the equivalent resistances between the terminals 1 and 2 is

$$R_{eq} = \frac{19}{16} \Omega$$

Alternatively :

The Y − between the terminals 1, 3 and 2 of Fig. 1.52 can be converted to its Δ equivalent and the steps in the reduction are shown in Fig. 1.52 (g), (h), (i), (j) and (k).

(g)

(h)

(i)

(j)
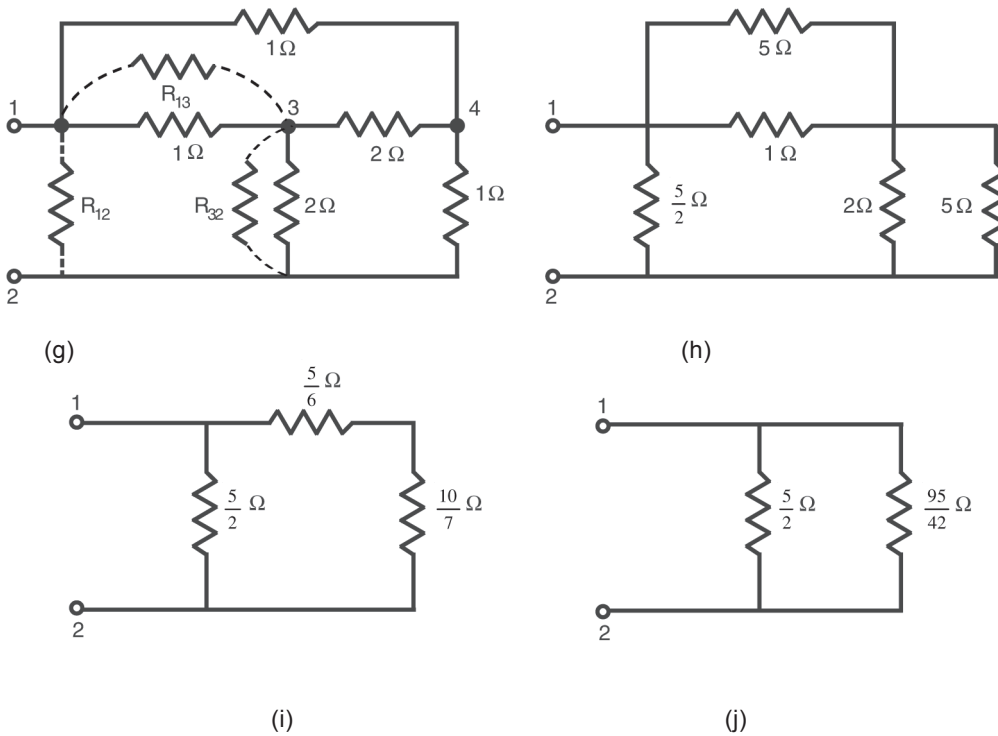
Fig. 1.52 Steps in the simplification of network of Fig. 2.24(a) by converting a star to delta

In Fig. 1.52(g)

$$R_{13} = \frac{2.1 + 2.1 + 1.1}{1} = 5 \ \Omega$$

$$R_{32} = \frac{2.1 + 2.1 + 1.1}{1} = 5 \ \Omega$$

$$R_{12} = \frac{2.1 + 2.1 + 1.1}{1} = \frac{5}{2} \Omega$$

The resulting network is shown in Fig. 1.52(h). Using series parallel combinations as shown in Fig. 1.52 (i) and (j) the equivalent resistance between terminals 1 and 2 is again obtained as

$$R_{eq} = \frac{19}{16} \ \Omega$$

(k)
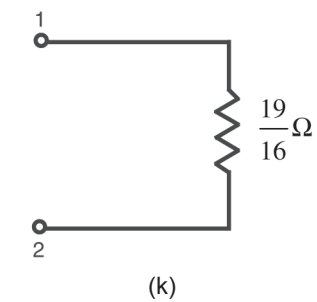
Fig. 1.52 Steps in the simplification of network of Fig. 2.24(a) by converting a star to delta

1.9   ANALYSIS OF NETWORKS

1.9.1 Node voltage method

Consider the network shown in Fig. 1.53. It is easy to identify the nodes of the
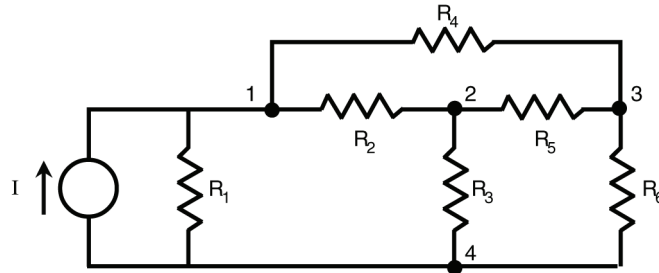
Fig. 1.53 A 4 node network

network. There are four nodes in this network. If we know the voltages across the branches of the network, we can find the currents in them using ohm's law. If instead of knowing voltage across each branch, if the voltage of every node, called as the node voltage, is known with respect to one of the nodes, called *datum node*, or *reference node*, the branch voltages can be easily calculated using Kirchhoff 's voltage law. For example, if node 4 is taken as reference in Fig. 1.53, and if the node voltages V_1, V_2 and V_3 are known with respect to node 4, the branch voltages can be easily calculated as shown in Fig. 1.54.
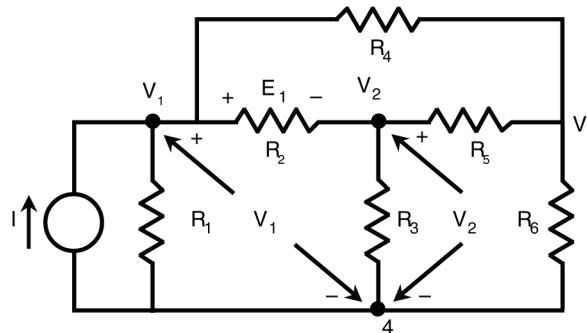
Fig. 1.54 Network with branch voltages and node voltages indicated

For simplicity, one branch voltage E_1 and two node voltages V_1 and V_2 only are shown in Fig. 1.54. Applying Kirchhoff's voltage law around the closed path

indicated we have

$$V_1 - E_1 - V_2 = 0$$

or $E_1 = V_1 - V_2$ .....(1.17)

Thus the branch voltage can be obtained as the difference of the two node voltage V_1 and V_2. In a similar manner other branch voltages can be obtained easily by knowing the node to datum voltages of all the nodes. If there are N nodes, there will be N − 1 nodes, whose

voltages have to be found with reference to the datum node. Thus to determine these $N-1$ node voltages we have to write $N-1$ equations. This can be accomplished by writing Kirchhoff 's Current Law equations for every node other than the reference node. In Fig. 1.54, the node voltages of nodes 1, 2 and 3 are designated as V_1, V_2, and V_3. These are the voltages with respect to the datum node arbitrarily chosen to be node 4.

Applying Kirchhoff 's current law at node 1, we get

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_4} - I = 0 \qquad .....(5.18)$$

It is necessary to reiterate that V_1, $V_1 - V_2$, $V_1 - V_3$ are the voltages across the branches containing resistors R_1, R_2 and R_4 respectively, and each of the terms in eq. (1.18) represents the current leaving the node in that branch. Without any loss of generality it is assumed that the voltage at the node at which KCL is written, is higher than the other nodes, so that the currents in all the branches connected to it leave the node. If, for example, the actual current R_2 is entering the node instead of leaving it, we would have written the equation as

$$\frac{V_1}{R_1} - \frac{V_2 - V_1}{R_2} + \frac{V_1 - V_3}{R_4} - I = 0 \qquad .....(1.19)$$

It is easy to observe the equivalence of eqs. (1.18) and (1.19). So it is customary to assume that the node at which we are writing the Kirchhoff 's current law equation, is at a higher potential compared to other nodes to which it is connected. Thus the other two equations at nodes 2 and 3 are,

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2 - V_3}{R_5} = 0 \qquad .....(1.20)$$

$$\frac{V_3 - V_1}{R_4} + \frac{V_3}{R_6} + \frac{V_3 - V_2}{R_5} = 0 \qquad .....(1.21)$$

The unknown voltages V_1, V_2 and V_3 can be obtained by solving eqs. (1.19), (1.20) and (1.21). Once the node voltages are known, the currents in various branches can be easily calculated.

Example 1.18

Find the node voltages in the circuit of Fig. 1.55.
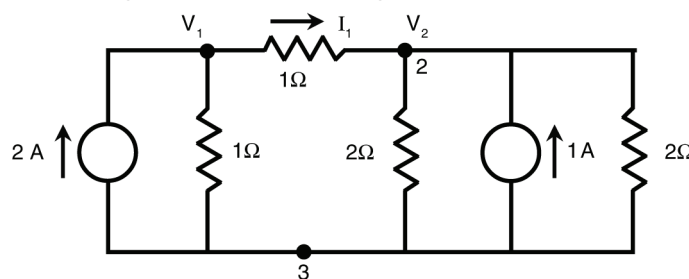
Fig. 1.55 Network for example 1.18

Solution :

There are 3 nodes and the node 3 is taken as reference. The node voltages are marked as V_1 and V_2. Writing the Kirchhoff 's current law equation at each of the nodes 1 and 2 we have

$$\frac{V_1}{1} + \frac{V_1 - V_2}{1} = 2 \qquad\qquad .....(1.22)$$

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_2}{2} = 1 \qquad\qquad .....(1.23)$$

Simplifying eqs. (1.22) and (1.23), we have.

$$2V_1 - V_2 = 2 \qquad\qquad .....(1.24)$$
$$-V_1 + 2V_2 = 1 \qquad\qquad .....(1.25)$$

Solving these two equations, we get

$$V_1 = \frac{5}{3} \text{ V}$$

$$V_2 = \frac{4}{3} \text{ V}$$

If in the above example, the current in 1 ohm resistor between nodes 1 and 2 is desired, then the voltage across this resistance is first calculated using the node voltages V_1 and V_2 and then the current can be easily calculated.

Thus the current through 1 Ω resistor connected between nodes 1 at 2, I_1, is given by

$$I_1 = \frac{V_1 - V_2}{1}$$

$$= \frac{\frac{5}{3} - \frac{4}{3}}{1} = \frac{1}{3} \text{ A}$$

The currents in other branches can be similarly calculated.

If a network consists of only current sources the application of node voltage method is direct and straight forward since KCL equation is used at each node. However if a voltage source is connected at a node, the current through the source is not directly available. If a resistance is present in series with this source,

this branch can be transformed into a current source in parallel with the resistor. This is shown in Fig. 1.56.

(a)   (b)
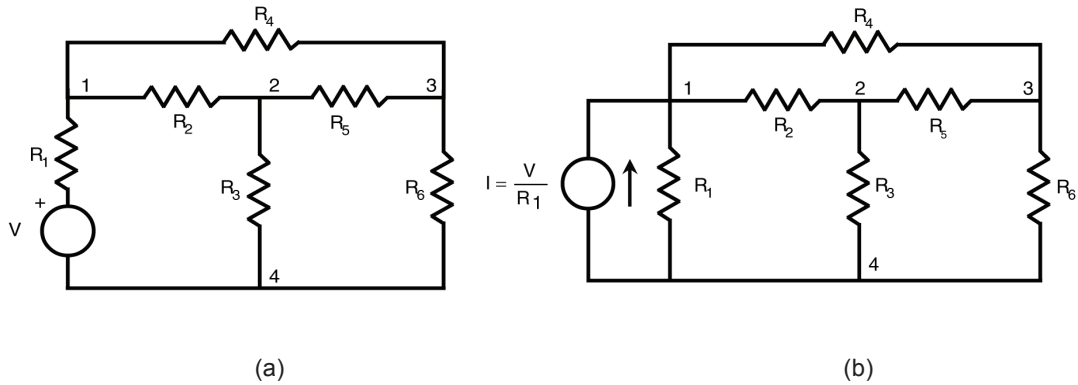
Fig. 1.56(a) A network with voltage source and
(b) the same network after source transformation

Now it is a straight forward procedure to apply the node voltage method.

It is often cumbersome to convert all voltage sources to current sources and hence it will be convenient to apply the node voltage method directly to the given network. Consider the network of Fig. 1.56(a). Let us designate the junction between R_1 and V as node 5 and its voltage with respect to reference node 4 as V_5. Now there is an additional node at which the KCL has to be applied. But as the voltage of this node is known and is equal to V, it is not necessary to write KCL equation at this node and

$$V_5 = V$$

Now applying the KCL at nodes 1, 2 and 3, we get

$$\frac{V_1 - V}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_4} = 0 \qquad \qquad .....(1.26)$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2 - V_3}{R_4} = 0 \qquad \qquad .....(1.27)$$

$$\frac{V_3 - V_2}{R_5} + \frac{V_3}{R_6} + \frac{V_3 - V_1}{R_4} = 0 \qquad \qquad .....(1.28)$$

The unknown voltages V_1, V_2 and V_3 can now be calculated using eqs. (1.26) to (1.28).

Example 5.2

Find the node voltages V_1, V_2 and V_3 in the network of Fig. 1.57 and find the current I_x.
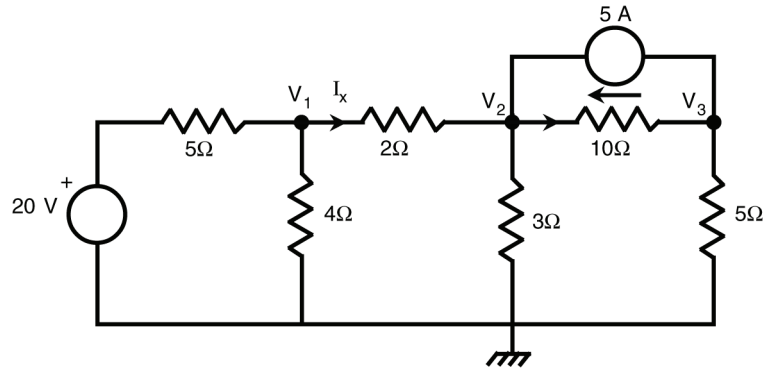
Fig. 1.57 Network for example 1.19

Solution :

Designating the node voltage as V_1, V_2 and V_3 with respect to the datum node, and writing the node voltage equations,

At node 1

$$\frac{V_1 - 20}{5} + \frac{V_1}{4} + \frac{V_1 - V_2}{2} = 0 \qquad .....(1.29)$$

$$V_1\left(\frac{1}{5} + \frac{1}{4} + \frac{1}{2}\right) - \frac{V_2}{2} = 4$$

$$19\,V_1 - 10\,V_2 = 80 \qquad .....(1.30)$$

At node 2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{3} + \frac{V_2 - V_3}{10} - 5 = 0 \qquad .....(1.31)$$

$$-\frac{V_1}{2} + V_2\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{10}\right) - \frac{V_3}{10} = 5$$

$$-15\,V_1 + 28\,V_2 - 3\,V_3 = 150 \qquad .....(1.32)$$

At node 3

$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} + 5 = 0 \qquad .....(1.33)$$

$$\frac{-V_2}{10} + V_3\left(\frac{1}{10} + \frac{1}{5}\right) = -5$$

$$-V_2 + 3\,V_3 = -50 \qquad .....(1.34)$$

Solving for V_1 from eqs. (1.30), (1.32) and (1.34)

$$V_1 = \frac{\begin{vmatrix} 80 & -10 & 0 \\ 150 & 28 & -3 \\ -50 & -1 & 3 \end{vmatrix}}{\begin{vmatrix} 19 & -10 & 0 \\ -15 & 28 & -3 \\ 0 & -1 & 3 \end{vmatrix}} = \frac{9480}{1089} = 8.7 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 19 & 80 & 0 \\ -15 & 150 & -3 \\ 0 & -50 & 3 \end{vmatrix}}{1089} = \frac{9300}{1089} = 8.54 \text{V}$$

and

$$V_3 = \frac{\begin{vmatrix} 19 & -10 & 80 \\ -15 & 28 & 150 \\ 0 & -1 & -50 \end{vmatrix}}{1089} = \frac{-15050}{1089} = -13.82 \text{V}$$

The current I_x can now be calculated

$$I_x = \frac{V_1 - V_2}{2} = \frac{8.7 - 8.54}{2}$$
$$= 0.08 \text{ A}$$

A special case occurs when a voltage source is connected at a node without any series resistance. An example is considered to illustrate this case.

Example 1.20

Find the node voltages V_1, V_2 and V_3 in Fig. 1.58.

Solution :

Applying KCL at node 1

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 1 \qquad\qquad .....(1.35)$$

$$\frac{3V_1}{2} - V_2 - \frac{V_3}{2} = 1$$

$$3 V_1 - 2 V_2 - V_3 = 2 \qquad\qquad .....(1.36)$$

If we try to apply KCL at node 2 or 3 we face a difficulty. The current in the branch containing the voltage source cannot be found and hence the KCL cannot be written at node 2 or node 3.

This difficulty can be overcome by considering the nodes 2 and 3 together and considering the combination as a '*super node*' as shown in Fig. 1.58(b). Writing KCL for this super node.
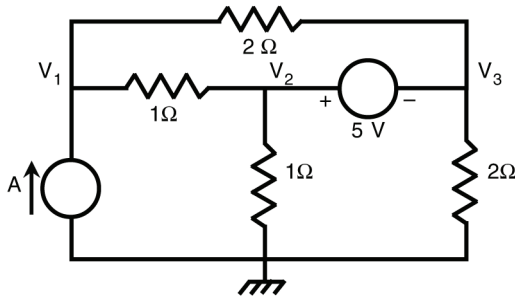
Fig. 1.58(a) Network for example 1.20   Fig. 1.58(b) Network of example 1.20 showing the supernode.
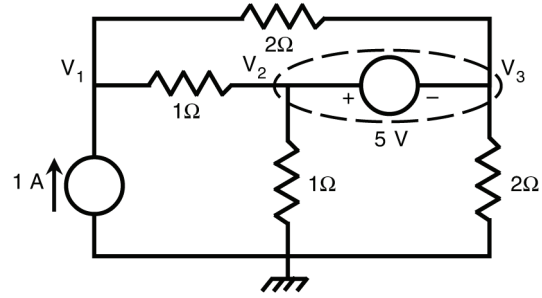
$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} + \frac{V_3}{2} + \frac{V_3 - V_1}{2} = 0 \qquad \qquad .....(1.37)$$

$$-V_1\left[1 + \frac{1}{2}\right] + V_2[1 + 1] + V_3\left[\frac{1}{2} + \frac{1}{2}\right] = 0$$

$$-3V_1 + 4V_2 + 2V_3 = 0 \qquad \qquad .....(1.38)$$

The third equation is obtained as

$$V_2 - V_3 = 5 \qquad \qquad .....(1.39)$$

Solving the eqs. (1.36), (1.38), and (1.39), we get

$$V_1 = \frac{4}{3}\ V \qquad \qquad V_2 = \frac{7}{3}\ V \qquad \qquad V_3 = \frac{-8}{3}\ V$$

This example illustrates the method to be used when voltage sources are connected between nodes without any series resistances.

1.9.2   Mesh current method

In a network if there are 'b' branches, there are 'b' currents and b voltages to be determined, i.e., a total of '2b' unknowns. Thus if we have to determine all the currents and voltages, we need to write '2b' linearly independent equations. By linear independence we mean, no equation, out of a set of 'n' equations written to determine 'n' unknowns, be a linear combination of other equations. For example, consider the set of three equations.

$$x + 2y + 3z = 4$$
$$2x - y - z = 6$$
$$8x + y + 3z = 26$$

The third equation is obtained by multiplying the first by 2 and the second by 3 and then adding the two equations. This set of equations have no unique solution.

Now coming back to the '2b' equations for the network, the 'b' voltages are related to the 'b' currents by means of the Ohm's law for the elements. Thus we need to know either b currents or b voltages only. Once the currents are known, voltages can be found or vice versa. We have seen in section 2.4 that if we choose node voltages instead of branch voltages as variables, we had to write N – 1 node voltage equations for a N – node network to solve for all the branch voltages and currents. N – 1 for a network is always less than 'b' as will be shown in Chapter 5.

Another method, by which we write fewer equations than 'b', is known as *'mesh current method'*. To understand the concept of mesh current method let us consider a network shown in Fig. 1.59.
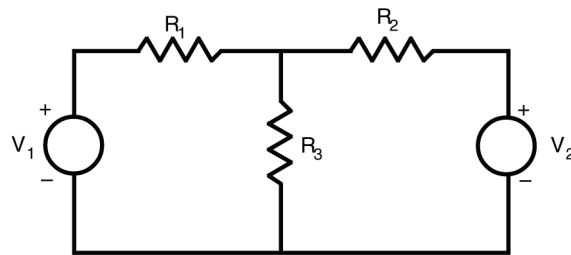
Fig. 1.59 Development of mesh current method

Let us consider the closed paths in the network in Fig. 1.60. One can identify 3 closed paths.

The closed paths containing the elements are :

1. V_1, R_1 and R_3 2. R_2, R_3 and V_2 and   3. V_1, R_1, R_2 and V_2

The first two closed paths are called either *loops* or *meshes* and the third is called only a *loop*. The mesh is defined as a loop which doesnot contain any other loop. The third closed path contains the first two loops and hence is not a mesh thus we can say every mesh is a loop but every loop is not a mesh.

Mesh is a property of a set of networks which are known as planar networks. Planar networks are those networks which can be drawn on paper without crossing of lines. Some planar networks may appear to be non planar if they are not drawn properly.

Thus Fig. 1.60(a) is an example of a planar network, Fig. 1.60(b) is a non-planar network, Fig. 1.60(c) is a planar network drawn to appear as a non planar network and Fig. 1.60(d) is the network of Fig. 1.60(c) drawn, avoiding the crossing of lines.
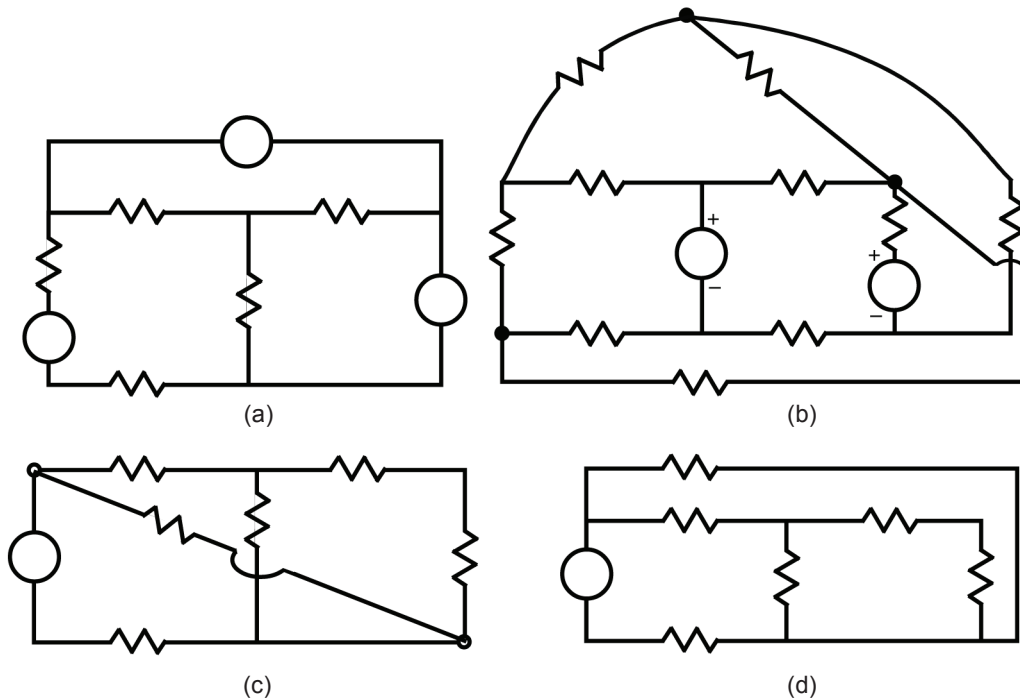
Fig. 1.60 Examples of (a) a planar network (b) non-planar network
(c) planar network drawn to look like non-planar network (d) network in (c) without crossing of lines

The mesh current method, to be developed in this section is applicable only to planar networks.

Returning to Fig. 1.59, redrawn in Fig. 1.61 for ready reference, let us identify the branch currents I_1, I_2, and I_3.

There are three branches and thus there are three unknown currents. But by applying Kirchhoff's Current Law at node 1, we observe that

$$I_3 = I_1 - I_2$$

Hence if I_1 and I_2 are known, I_3 could be easily found out.

Consider imaginary currents I_1 and I_2 circulating in meshes 1 and 2 as shown in Fig. 1.62.
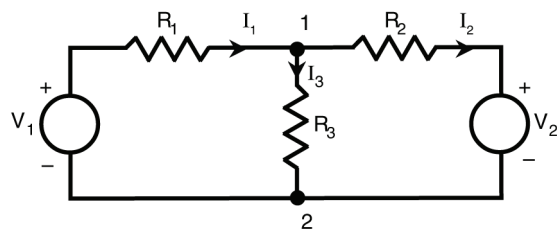
Fig. 1.61 Network in Fig. 1.59 redrawn
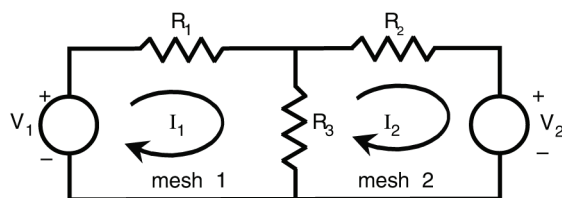
Fig. 1.62 Network showing mesh currents

These currents are customarily taken to flow in the meshes in a clockwise direction. If the branch currents are compared for the two networks in Fig. 1.60 and 1.61, they are exactly same in both the networks.

Branch current in R_1 is same as mesh current I_1

Branch current in R_2 is same s mesh current I_2

Branch current in R_3 is the difference of mesh currents I_1 and I_2.

Applying K V L to the two loops.

$$I_1 R_1 + (I_1 - I_2) R_3 = V_1 \qquad\qquad .....(1.40)$$

$$I_2 R_2 + (I_2 - I_1) R_3 = - V_2 \qquad\qquad .....(1.41)$$

Observe that the current in R_3 in eq. (1.41) is taken as $(I_2 - I_1)$ flowing from bottom to top, so that in the direction of mesh current, voltage across R_3 is a drop and hence a positive sign can be taken in that equation for the term $(I_2 - I_1) R_3$. If the current in R_3 is taken as $(I_1 - I_2)$ as in eq. (1.40), the term $(I_1 - I_2) R_3$ will be taken with a – ve sign and is same as $(I_2 - I_1) R_3$.

The two equations (1.40) and (1.41) are linearly independent and therefore they have a unique solution.

Thus if we take mesh currents as unknown variables rather than branch currents, we will have a set of linearly independent equations and less number of equations too. It will be shown later in chapter 5 that the number of independent equations to be written is equal to b – n + 1 where 'b' is the total number of branches and 'n' is the number of nodes in the network.

How does one recognise the meshes in a network ? A planar network looks like a window with number of panes of glass and a mesh can be identified with each single pane of glass in the window, as shown in Fig. 1.63.
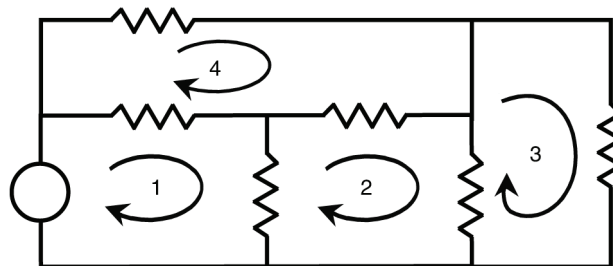
Fig. 1.63 Network with meshes identified as panes is a window

Now assign a mesh current for each of the meshes, usually taken in clockwise direction, and write KVL for each mesh. This rule is known as 'window pane rule'. Since we are writing Kirchhoff 's voltage law equations, voltage sources pose no problem as can be seen from the example 1.21.

Example 1.22

Find the current in 2 Ω resistor using mesh current method in the network of Fig. 1.64.

Solution :

There are two meshes in the network. Assigning two mesh currents I_1 and I_2 in clockwise directions and writing KVL equations for the two meshes, we have

Fig. 1.64 Network for example 1.22

For mesh 1

$$I_1 (1) + (I_1 - I_2)2 = 5$$
$$3 I_1 - 2 I_2 = 5 \qquad\qquad .....(1.42)$$

For mesh 2

$$I_2 (1) + (I_2 - I_1)2 = 2$$
$$- 2 I_1 + 3 I_2 = 2 \qquad\qquad .....(1.43)$$

Solving for I_1 and I_2 from eqs. (1.42) and (1.43) we get

$$I_1 = \frac{19}{5} \text{ A} \quad \text{and} \quad I_2 = \frac{16}{5} \text{ A}$$

The current in 2Ω resistor is

$$I_1 - I_2 = \frac{19}{5} - \frac{16}{5}$$

$$= \frac{3}{5} \text{ A}$$

As can be seen from this example, voltage sources do not cause any problems. But if the network contains current sources, writing KVL in the loop containing the current source is not possible since the voltage across the current source is not known.

1.9.3 Dummy mesh current method

In order to overcome the problem of current sources, the mesh currents are assigned for the network making the current source equal to zero, or open circuiting the current source. Then the current source branch is reintroduced and an additional dummy mesh is formed with the current source branch and other branches. This dummy mesh current is equal to the current of the current source itself. The procedure will be clear from the example 1.23.

Example 1.23

Obtain the current I_x in the network of Fig. 1.65 using mesh current method.
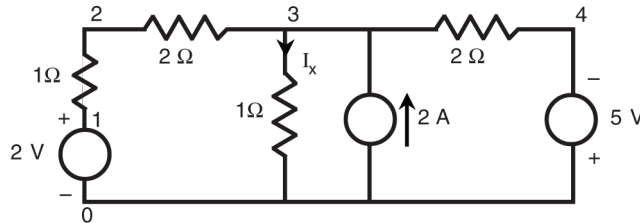
Fig. 1.65 Network for example 1.23

Solution :

Open circuiting the current source and indicating the mesh currents I_1 and I_2 as shown in Fig. 1.66, we have
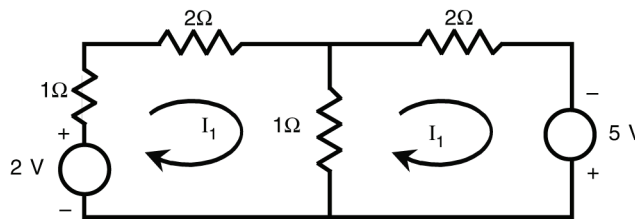
Fig. 1.66 Network with current source open circuited

Reintroducing the current source a dummy mesh is formed with mesh current equal to 2A in the direction indicated in Fig. 1.67.
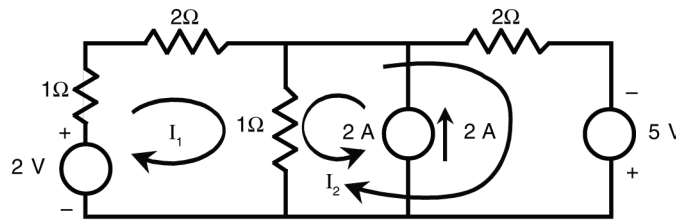
Fig. 1.67 Network with dummy mesh current

Now, writing the mesh equations for the two meshes, we have

For the mesh 1

$$3 I_1 + (I_1 - I_2 + 2)1 = 2 \qquad \qquad .....(1.44)$$

We note here that the current in the branch common to the 2 meshes and dummy mesh is given by $(I_1 - I_2 + 2)$.

Simplifying eq. (1.44)

$$4 I_1 - I_2 = 0 \qquad\qquad\qquad .....(1.45)$$

For mesh 2

$$2 I_2 + (I_2 - I_1 - 2)1 = 5$$
$$-I_1 + 3 I_2 = 7 \qquad\qquad\qquad .....(1.46)$$

Solving (1.45) and (1.46) for I_1 and I_2 we have

$$I_1 = \frac{7}{11} \text{ A} \qquad\qquad I_2 = \frac{28}{11} \text{ A}$$

and
$$I_x = I_1 - I_2 + 2$$
$$= \frac{7}{11} - \frac{28}{11} + 2$$
$$= \frac{1}{11} \text{ A.}$$

Important note

A clear understanding of the difference between the branch currents and mesh currents is in order here. Branch currents are the actual currents that are flowing in the branches. Mesh currents are imaginary circulating currents in the meshes. If in a branch only one mesh current is flowing, then the branch current is equal to the mesh current. If more than one mesh currents are flowing in a branch which is common to two or more meshes, the branch current is equal to algebraic sum of the mesh currents flowing in that branch. In Fig. 1.67 the branch current in 1Ω and 2Ω resistors in series with 2V source is equal to the mesh current I_1 and the branch current in 2Ω resistor in series with the 5V source is equal to the mesh current I_2 where as the current in 1Ω resistor common to the three meshes is the algebraic sum of the currents I_1, I_2 and 2A. Further, only one current, i.e., 2A is flowing through the branch containing the current source and therefore the current in the dummy mesh is assigned a current of 2A.

Problems

1.1 Ten Coulombs of positive charge per second are passing through a wire in a direction from 1 to 2.

(a)   What is the current if the assumed reference direction is from 1 to 2 ?

(b)   What is the current if the assumed reference direction is from 2 to 1 ?

(c)   What would be the answer for (a) and (b) if the charge is negative instead of positive ?

1.2 A Coulomb of charge changes its energy by 30 Joules in moving from point 1 to 2. What is the voltage of point 1 with respect to point 2 if

(a)   the charge is positive and the energy is lost.

(b)   the charge is negative and the energy is lost.

(c)   the charge is positive and the energy is gained

and   (d)   the charge is negative and the energy is gained.

1.3 A 2 terminal device has a positive current of 15 A entering at terminal 1 and leaving at terminal 2. What is the power absorbed in the device ? When

(a)   the voltage at 1 is 5 V positive with respect to 2.

(b)   the voltage at 1 is 5 V negative with respect to 2

and   (c)   the voltage at 2 is $-$ 10 V with respect to 1.
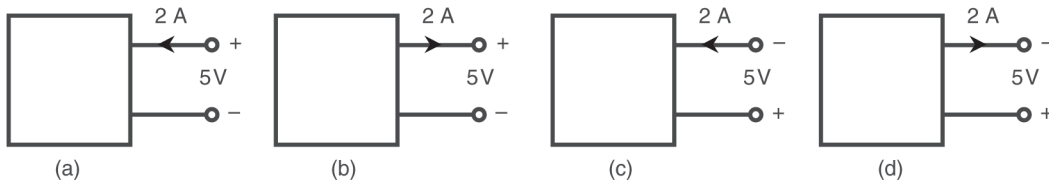
Fig. P. 1.1

1.5 A resistor of value 2 Ω is connected across a voltage whose waveform is shown in Fig. P. 1.2. Draw the waveforms of current and power.
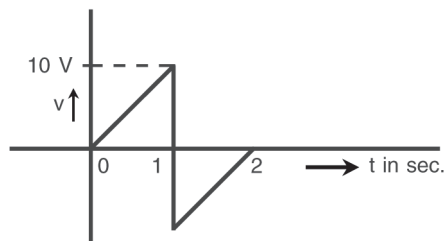
Fig. P. 1.2

1.6   The volt – ampere characteristic of a device is shown in Fig. P. 1.3. What type of device is it and what is its value ?
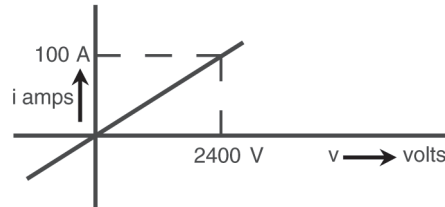
Fig. P. 1.3

1.7   A current $i = 10\,e^{-t}$ is applied to

(a) a 3Ω resistor (b) a 2H inductor and (c) a 0.1 F capacitor.

What are the respective voltages ? Write down the expressions for power in each case.

1.8   A number of waveforms are shown in Fig. P. 1.4(a). Similarly a number elements are shown in Fig. P. 1.4(b). Write down the waveforms of the response for each of these elements for each of the waveforms in Fig. P. 1.4(a) where

(a) f(t) is a current and the response is the voltage and

(b) f(t) is voltage and the response is the current.

Fig. P. 1.4(a)

$$R = 2\Omega \qquad L = 2H \qquad C = \frac{1}{2}\ F$$
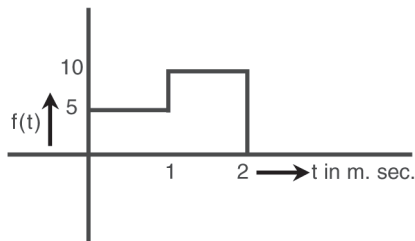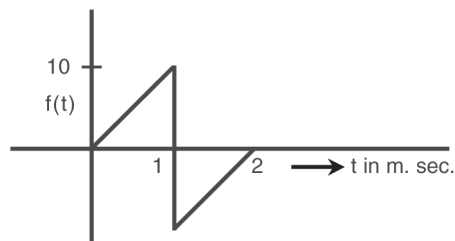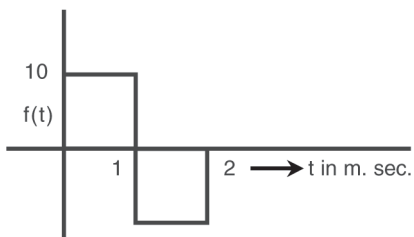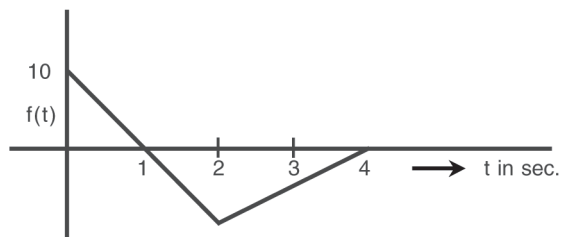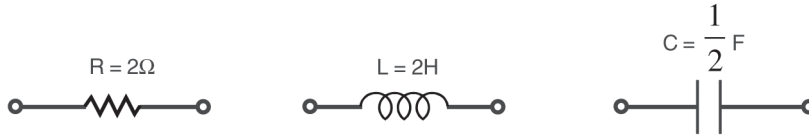
Fig. P. 1.4(b)

1.9 A voltage $v(t) = 200\ Sin\ 100\ \pi t$ volts is applied across a capacitor of value $C = 0.05$ farads. Find the expression for the current $i(t)$ and the energy stored in the capacitor.

1.10 A voltage $v(t) = 25\ Sin\ 1000\ t$ volts is applied across a 5 mH inductor at $t = 0$ when the current in it is 2 A. Find the expression for the current $i(t)$ for $t \geq 0$.

1.11 The quantity $\int_{-\infty}^{t} idt$ in a capacitor is called the *charge* 'q'. In a capacitor of value $\frac{1}{2}$ F and charged to a voltage of 1 V, a current of the from $i(t) = e^{-2t}$, $t > 0$ is flowing. What is the value of the current at $t = 1$ sec. ? At $t = 1$ sec.

  (a) What is the total charge accumulated in the capacitor ?

  (b) What is the rate of change of voltage across the capacitor ?

  (c) What is the voltage across the capacitor ?

and (d) At what rate is energy being taken from the electric field of the capacitor ?

1.12 A nonlinear capacitor has its v Vs q characteristic shown in Fig. P. 1.5. If a voltage $v(t) = 10\ Sin\ 2t$ is applied to such a capacitor what is the expression for the current through it ?
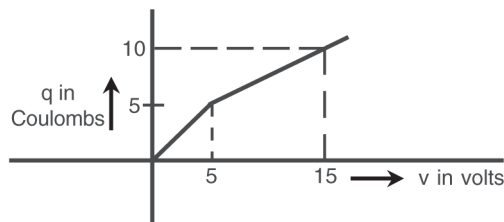
Fig. P. 1.5

1.13 By using series parallel combinations, find the equivalent resistance in Fig.P. 1.6 at the terminals a, b.
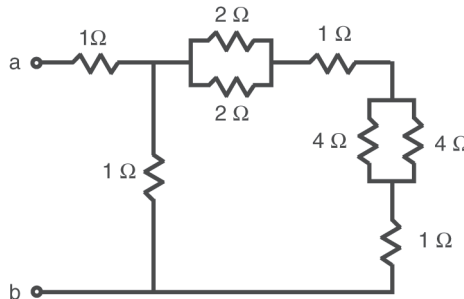
Fig. P. 1.6

1.14 Find the input resistance at terminals a, b of the network of Fig. P. 1.7.
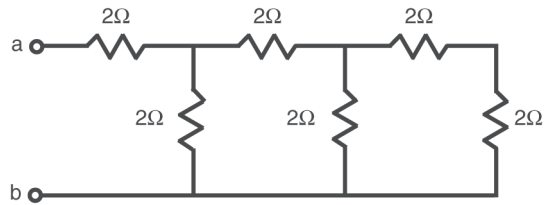
Fig. P. 1.7

1.15 Write the v – i equation of the branches shown in Fig. P. 1.8.
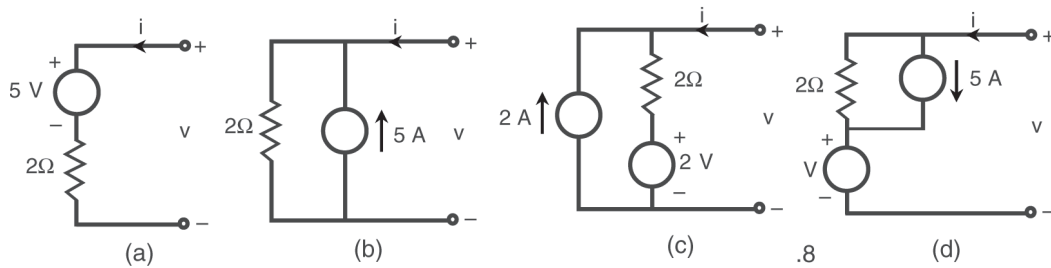
Fig. P. 1.8

1.16 Reduce the branches in Fig. P. 1.9 to single sources.
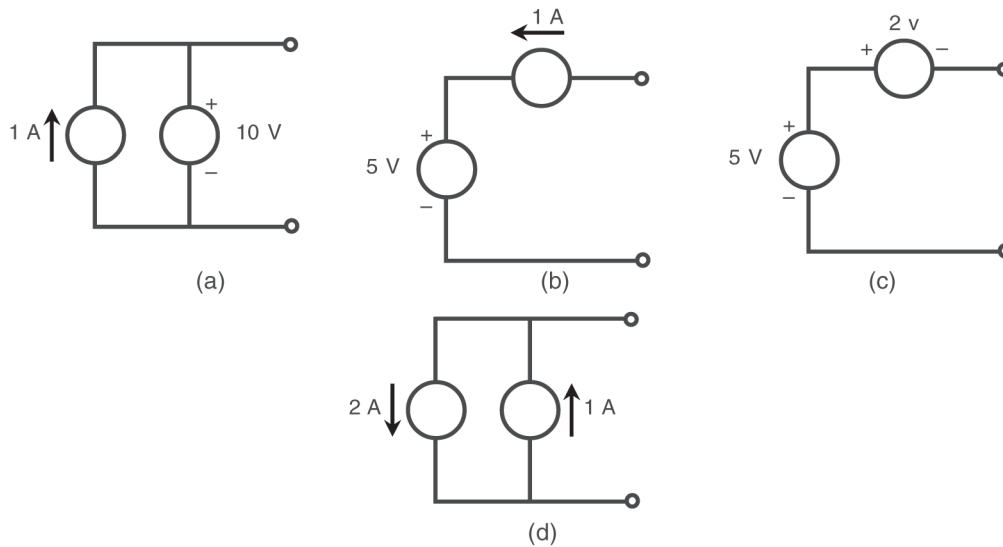
Fig. P. 1.9

1.17 Convert the voltage source branches in Fig. P. 1.10 to equivalent current source branches.
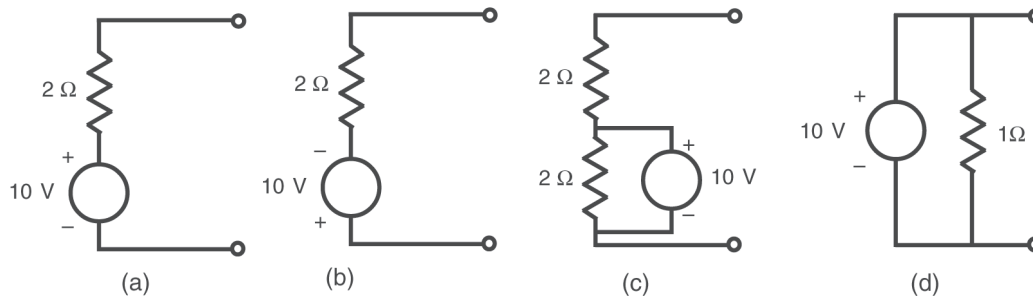
Fig. P. 1.10

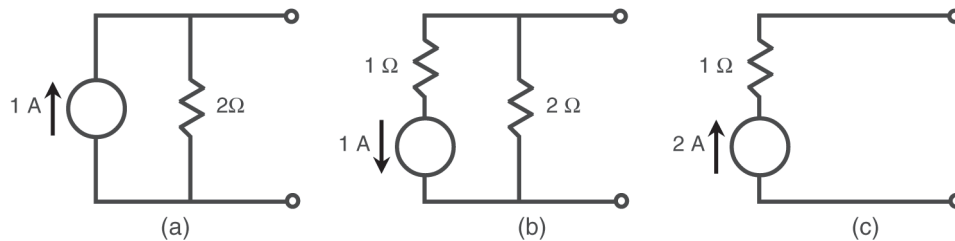**1.18** Convert the current source branches in Fig. P.1.11 to equivalent voltage source branches.

Fig. P. 1.11

1.19 Convert the branches in Fig. P. 1.12 to single voltage sources in series with single resistors.
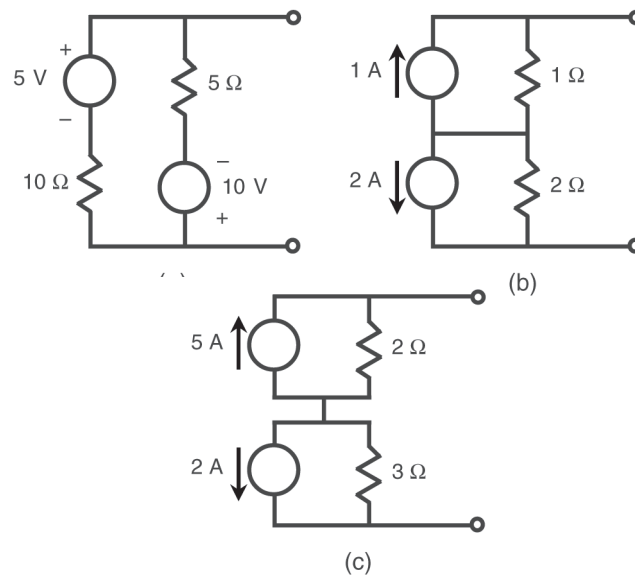
Fig. P. 1.12

1.20 Using network reduction, find the voltage V in Fig. P. 1.13.
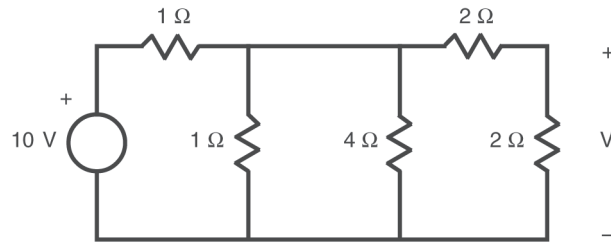
Fig. P. 1.13

1.21 Find V in Fig. P. 1.14 using the network reduction.
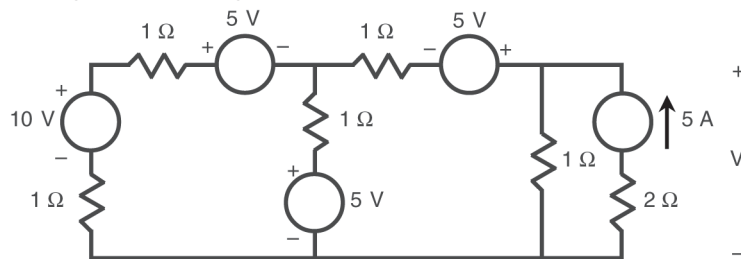
Fig. P. 1.15

1.22 Find the equivalent resistance between the terminals 1, 2 of network shown in Fig. P. 1.16. Resistor values are in ohms.
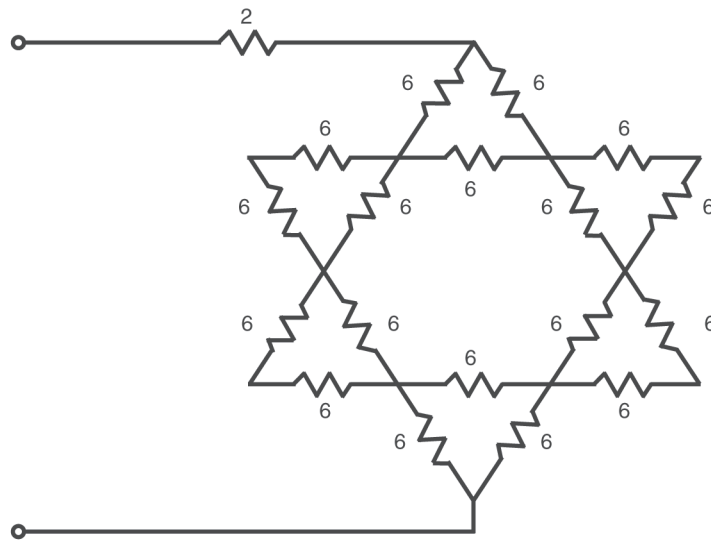
Fig. P. 1.16

1.23 Find equivalent resistance R$_{ab}$ in Fig. P. 1.17. Resistor values are in ohms.
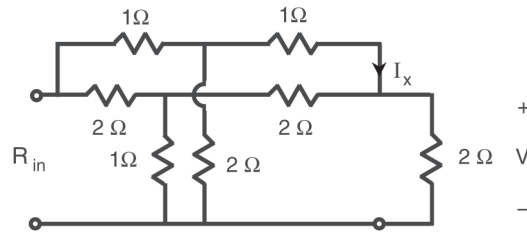
Fig. P. 1.17

1.24 Find R$_{eq}$ in Fig. P. 1.18. Resistor values are in ohms.
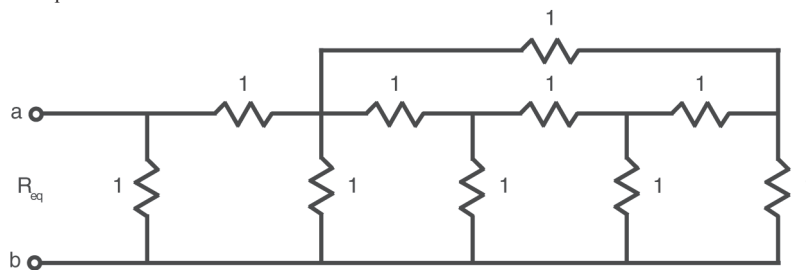
Fig. P. 1.18

1.25 1 set of twelve 1 Ω resistors are connected to form a cube as shown in Fig. P. 1.19. Find R$_{eq}$.
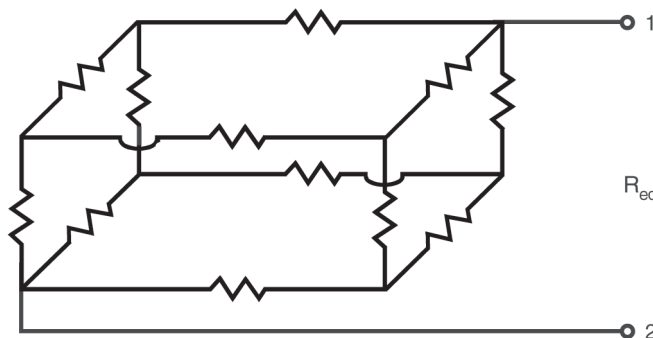
Fig. P. 1.19

1.26 The network in Fig. P. 1.19 can be written in the form shown in Fig. P. 1.20. Find the R_{eq}. Resistor values are in ohms.
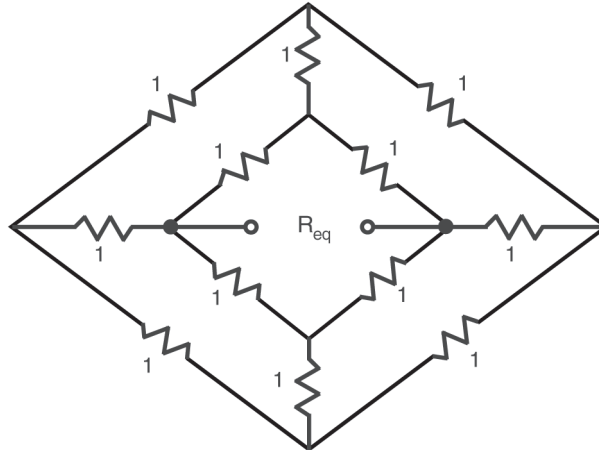

Fig. P. 1.20

1.27 Find the input resistance of the circuit in Fig. P. 1.25. (Hint; Assume V = 2V and I_x = x amps and solve for x.)
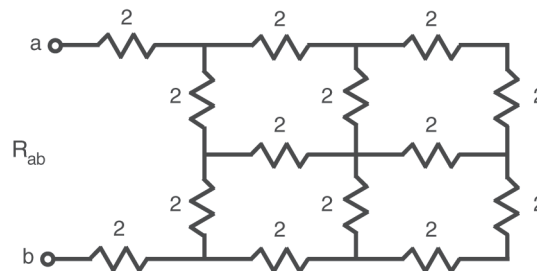
Fig. P. 1.21

1.28 Find the voltage V in Fig. P. 1.22. Using node voltage method. Redraw the figure so that 2 Ω branch connected between the two current sources is drawn above the T network consisting of 1 Ω, 1 Ω and 4 Ω resistors. Find 'V' again by inspection.
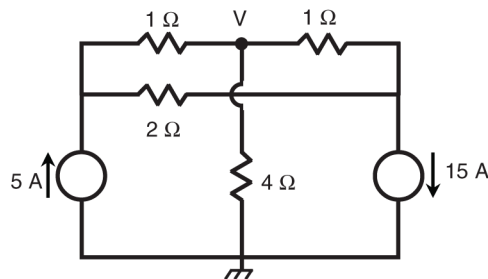

Fig. P. 1.22

1.29 Use nodal analysis to find the voltages V_1 and V_2 in Fig. P. 1.23.
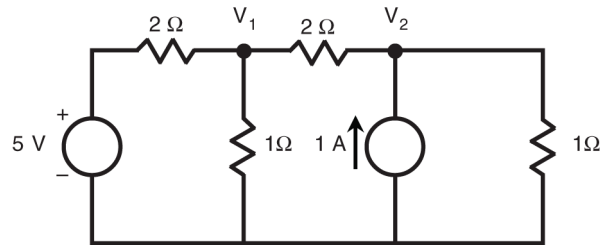

Fig. P. 1.23

1.30 Use nodal analysis to find the current 'I' in Fig. P. 1.24. What is the power delivered by the 1 A current source.
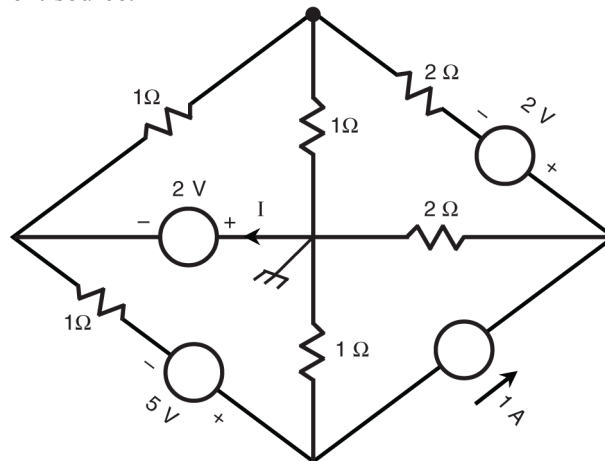

Fig. P. 1.24

1.31 Use nodal analysis to find the power delivered by the 2 V source in Fig. P. 1.25.
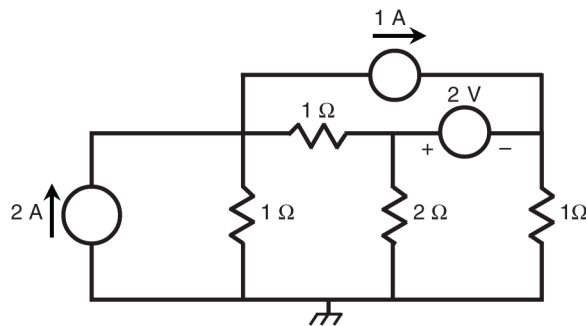

Fig. P. 1.25

1.32 Use mesh analysis to solve problem 1.30.

1.33 Use mesh analysis to solve problem 1.31.